61A Lecture 7

Announcements

Hog Contest Rules

`cs61a.org/proj/hog_contest`

Hog Contest Rules

- Up to two people submit one entry;
  Max of one entry per person

cs61a.org/proj/hog_contest

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- Your score is the number of entries against which you win more than 50.00001% of the time

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Fall 2011 Winners

Kaylee Mann
Yan Duan & Ziming Li
Brian Prike & Zhenghao Qian
Parker Schuh & Robert Chatham

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Alan Tong & Elaine Zhao
Zhenyang Zhang
Adam Robert Villaflor & Joany Gao
Zhen Qin & Dian Chen
Zizheng Tai & Yihe Li

cs61a.org/proj/hog_contest

Hog Contest Winners

Spring 2015 Winners

Sinho Chewi & Alexander Nguyen Tran
Zhaoxi Li
Stella Tao and Yao Ge

Fall 2015 Winners

Micah Carroll & Vasilis Oikonomou
Matthew Wu
Anthony Yeung and Alexander Dai

Spring 2016 Winners

Michael McDonald and Tianrui Chen
Andrei Kassiantchouk
Benjamin Krieges

Your name could be here FOREVER!

Spring 2017 Winners

Cindy Jin and Sunjoon Lee
Anny Patino and Christian Vasquez
Asana Choudhury and Jenna Wen
Michelle Lee and Nicholas Chew

Fall 2017 Winners

Order of Recursive Calls

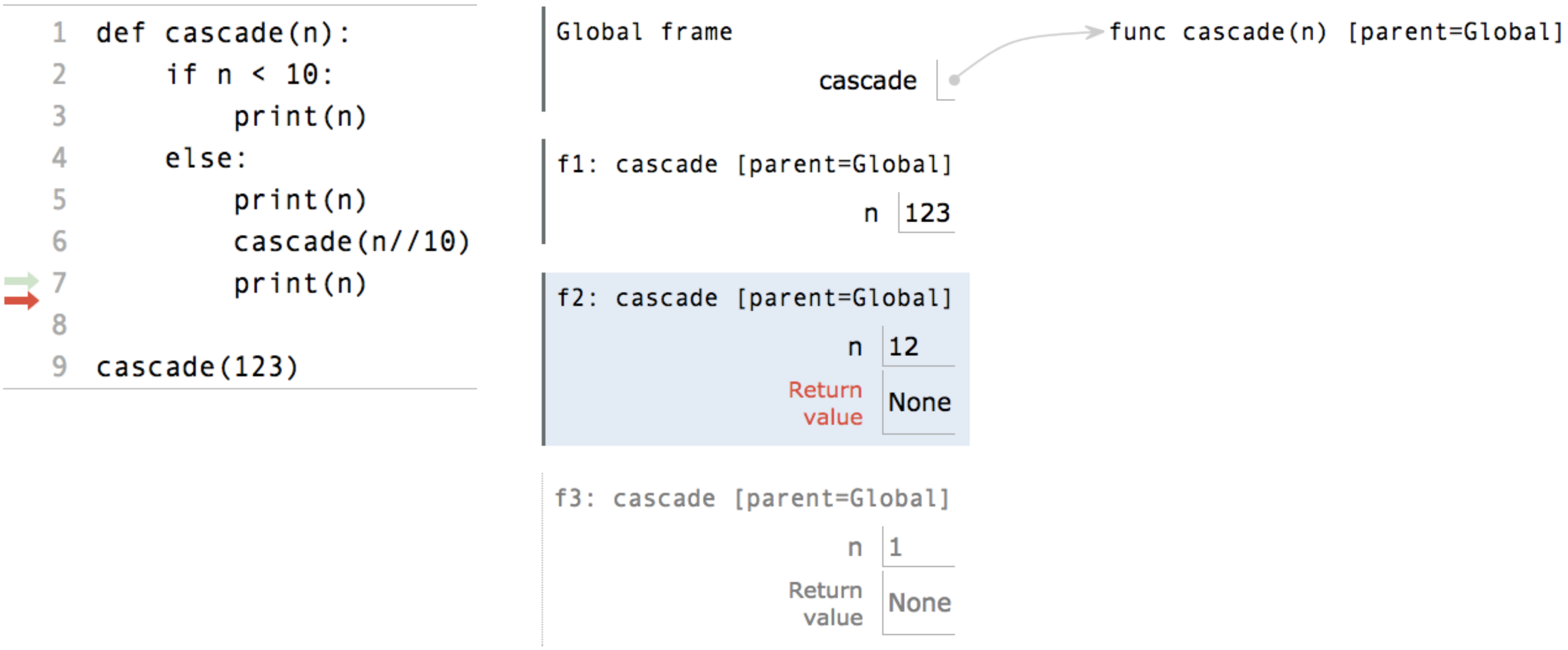
The Cascade Function

(Demo)

<u>Interactive Diagram</u>

The Cascade Function

(Demo)



```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
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cascade(123)
```

Global frame

cascade → func cascade(n) [parent=Global]

f1: cascade [parent=Global]

n 123

f2: cascade [parent=Global]

n 12

Return value None

f3: cascade [parent=Global]

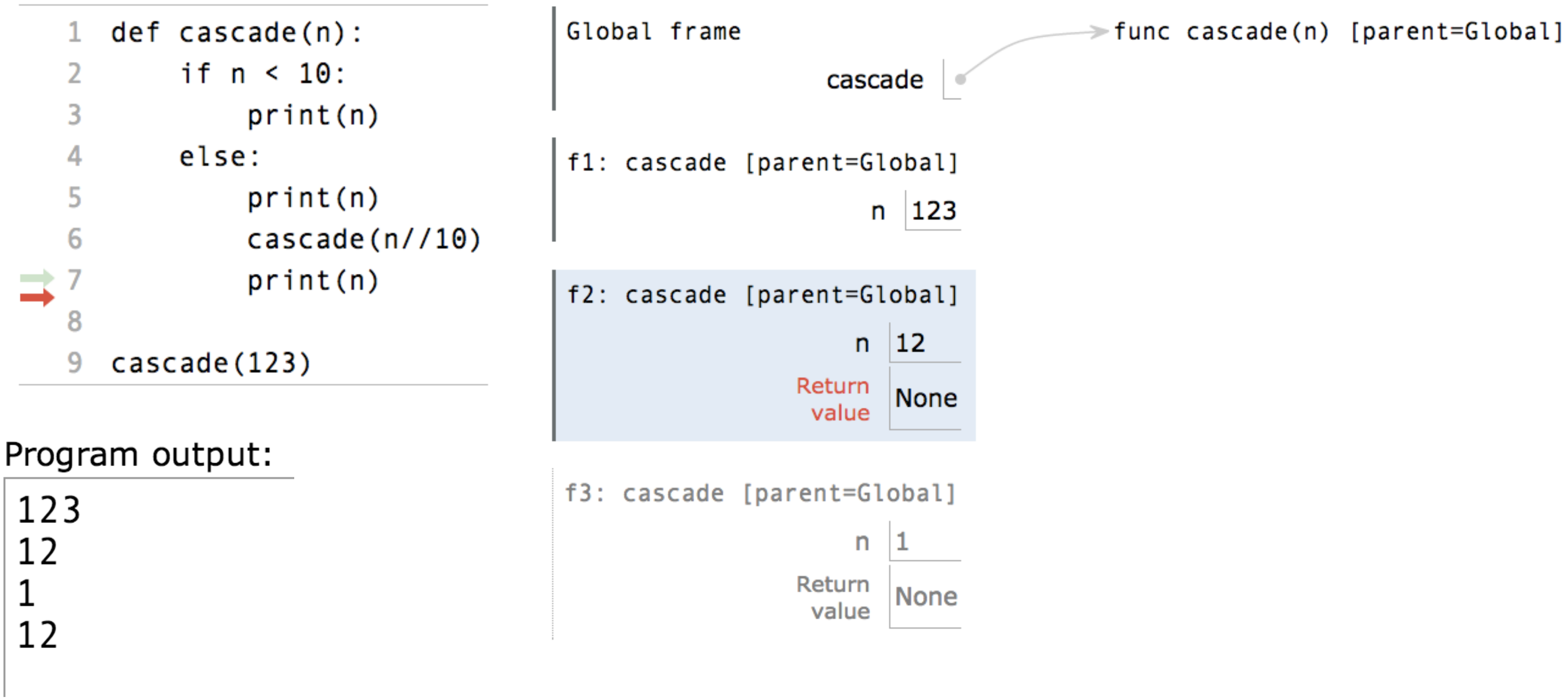
n 1

Return value None

Interactive Diagram

The Cascade Function

(Demo)



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Program output:

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1
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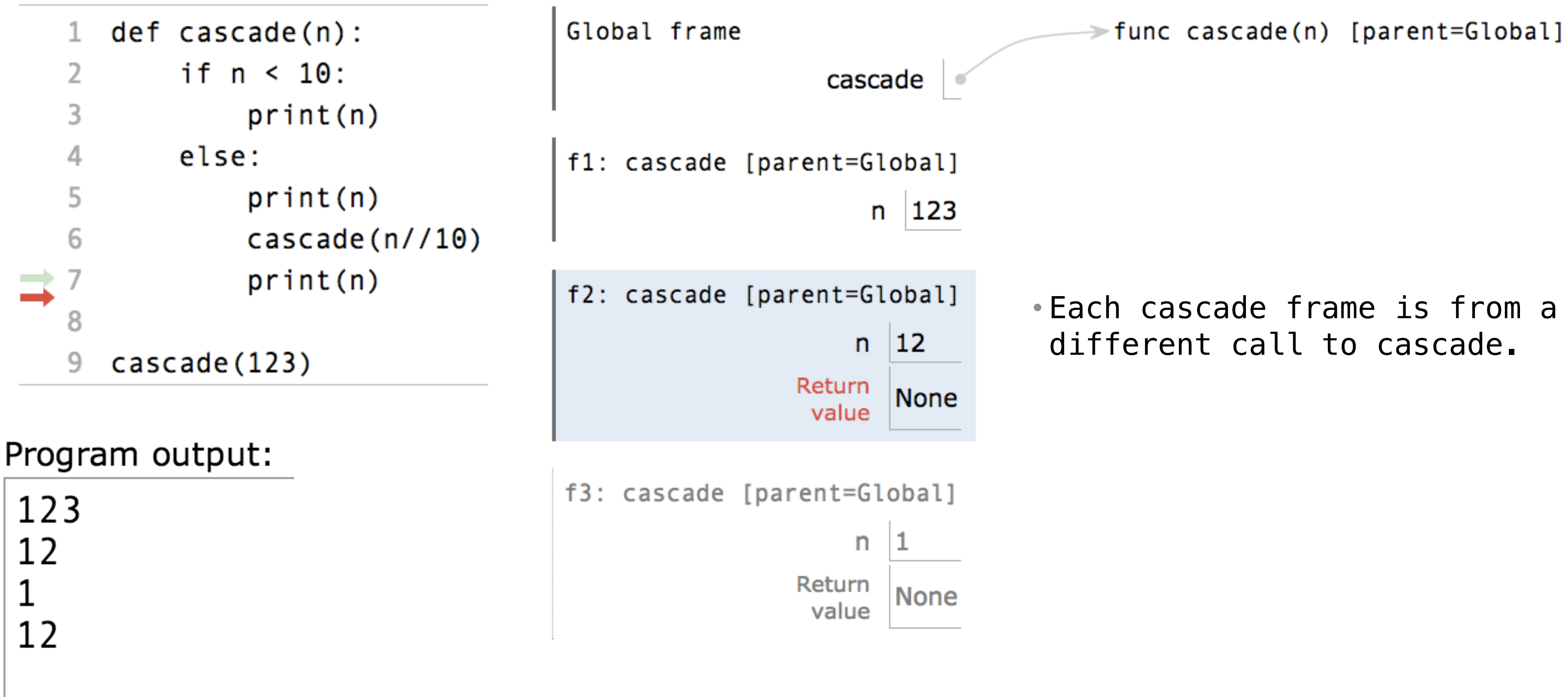

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- Each cascade frame is from a different call to cascade.

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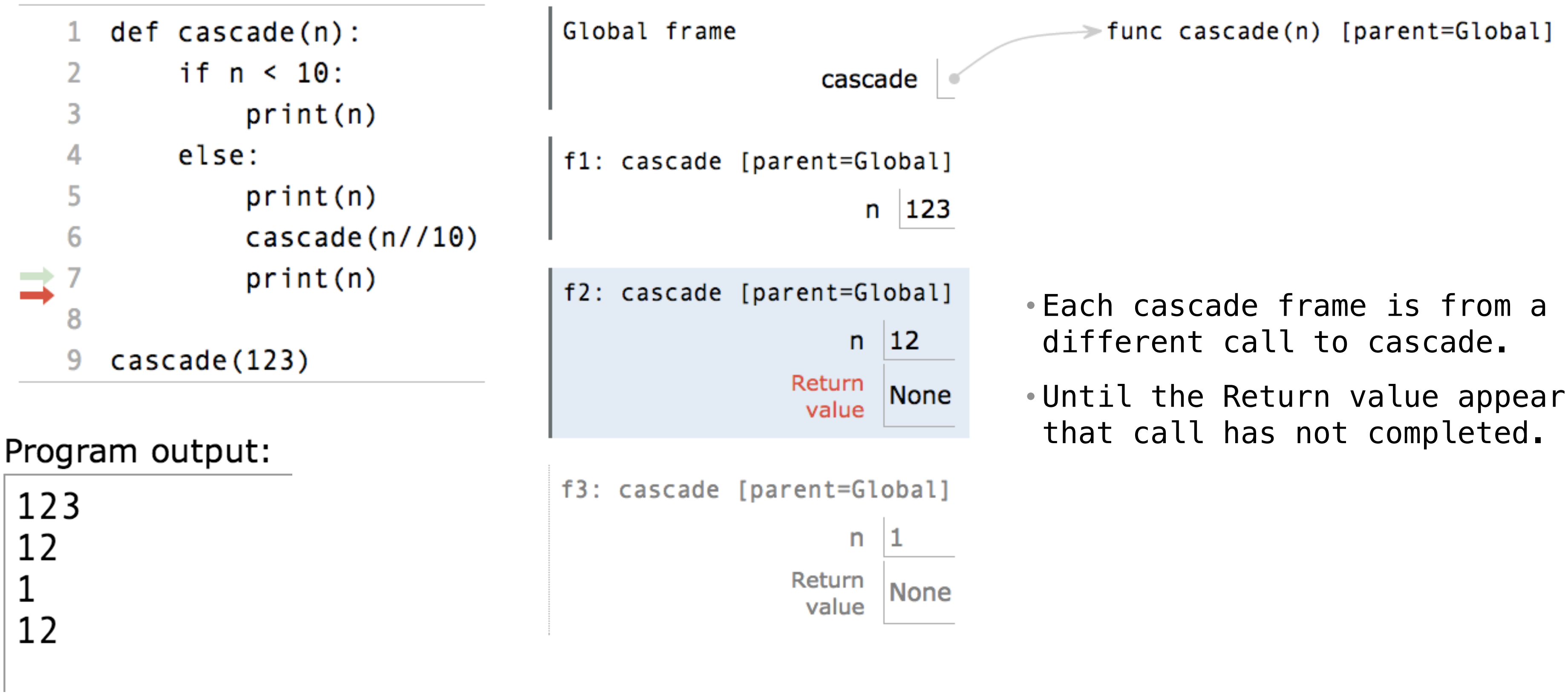

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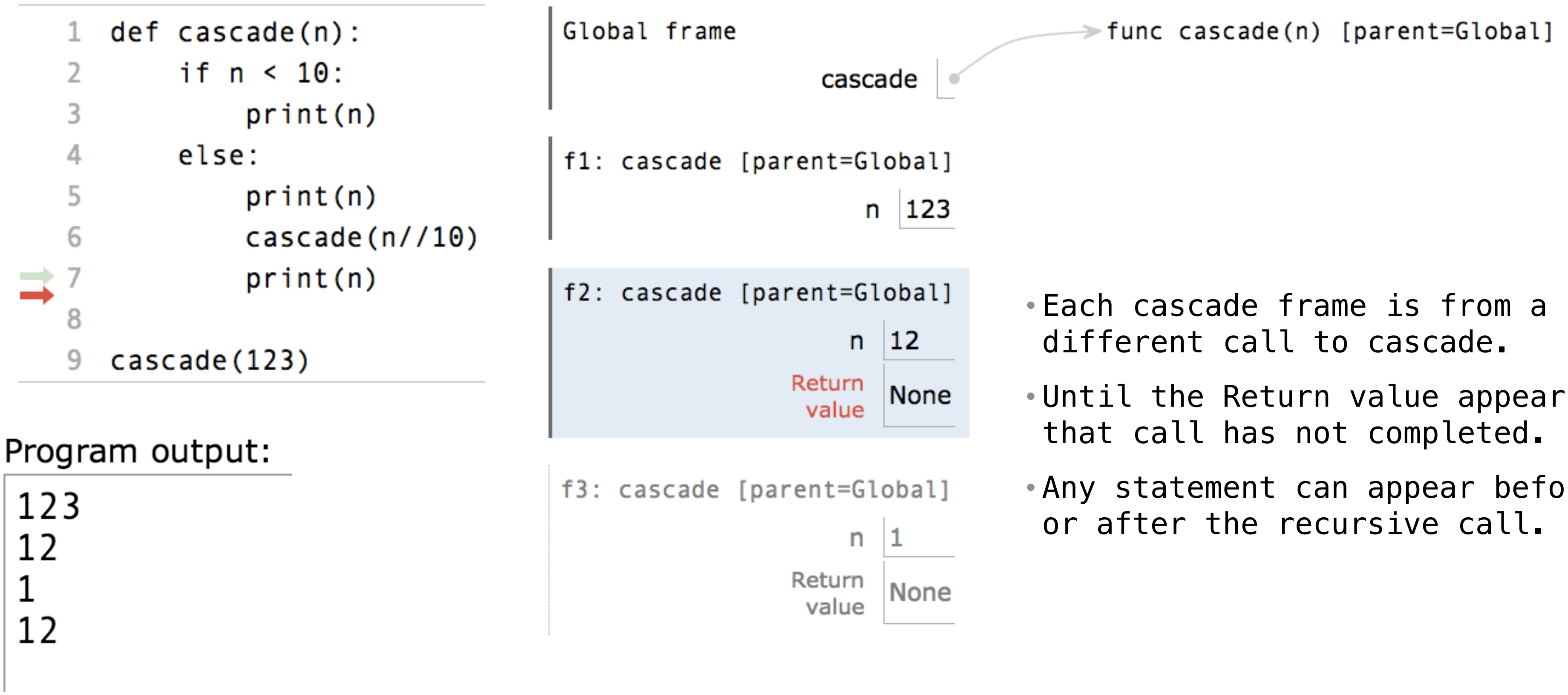

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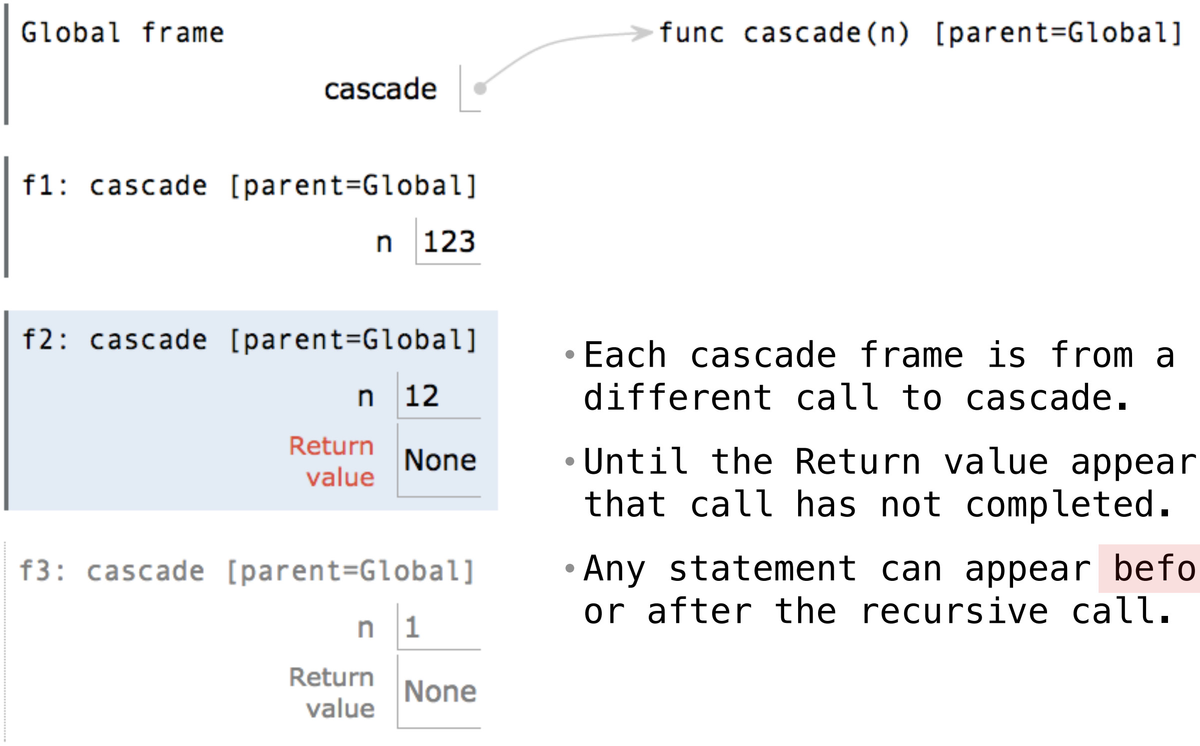

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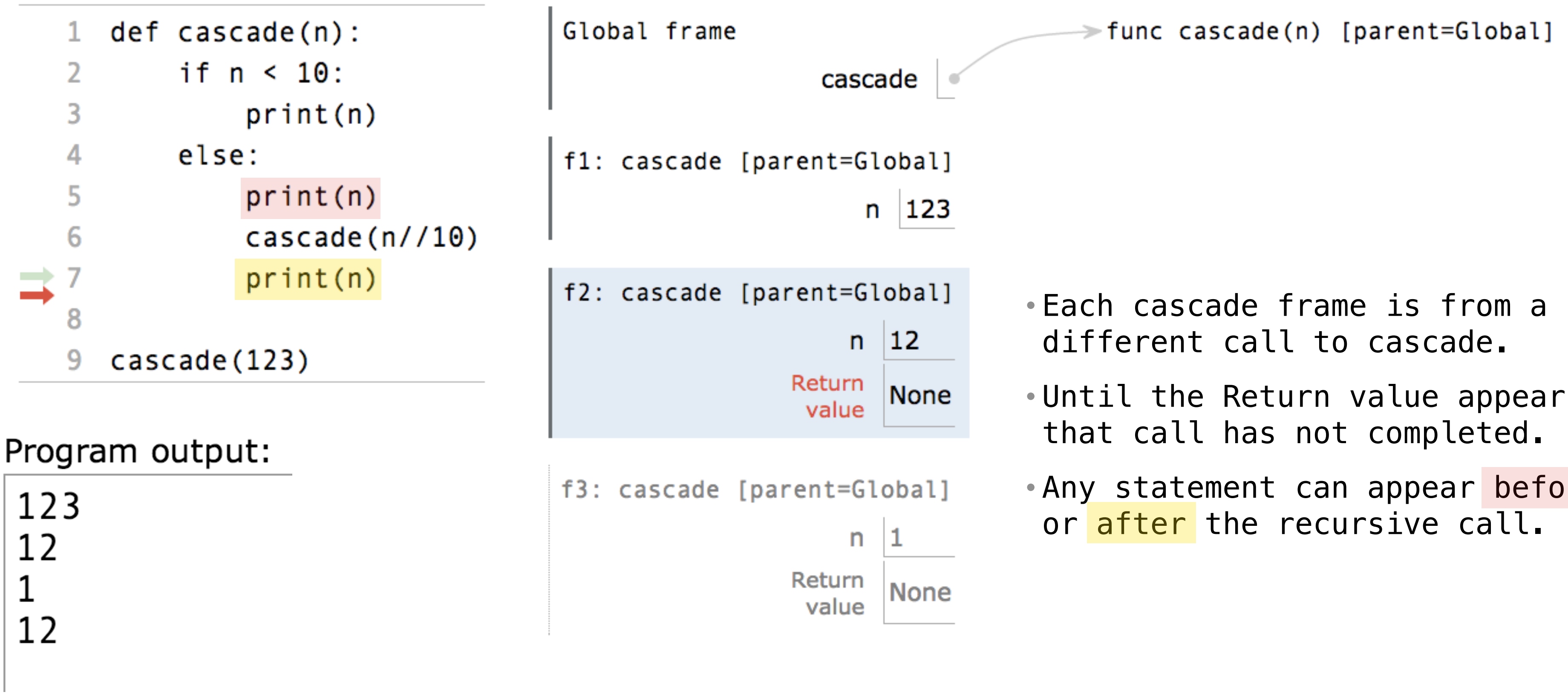

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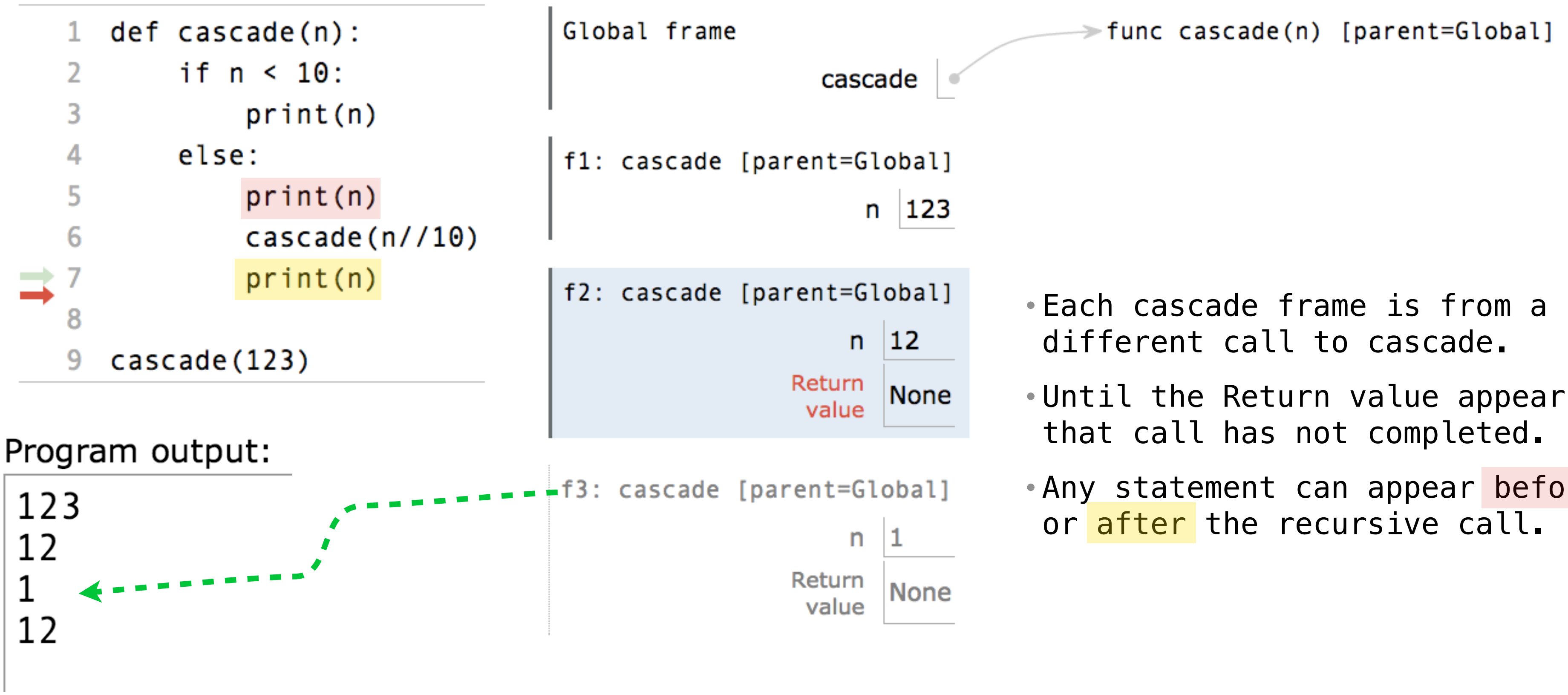

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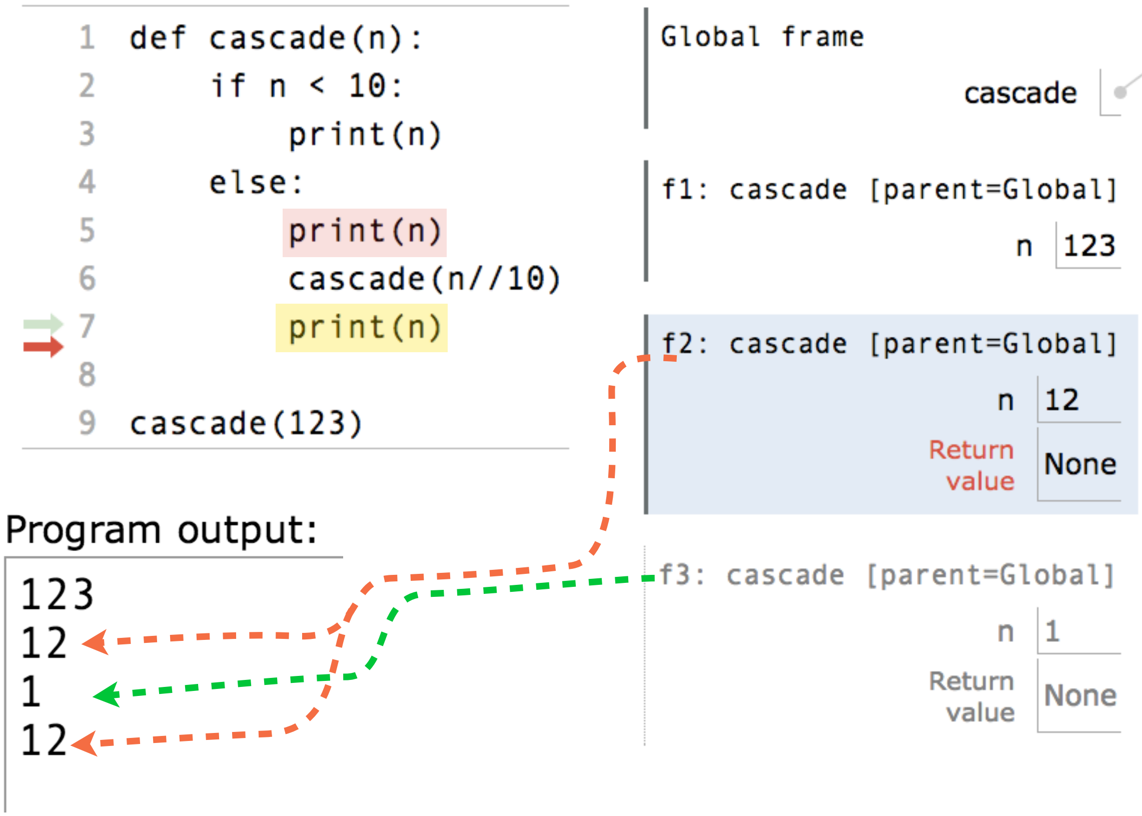

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Interactive Diagram

Two Definitions of Cascade

`(Demo)`

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def cascade(n):
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    else:
        print(n)
        cascade(n//10)
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```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

Two Definitions of Cascade

(Demo)

```
def cascade(n):
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        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better

Two Definitions of Cascade

(Demo)

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    if n >= 10:
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```

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- In this case, the longer implementation is more clear (at least to me)

Two Definitions of Cascade

(Demo)

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    if n >= 10:
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```

- If two implementations are equally clear, then shorter is usually better
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- When learning to write recursive functions, put the base cases first

Two Definitions of Cascade

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def cascade(n):
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- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
- Both are recursive functions, even though only the first has typical structure

Example: Inverse Cascade

Inverse Cascade

Write a function that prints an inverse cascade:

Inverse Cascade

`Write a function that prints an inverse cascade:`

```
1
12
123
1234
123
12
1
```

Inverse Cascade

`Write a function that prints an inverse cascade:`

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
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```

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def inverse_cascade(n):
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    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
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        g(n)

grow =   lambda n: f_then_g(                    )
shrink = lambda n: f_then_g(                    )
```

Inverse Cascade

`Write a function that prints an inverse cascade:`

```
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12
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def inverse_cascade(n):
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def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)


grow =   lambda n: f_then_g(grow,  print,  n//10)
shrink = lambda n: f_then_g(print, shrink, n//10)
```

Tree Recursion

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Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

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n: 0, 1, 2, 3, 4, 5, 6, 7, 8,

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n:	0, 1, 2, 3, 4, 5, 6, 7, 8,
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21,

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Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n: 0, 1, 2, 3, 4, 5, 6, 7, 8, ... , 35

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fib(n): 0, 1, 1, 2, 3, 5, 8, 13, 21, ... , 9,227,465

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```
def fib(n):
    if n == 0:
```

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```
def fib(n):
    if n == 0:
        return 0
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```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
```

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    else:
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```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

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A Tree-Recursive Process

The computational process of fib evolves into a tree structure

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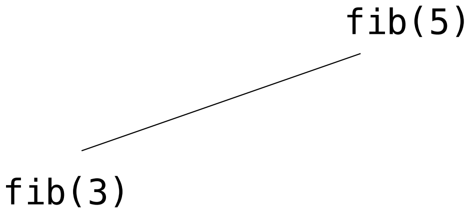
`The computational process of fib evolves into a tree structure`

`fib(5)`

A Tree-Recursive Process

The computational process of fib evolves into a tree structure



fib(5)

fib(3)

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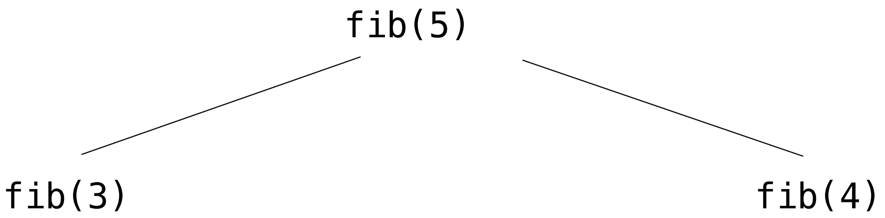
The computational process of fib evolves into a tree structure



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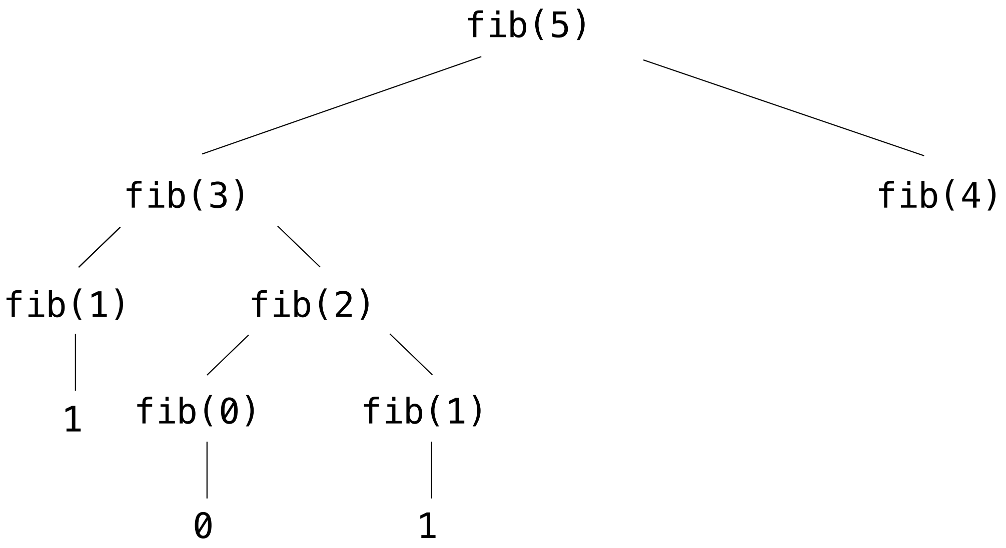
The computational process of fib evolves into a tree structure


fib(5)

fib(3)

fib(4)

fib(1)

fib(2)

1

fib(0)

fib(1)

0

1

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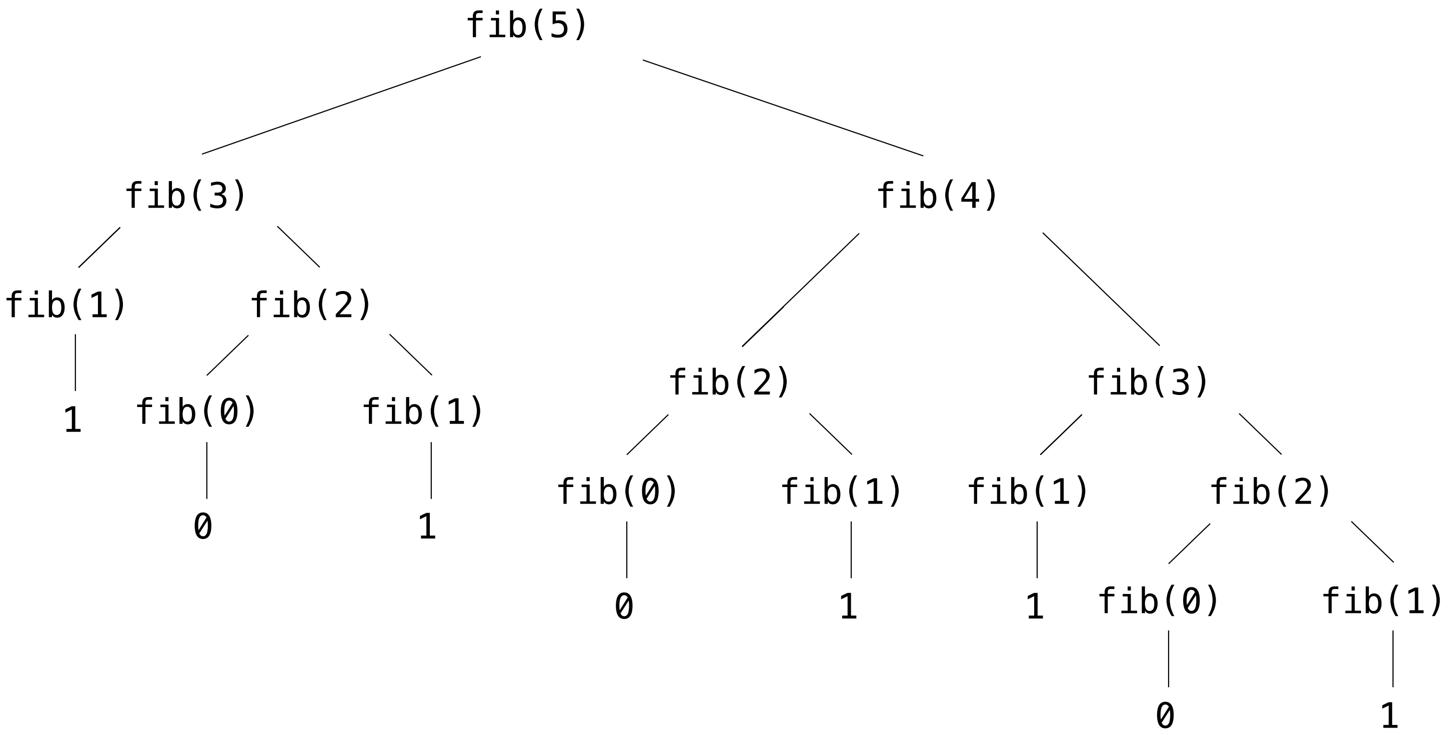
`The computational process of fib evolves into a tree structure`



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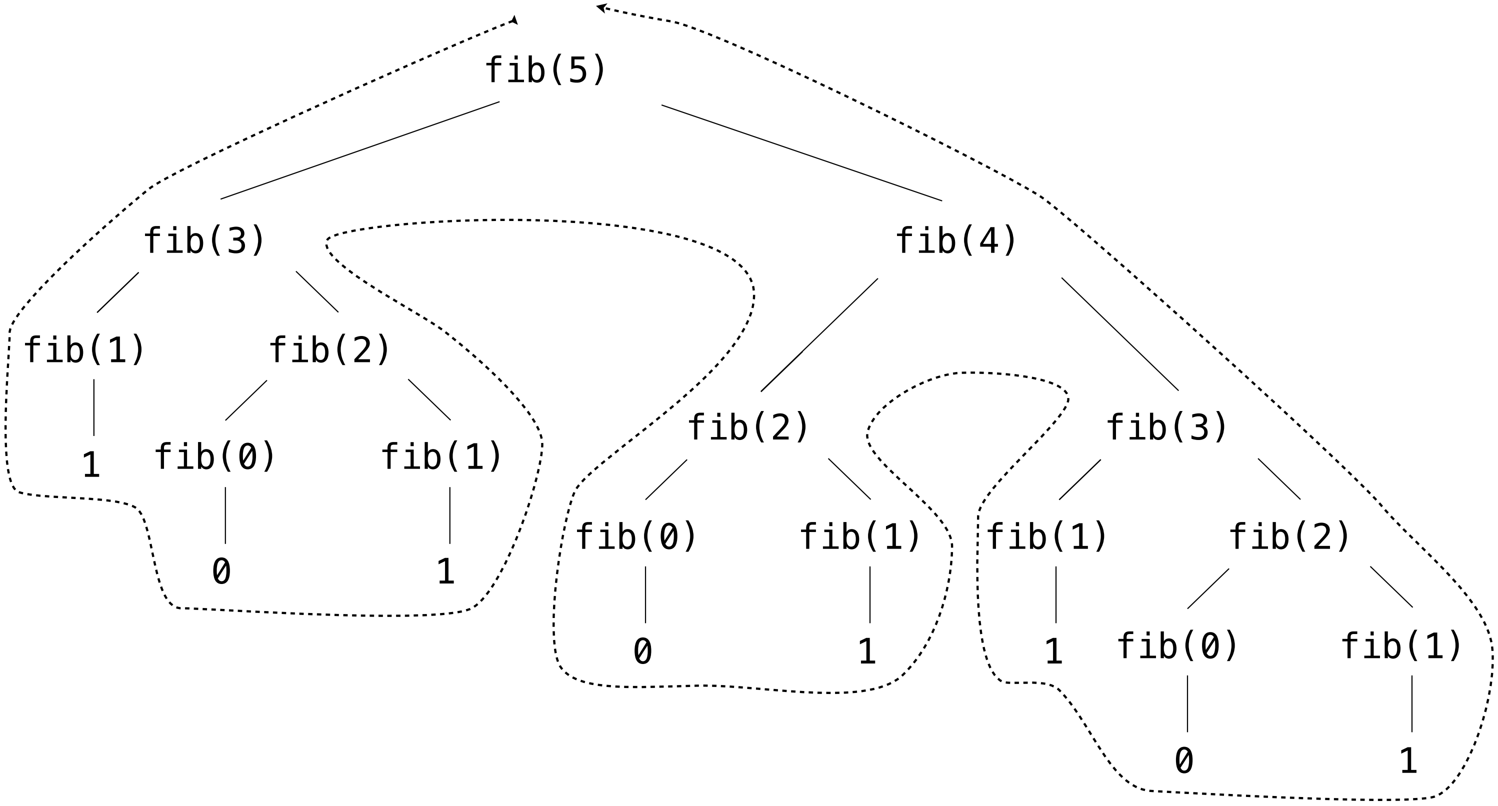

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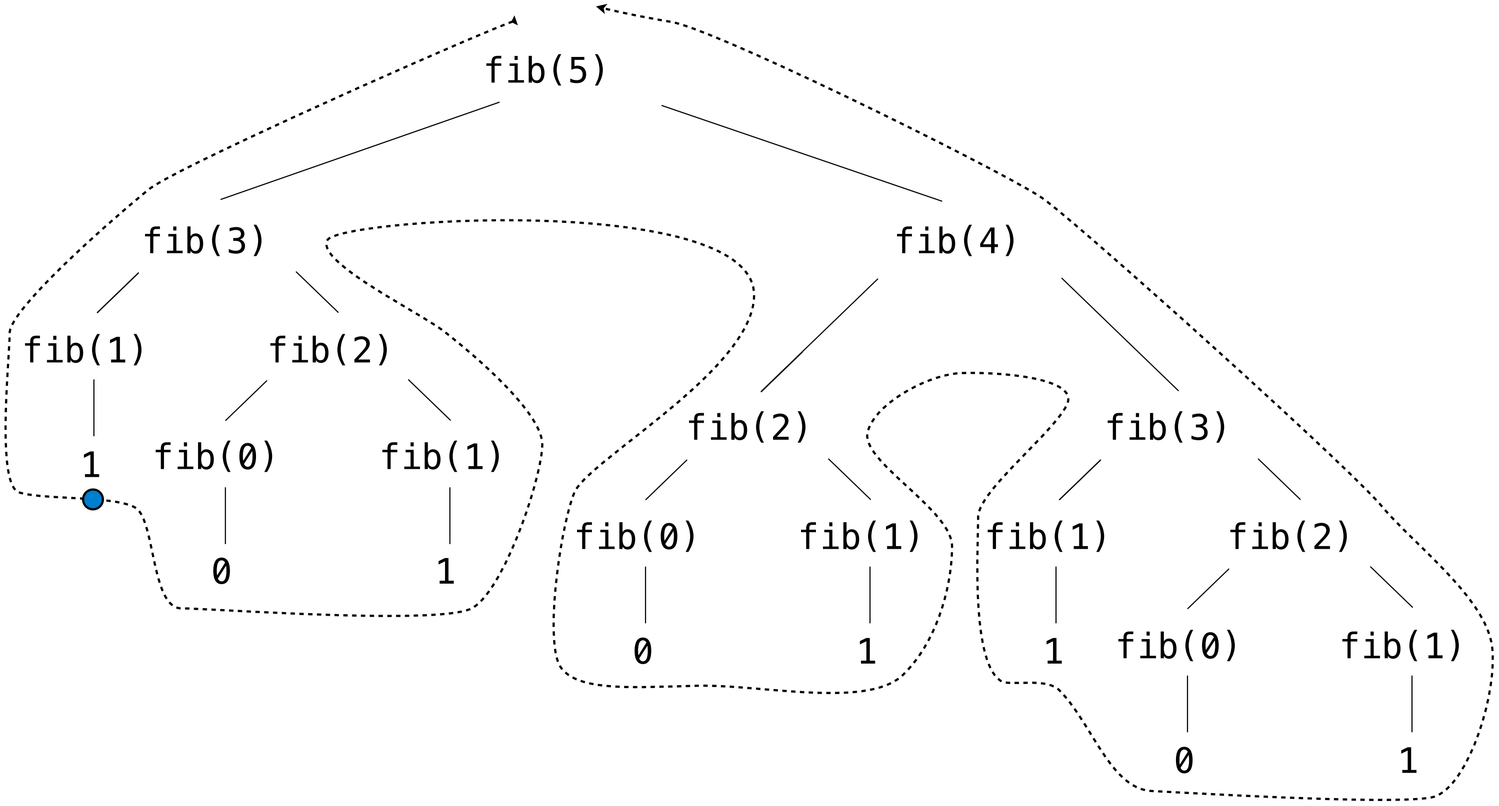

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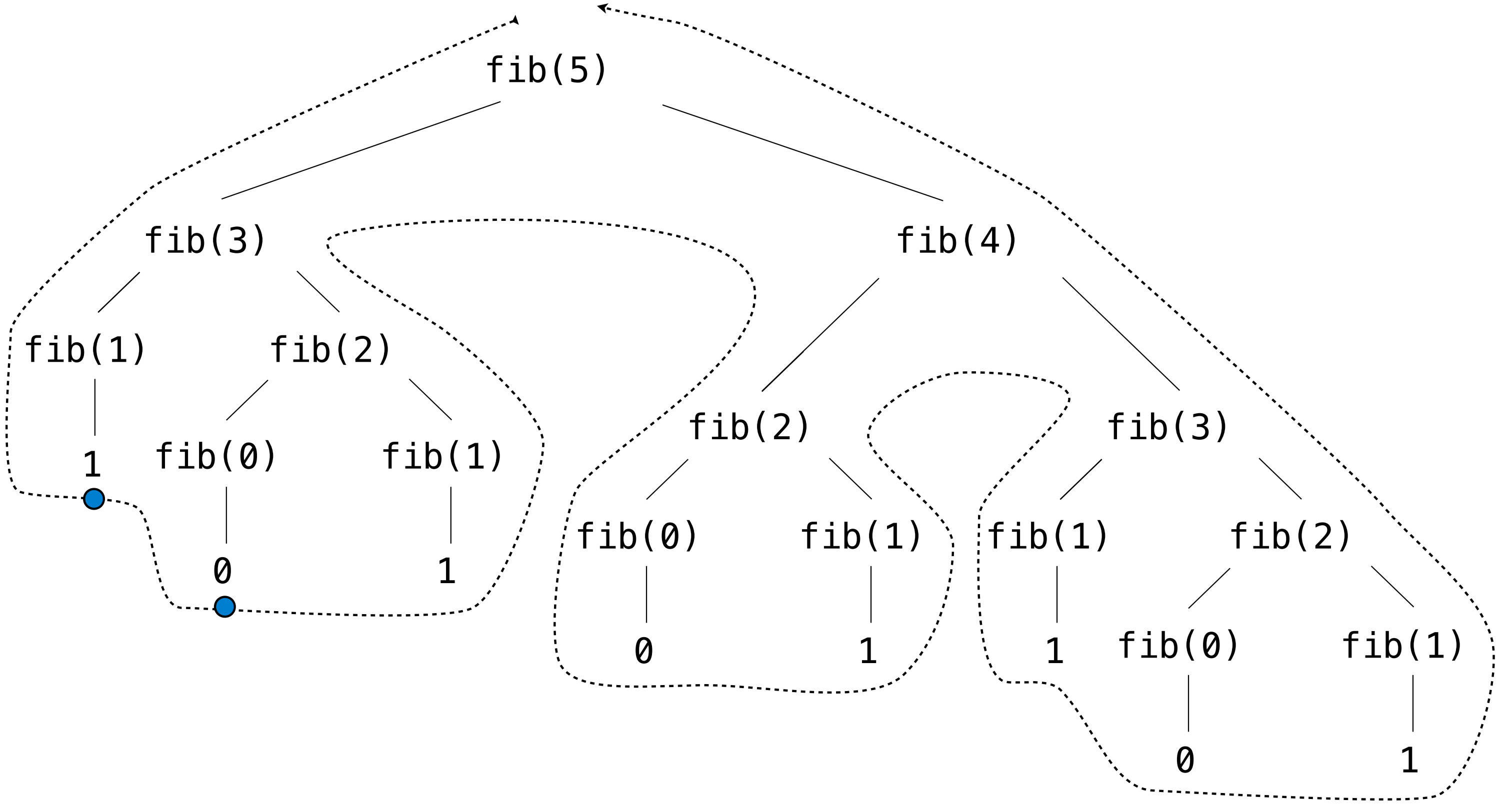

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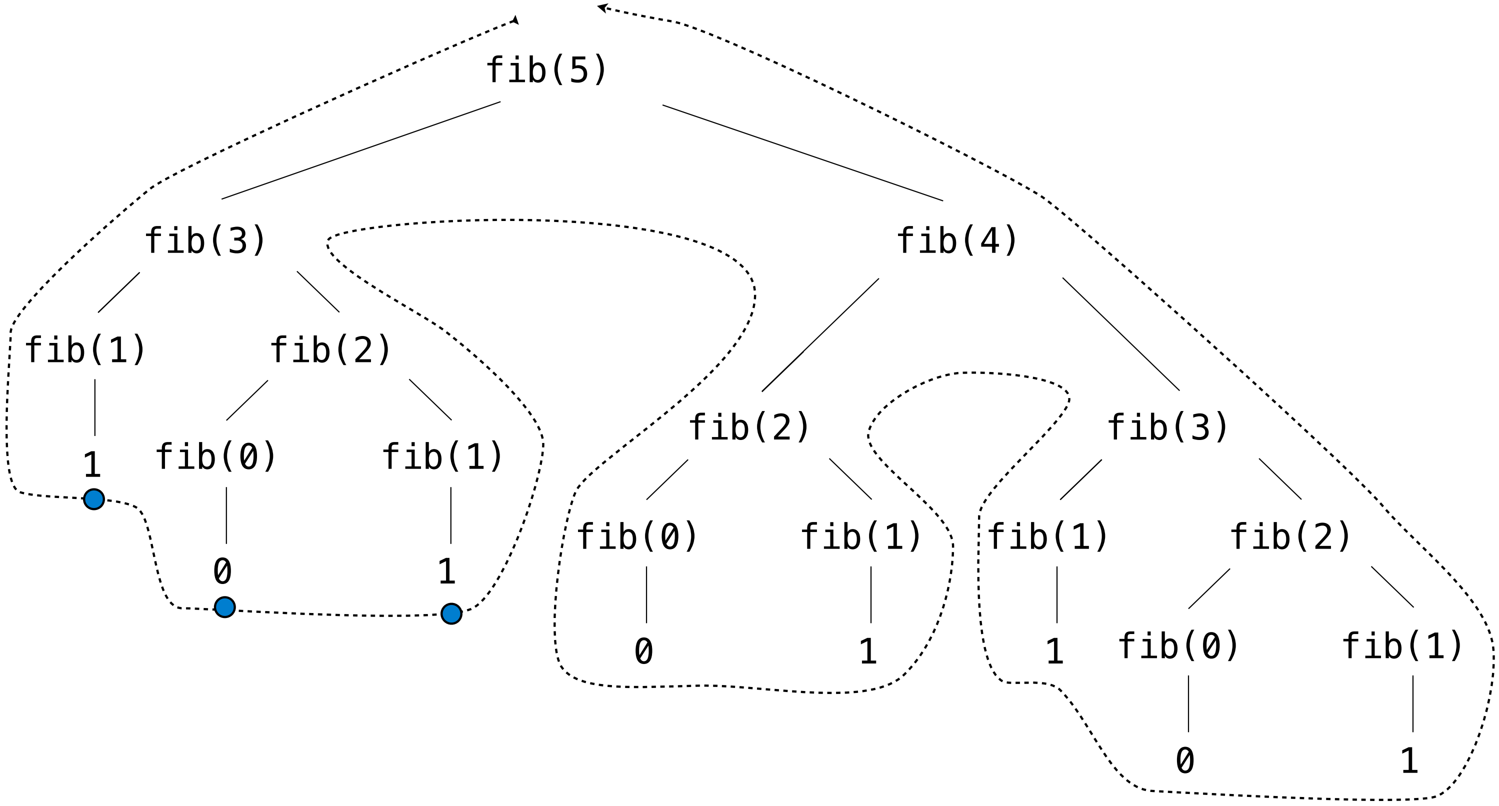

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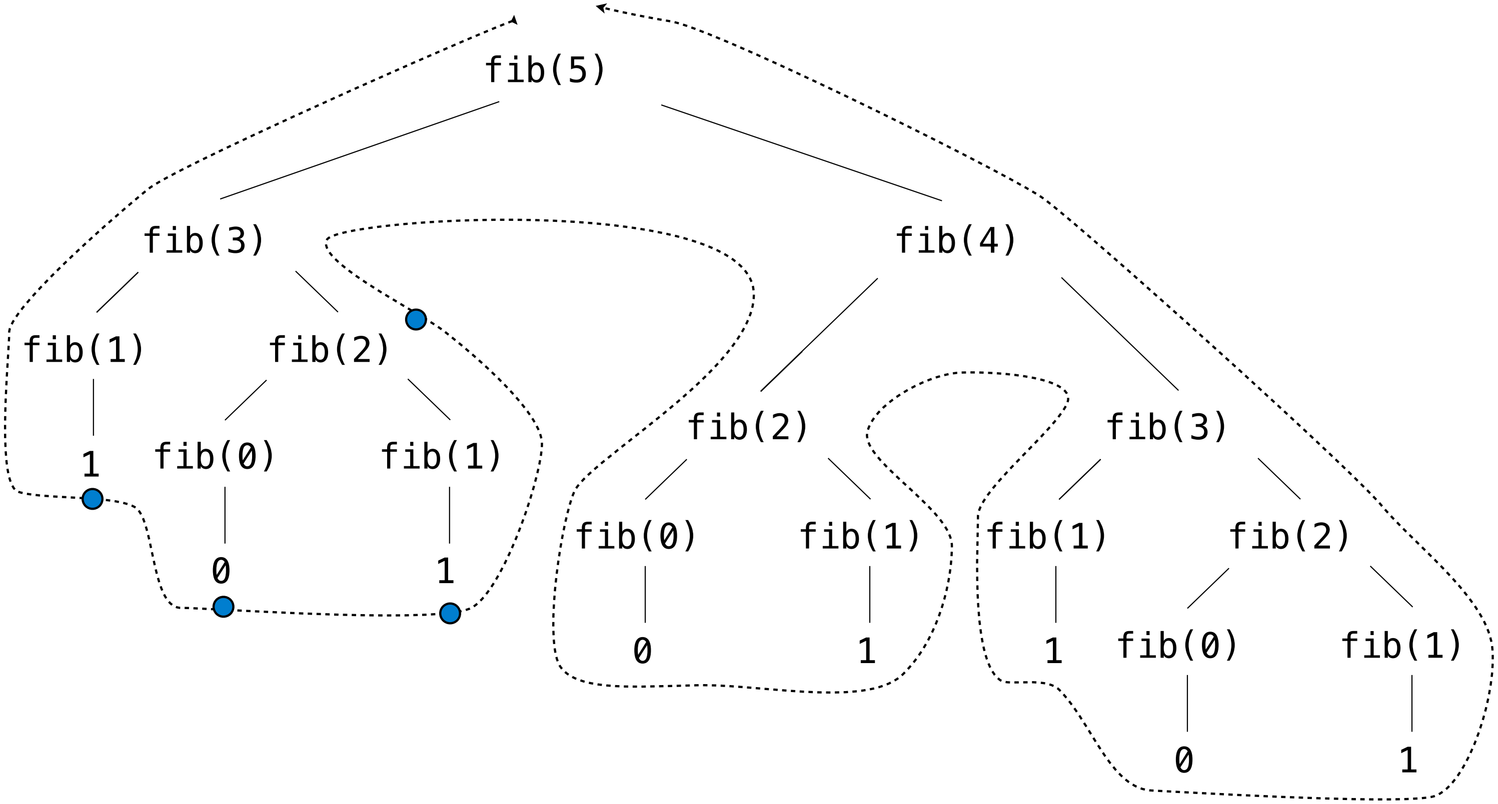

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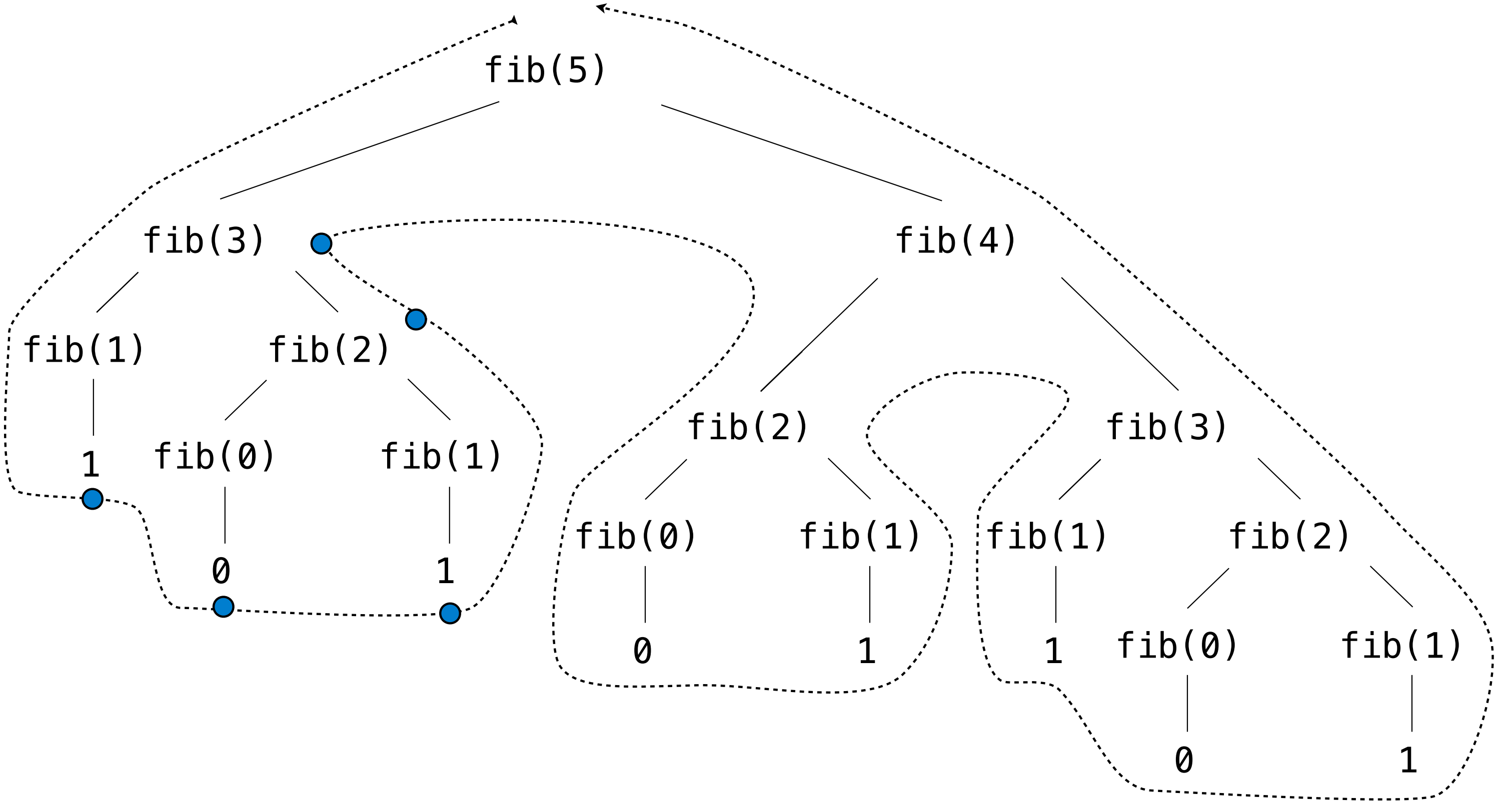

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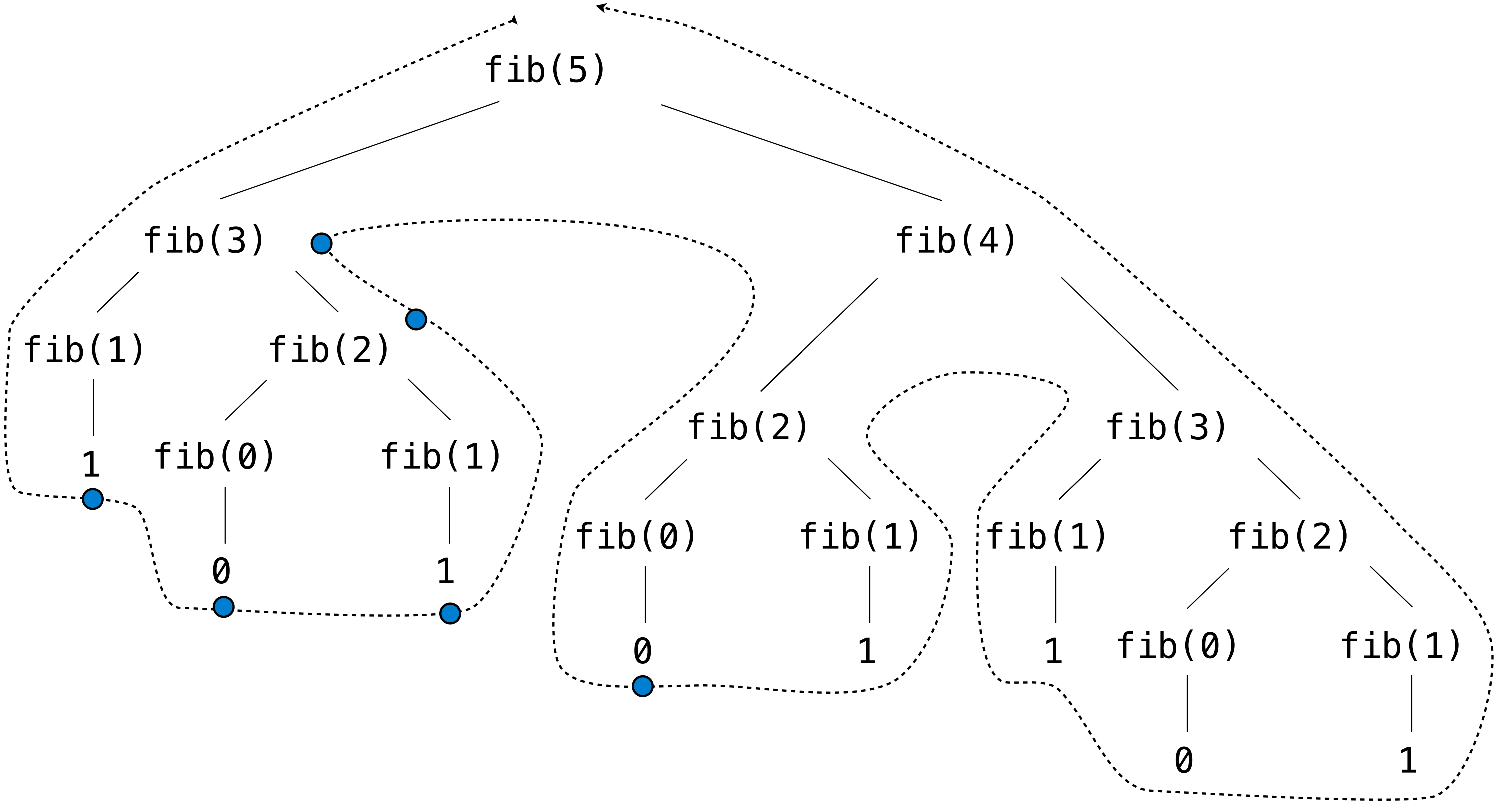

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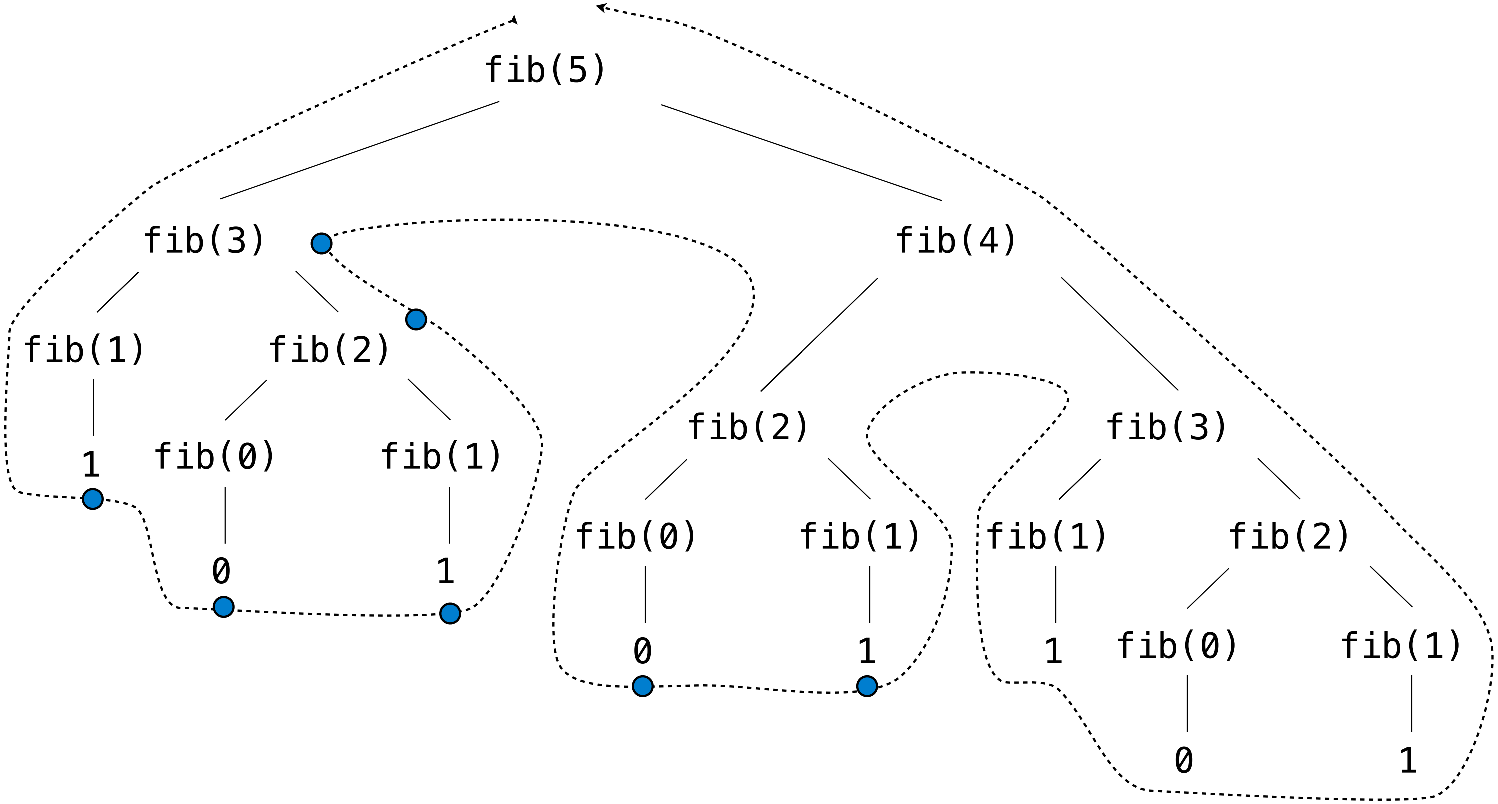

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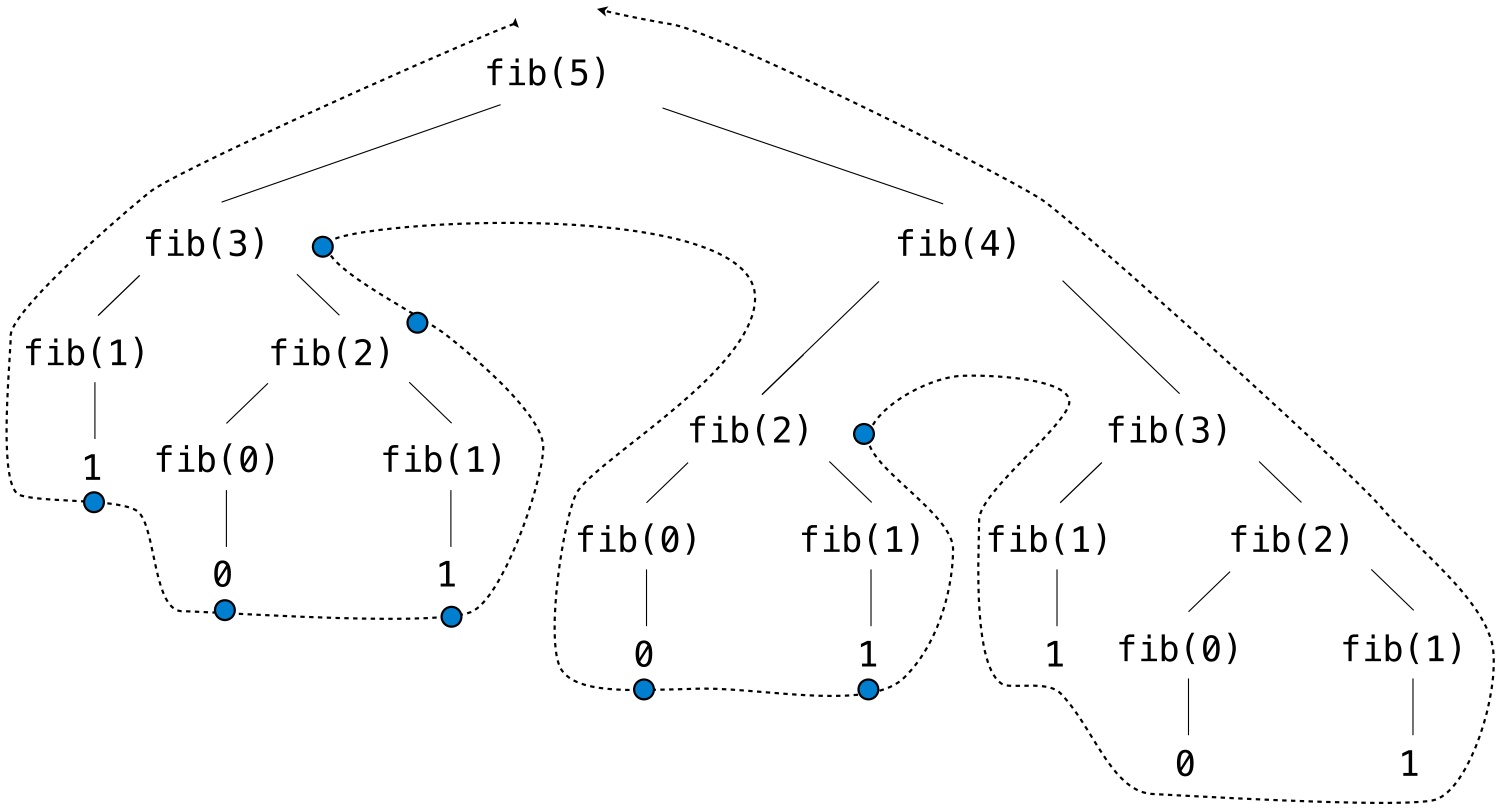

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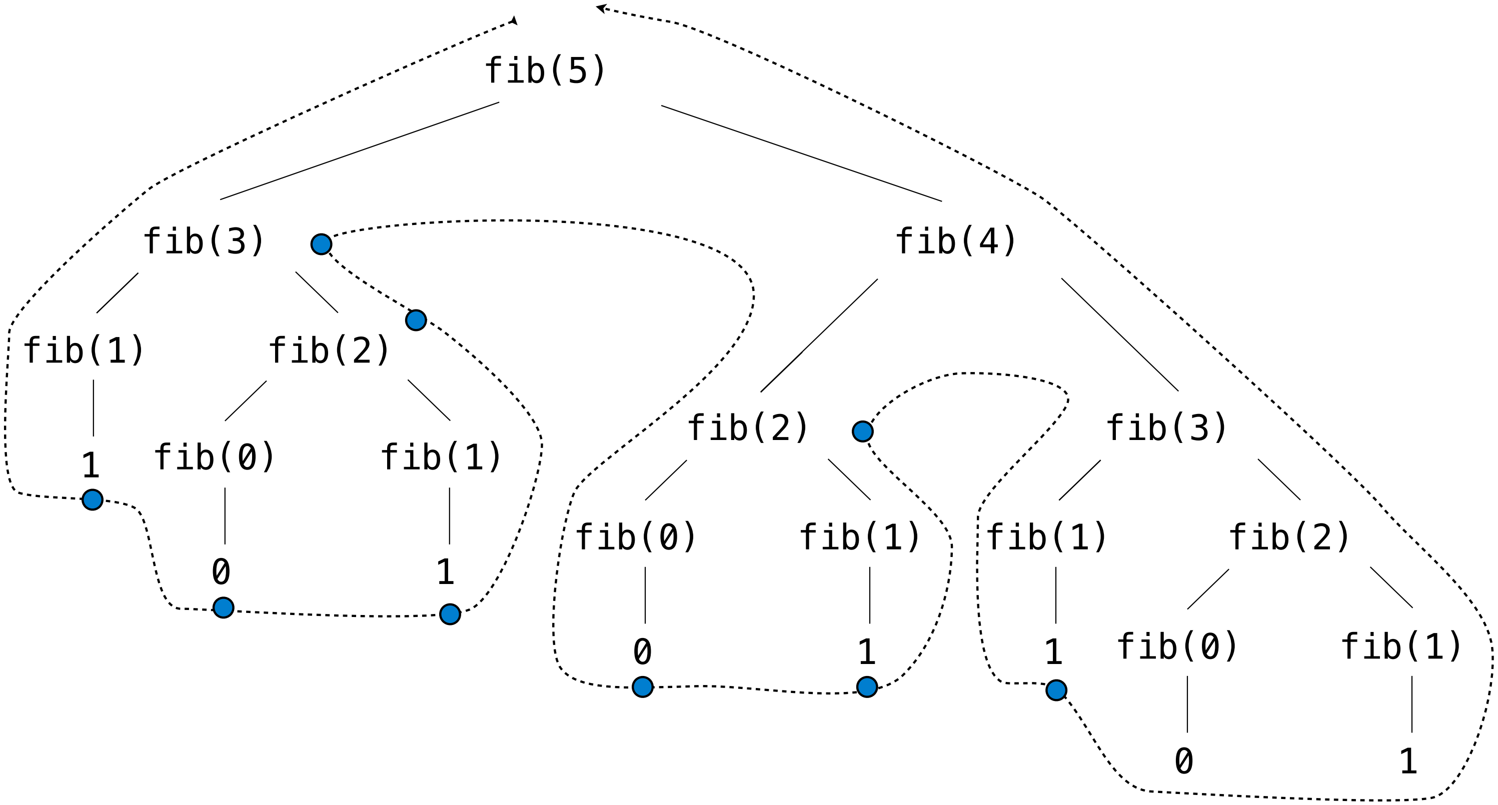

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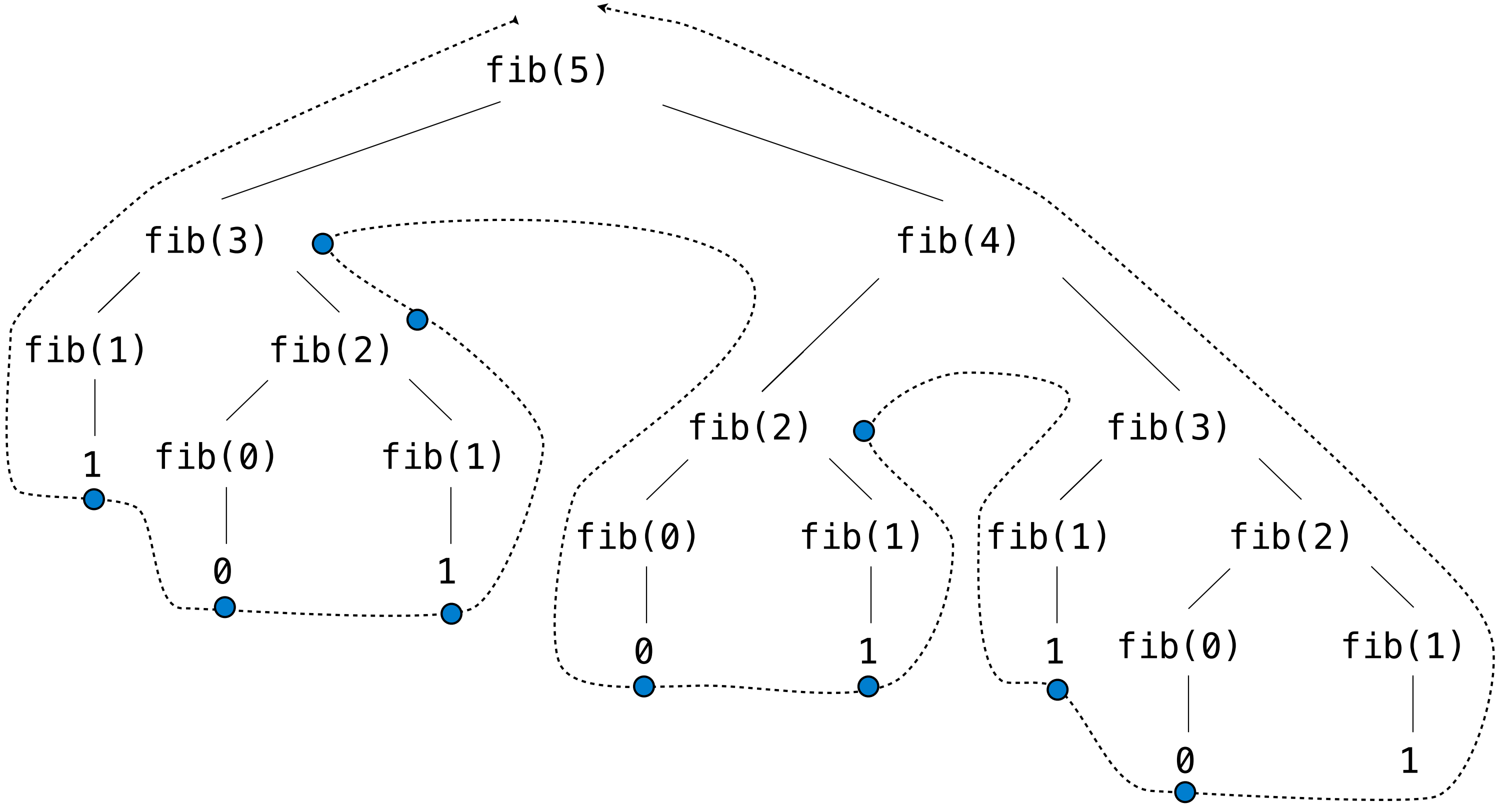

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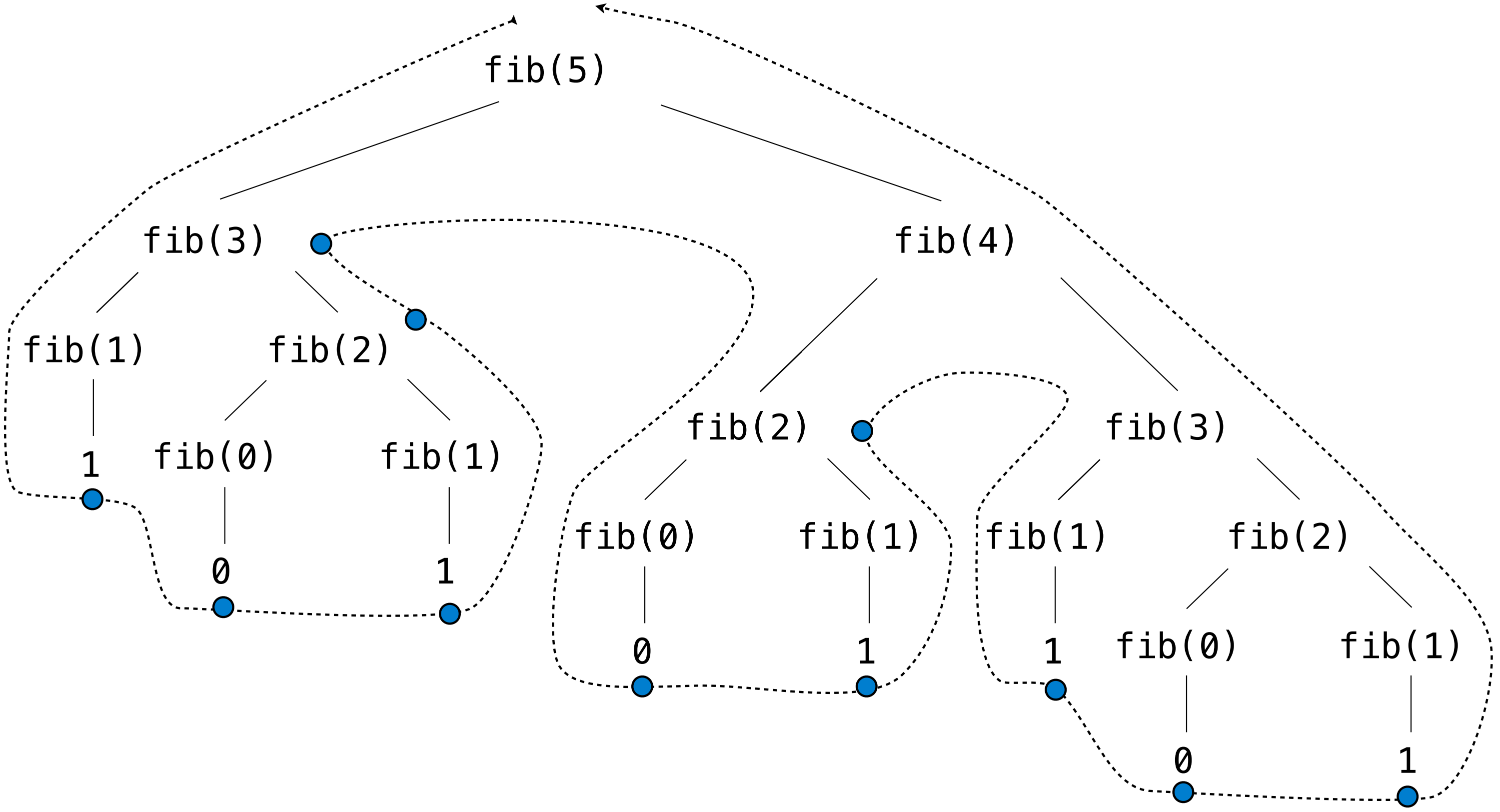

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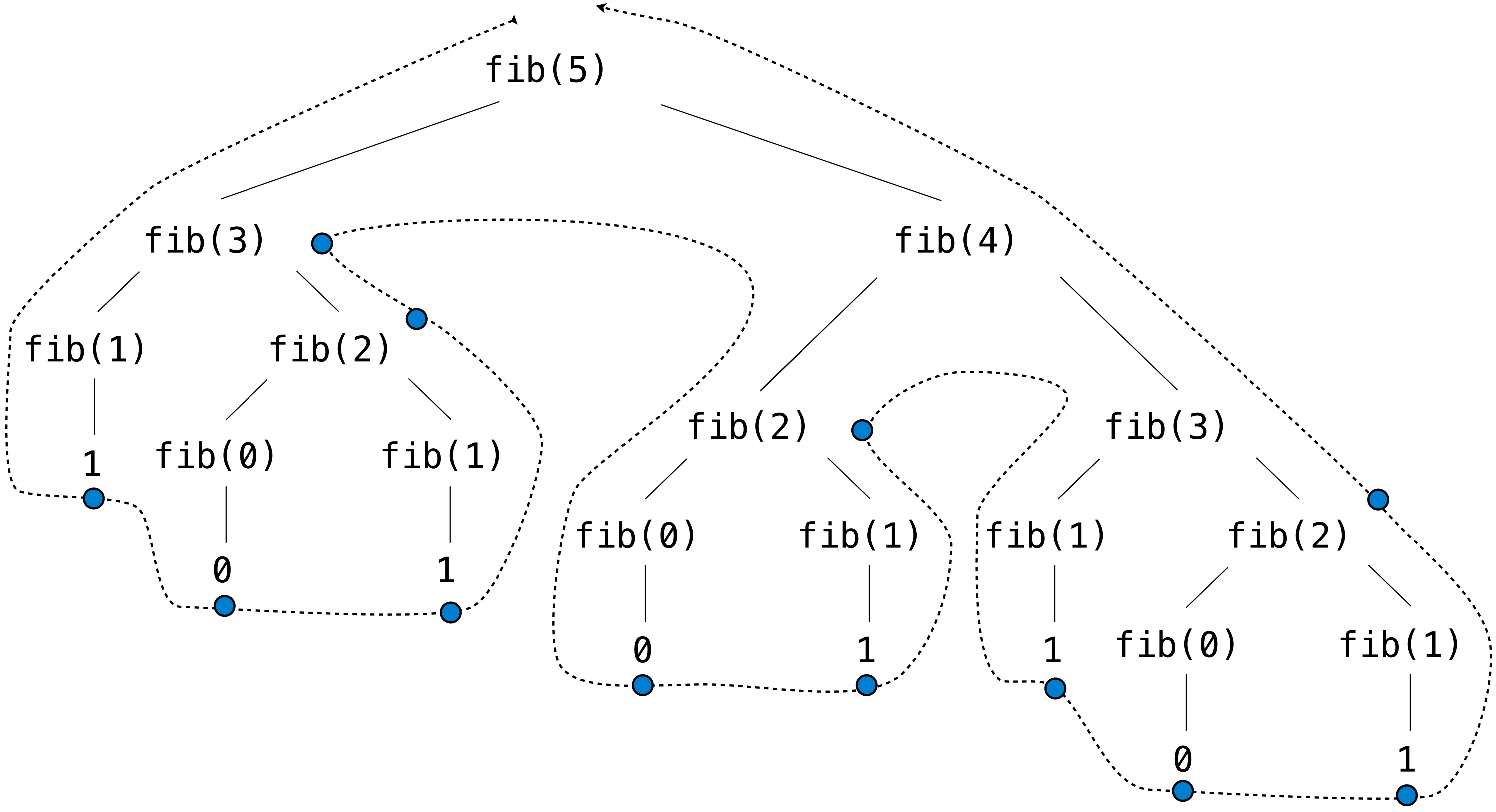

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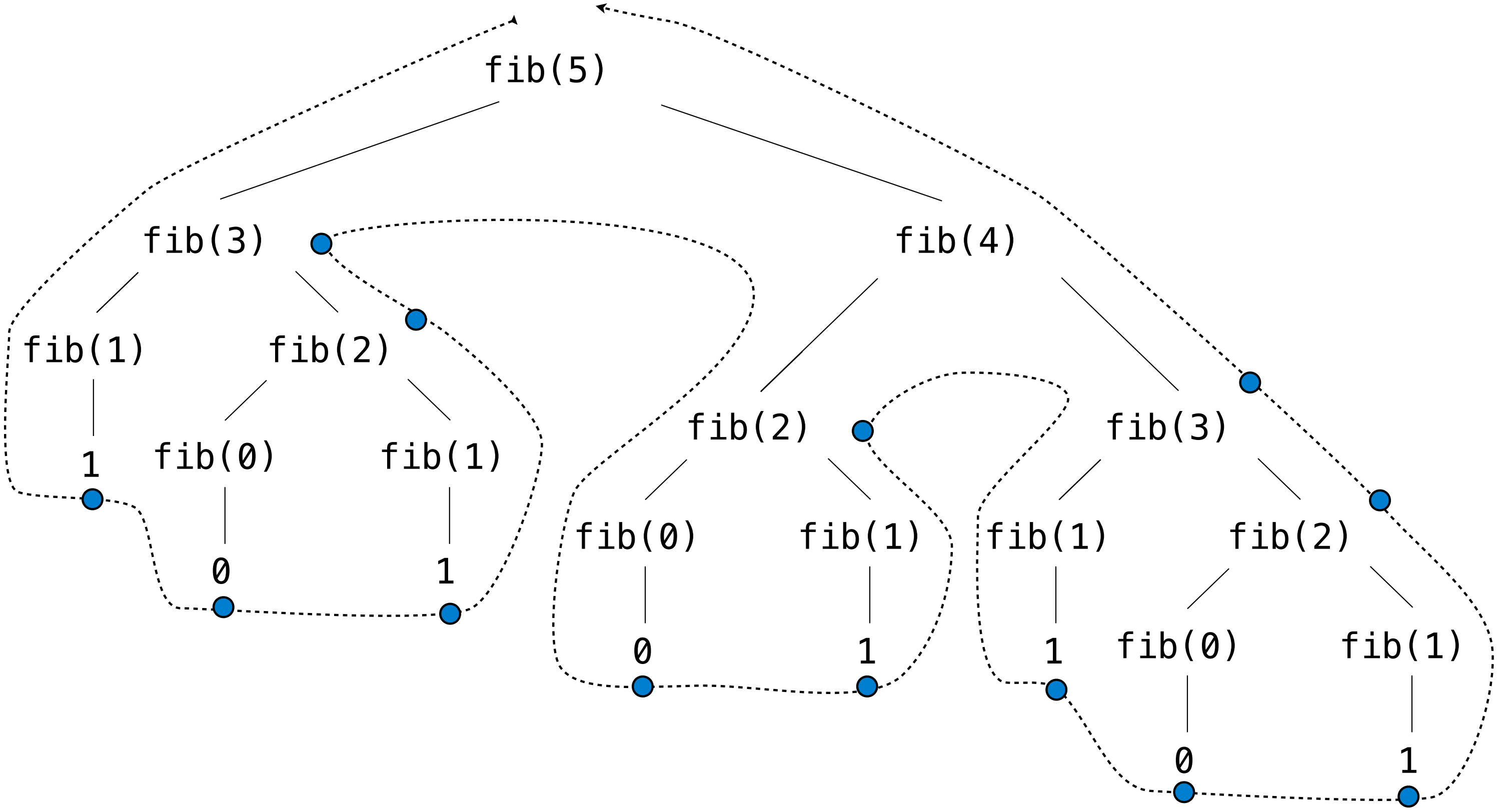

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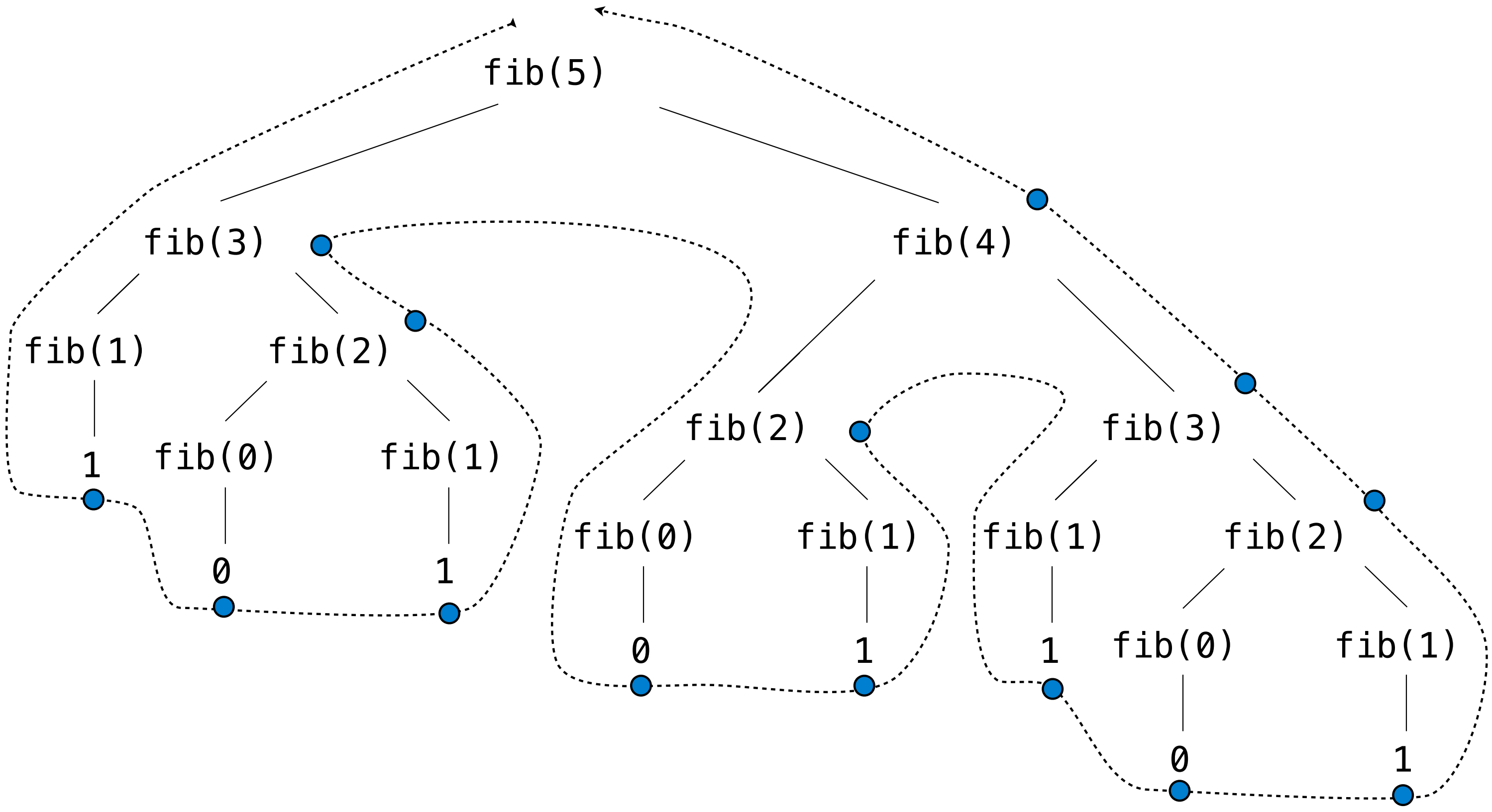

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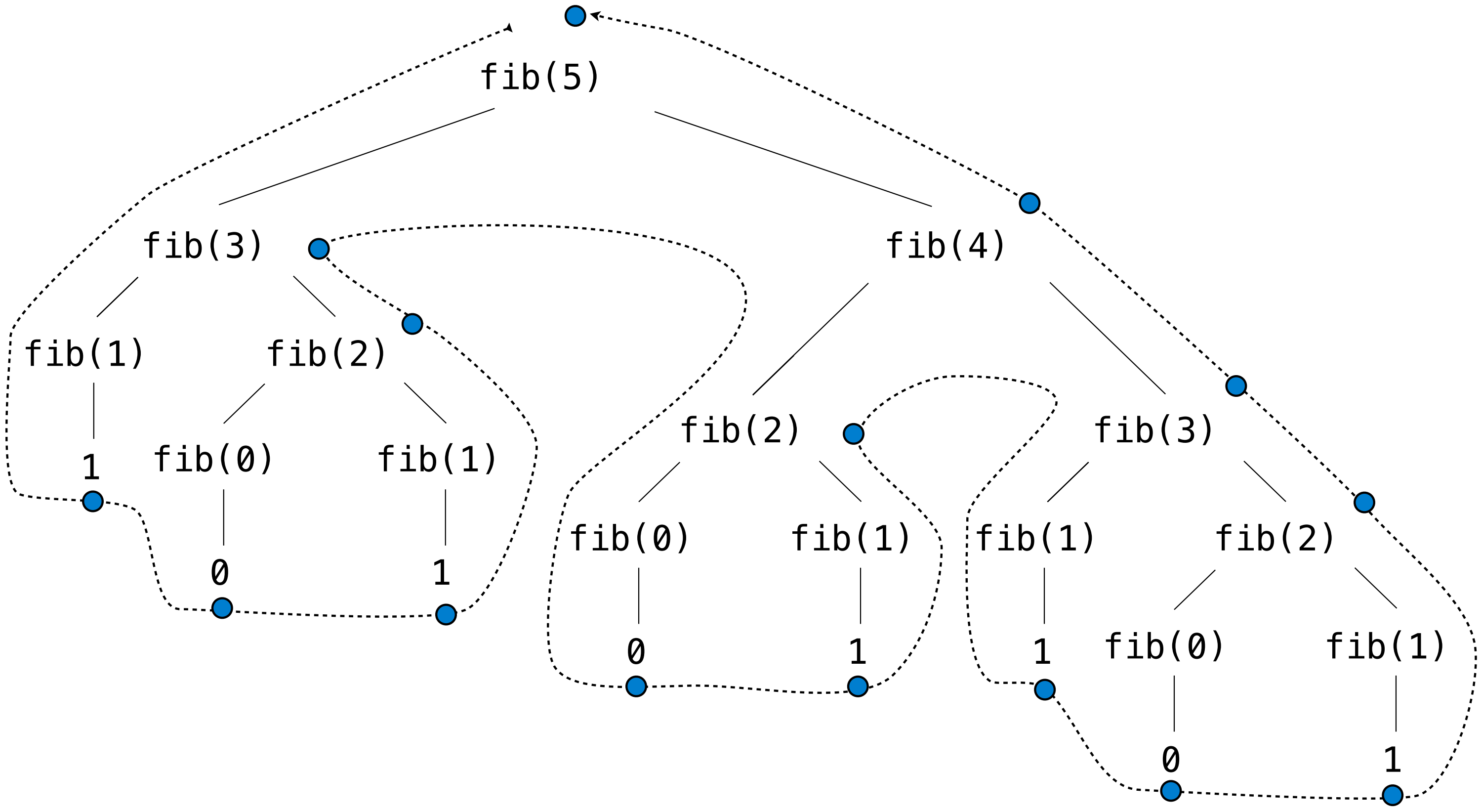

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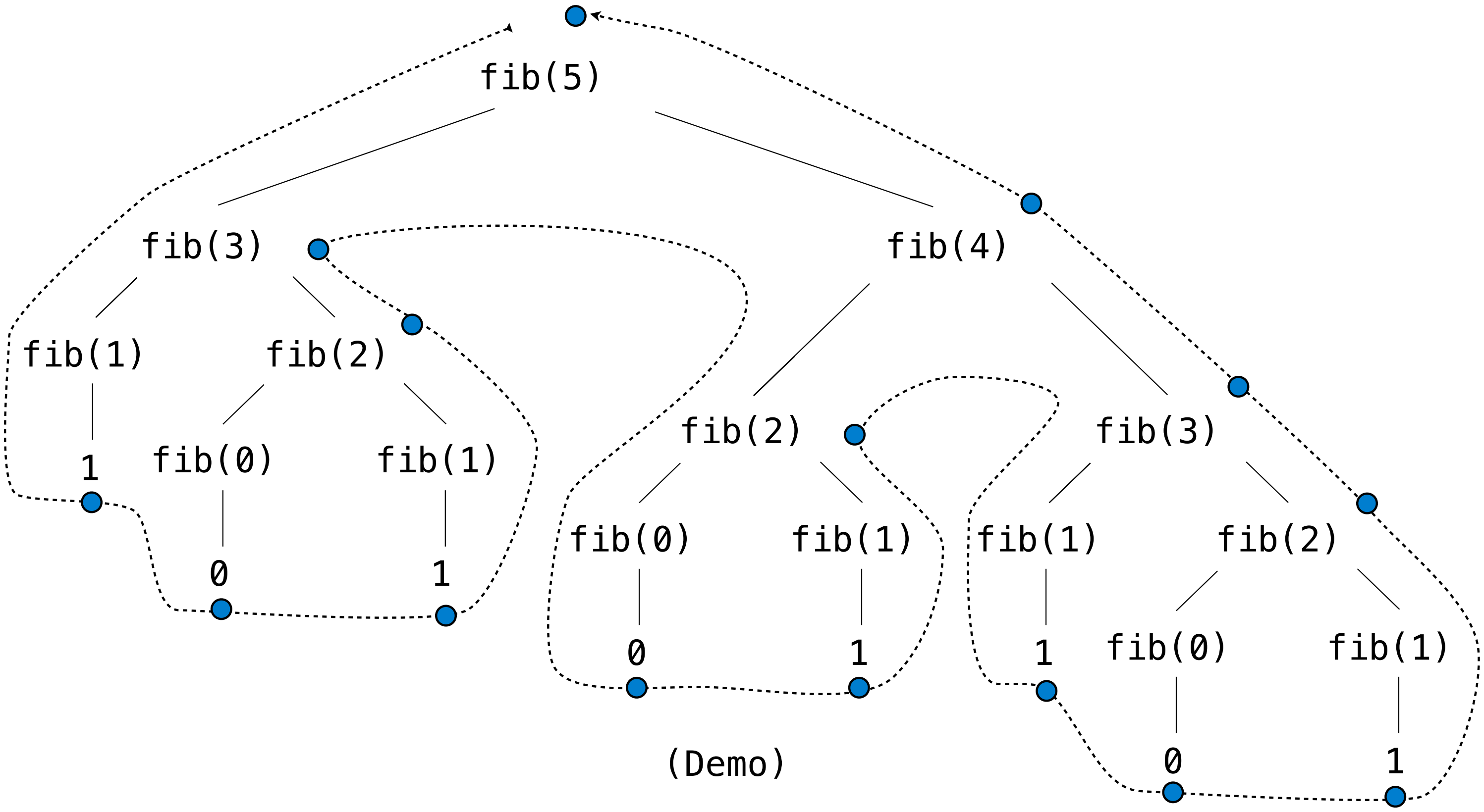

fib(5)

fib(3) fib(4)

fib(1) fib(2)

1 fib(0) fib(1) fib(2) fib(3)

0 1

fib(0) fib(1) fib(1) fib(2)

0 1 1 fib(0) fib(1)

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(Demo)

Repetition in Tree-Recursive Computation

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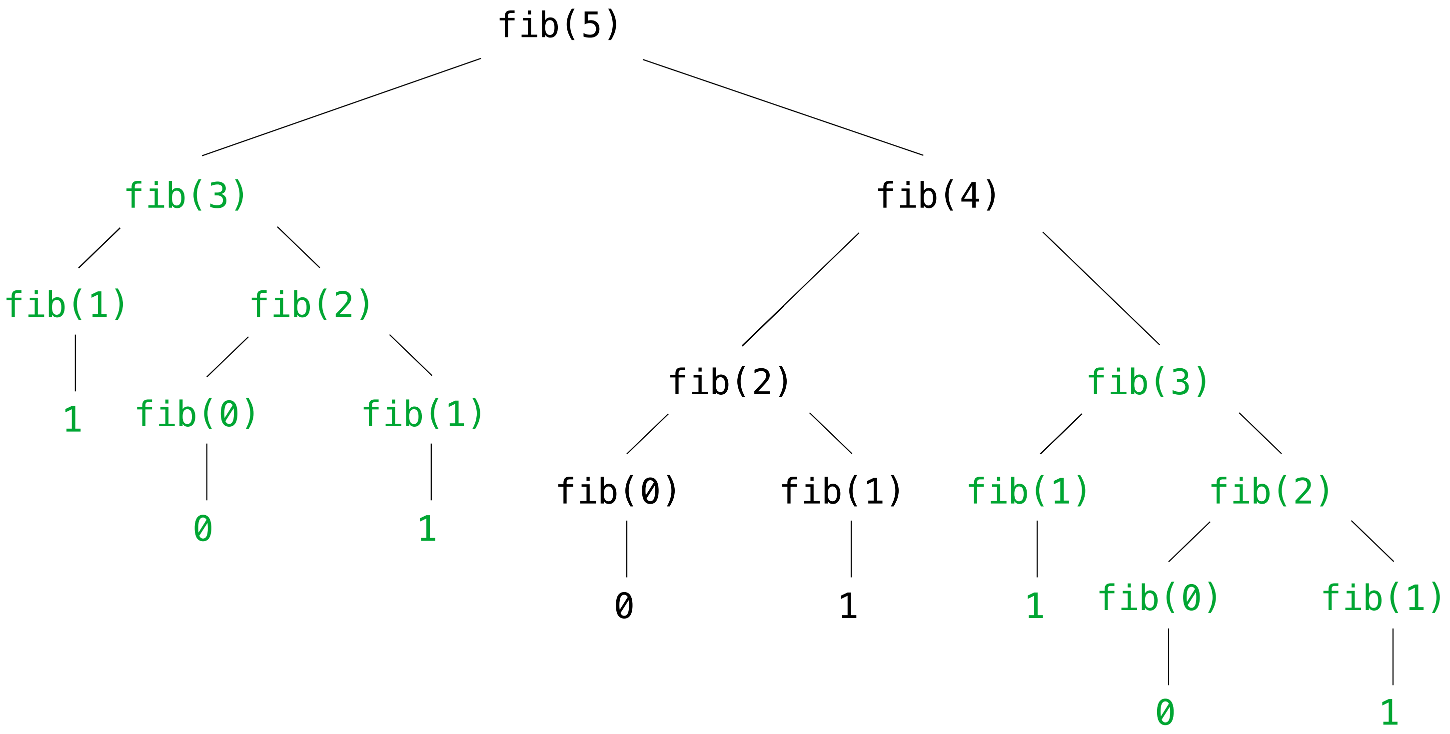

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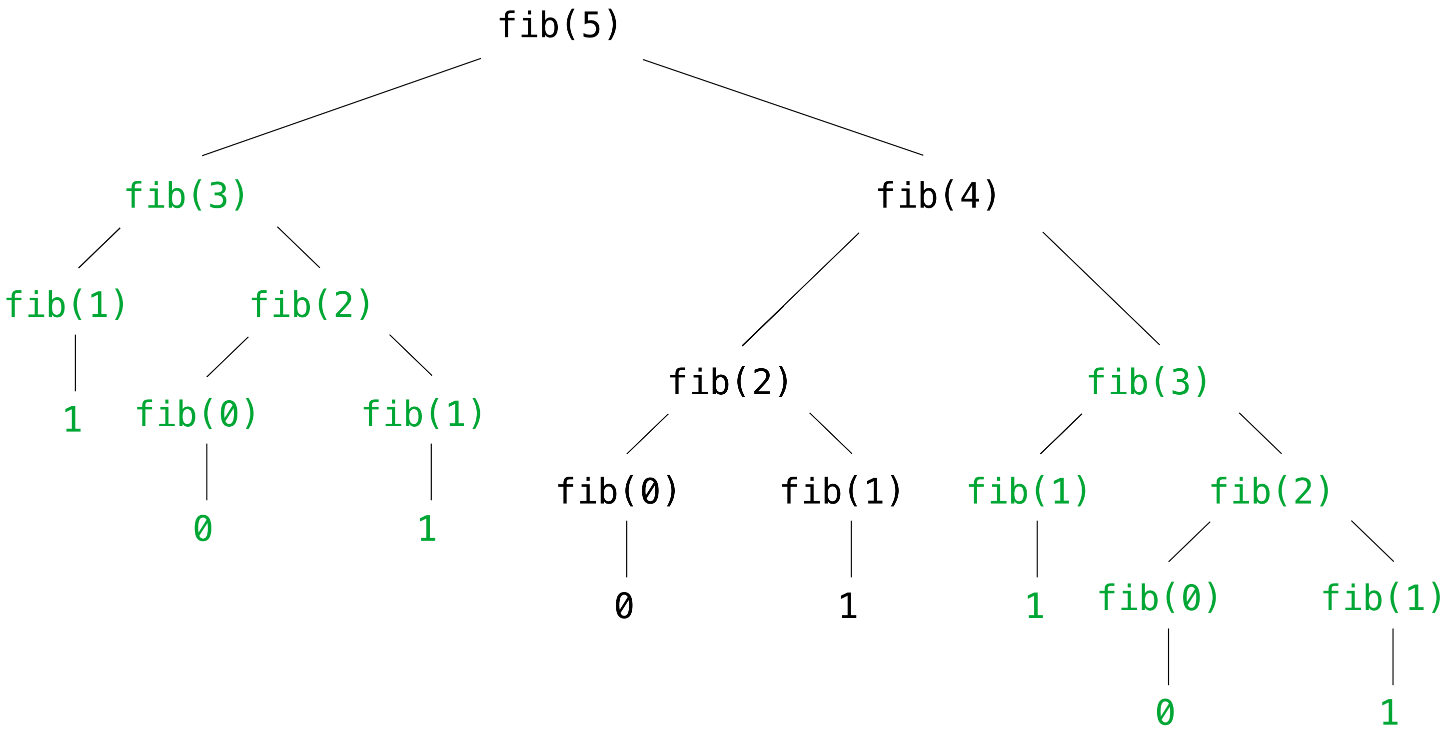

(We will speed up this computation dramatically in a few weeks by remembering results)

Example: Counting Partitions

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1 + 2 + 3 = 6

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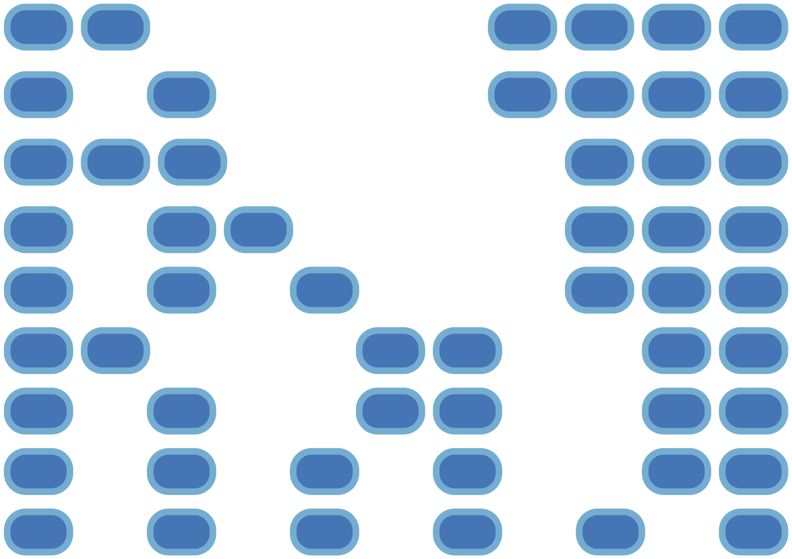

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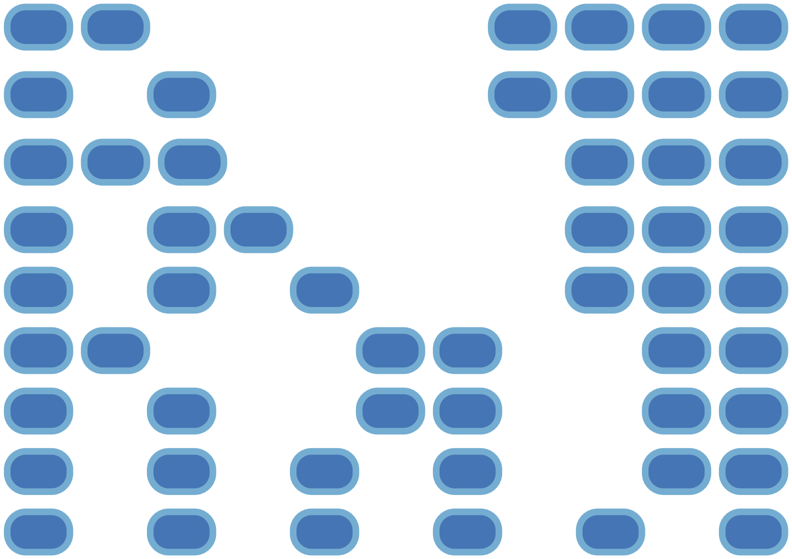

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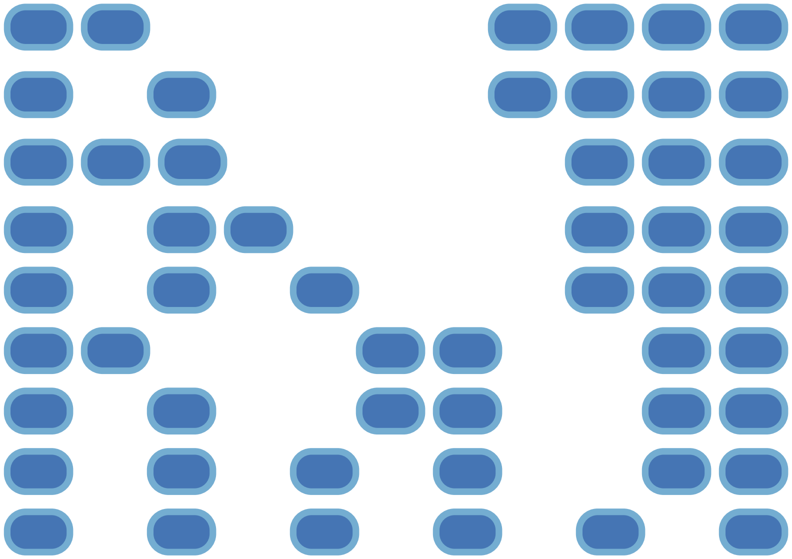

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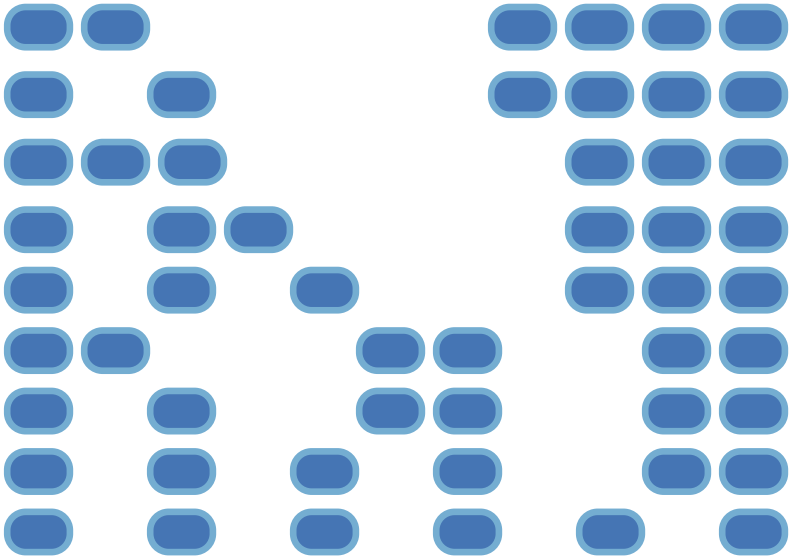

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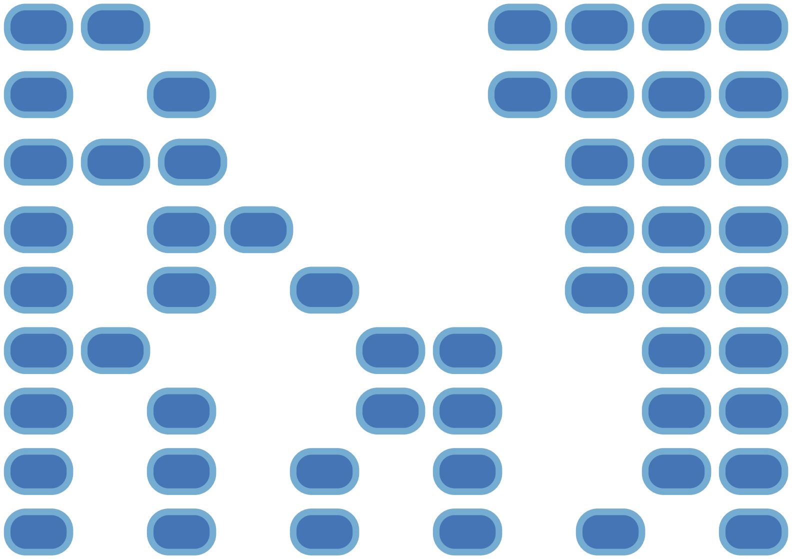

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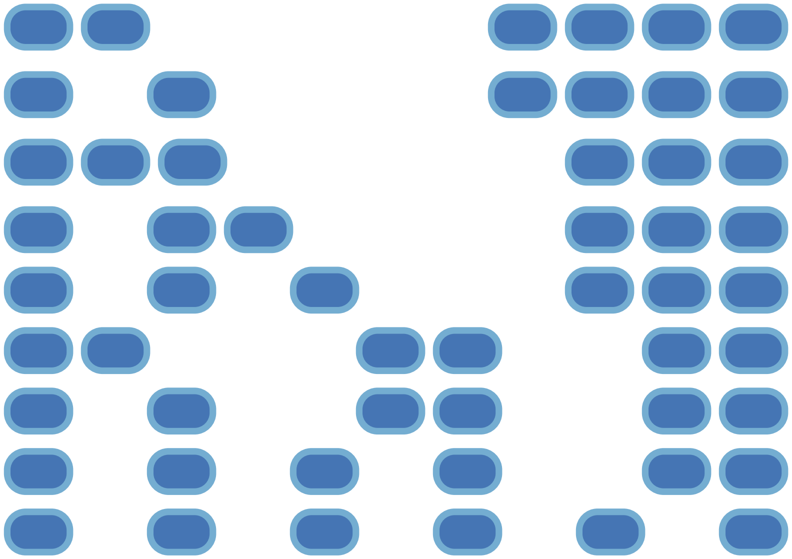

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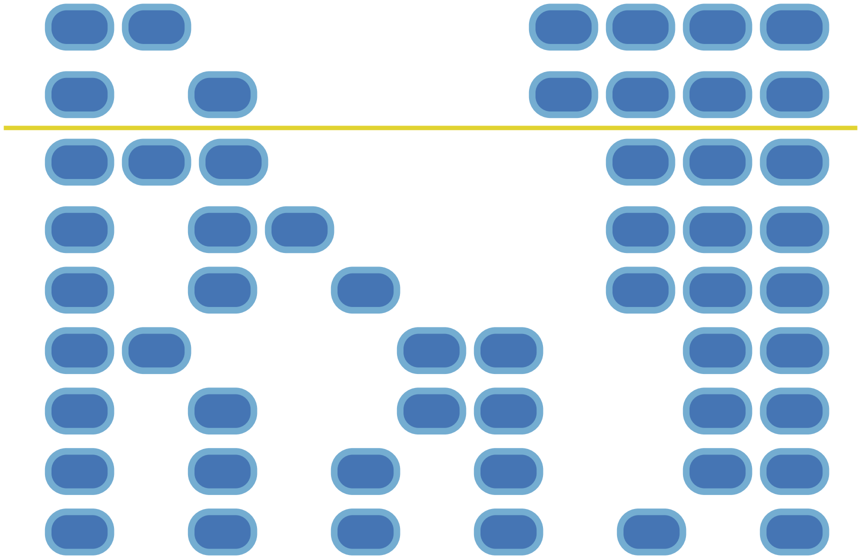

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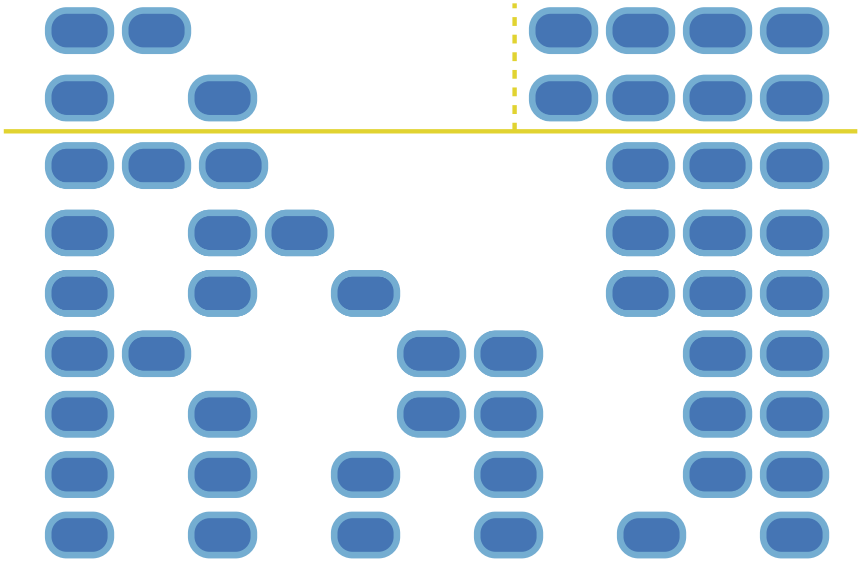

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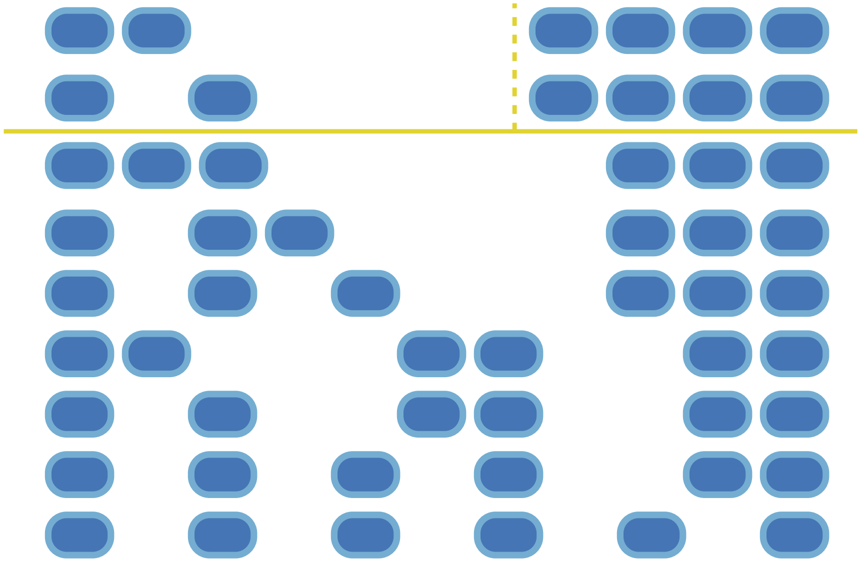

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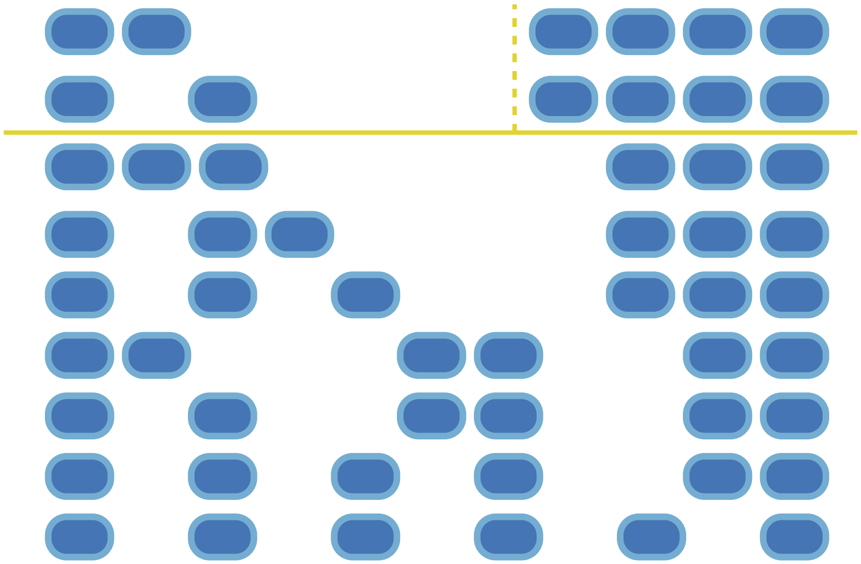

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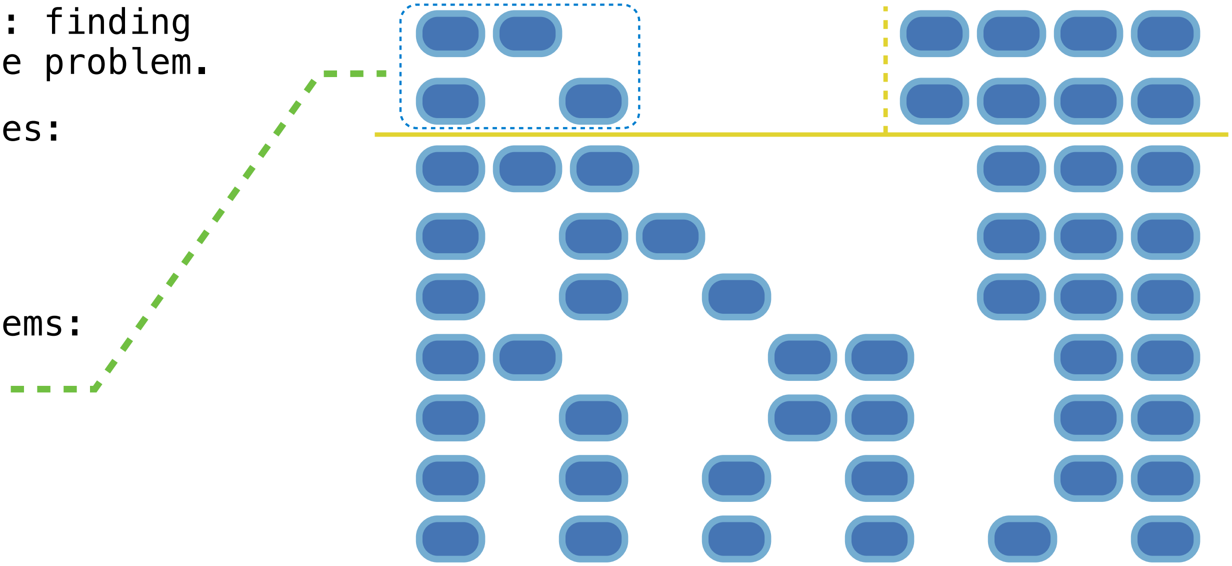

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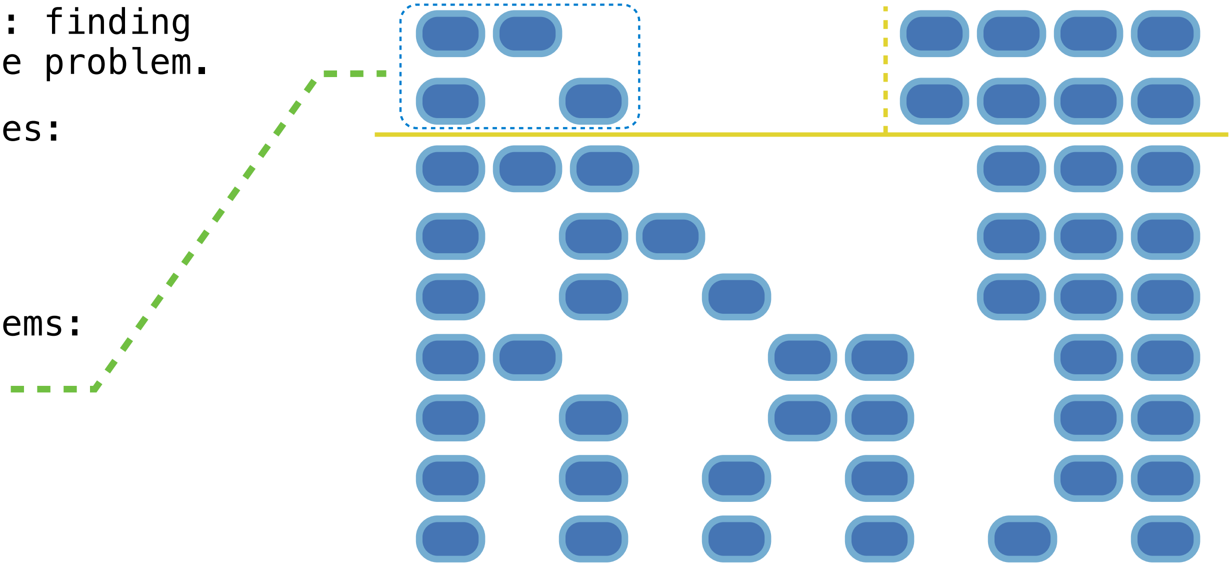

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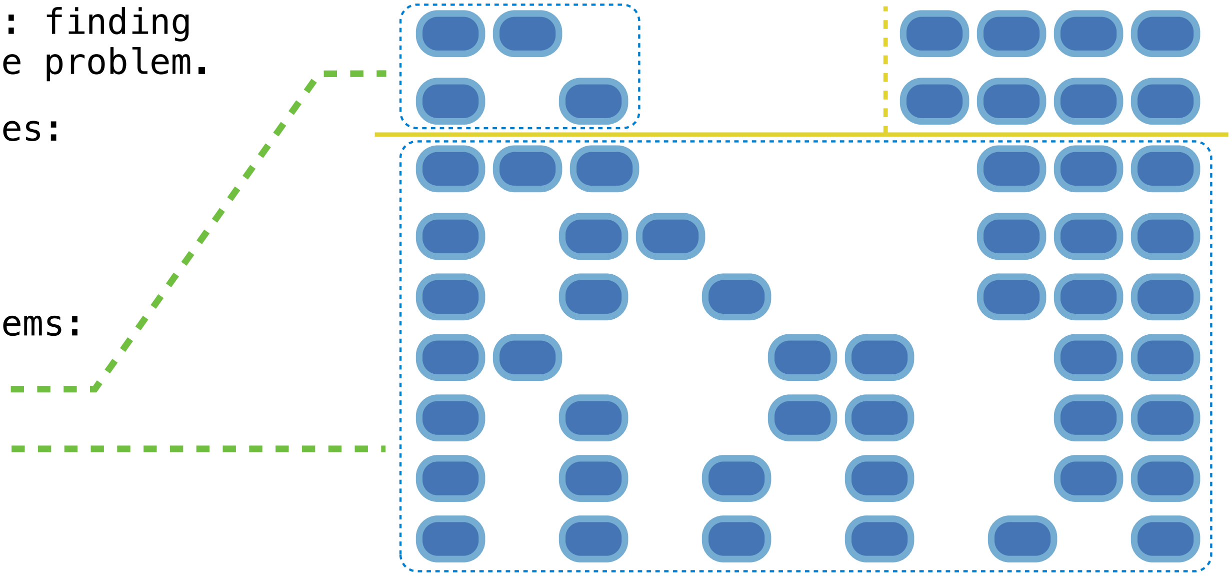

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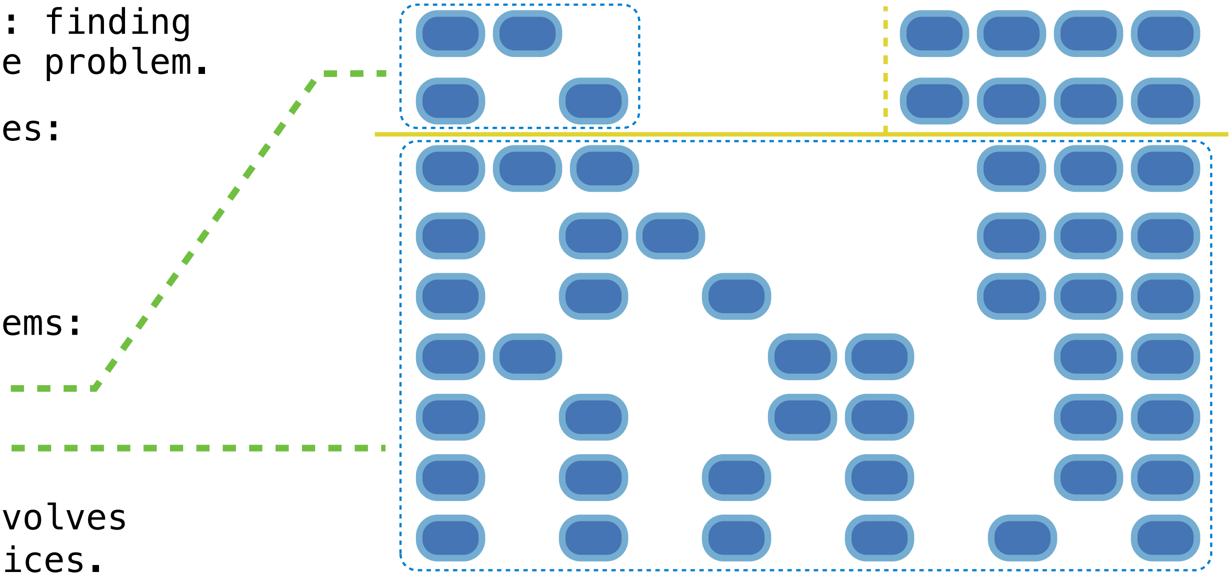

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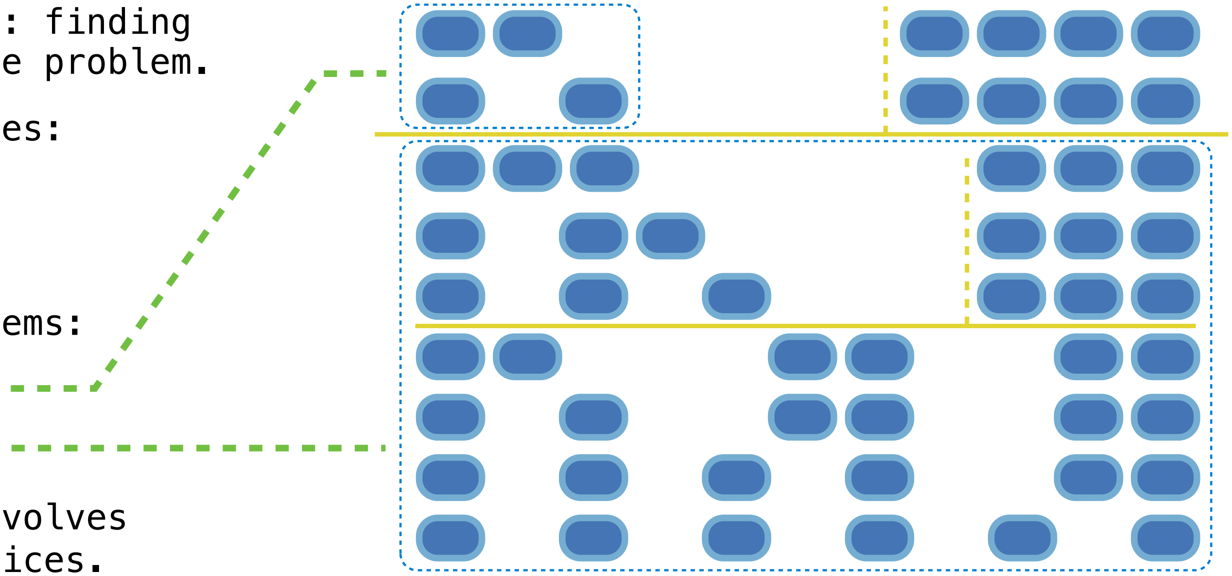

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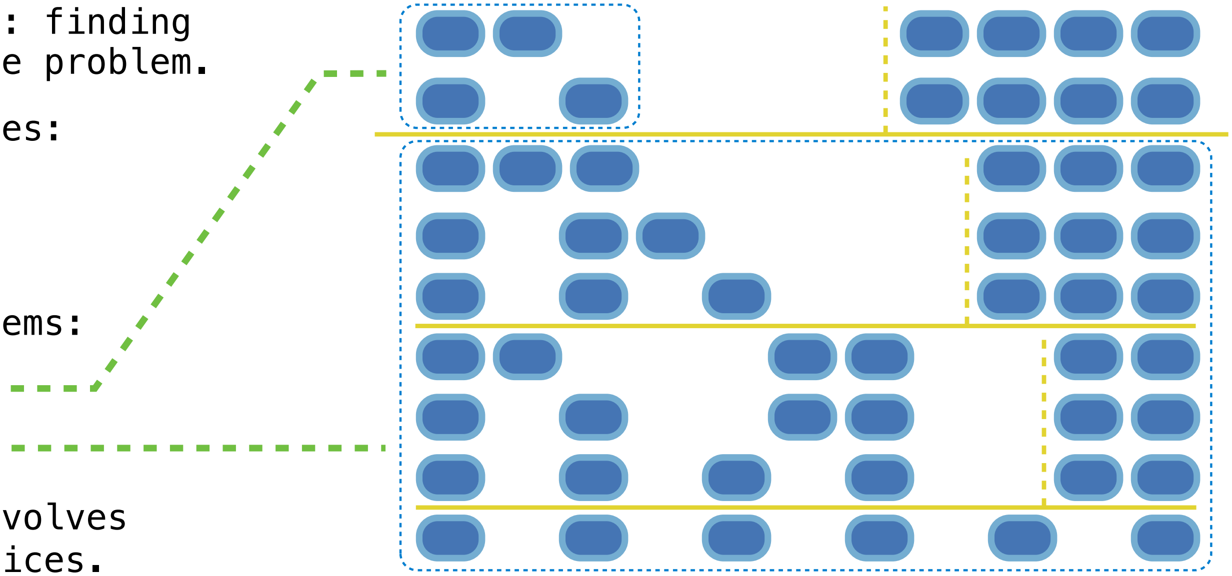

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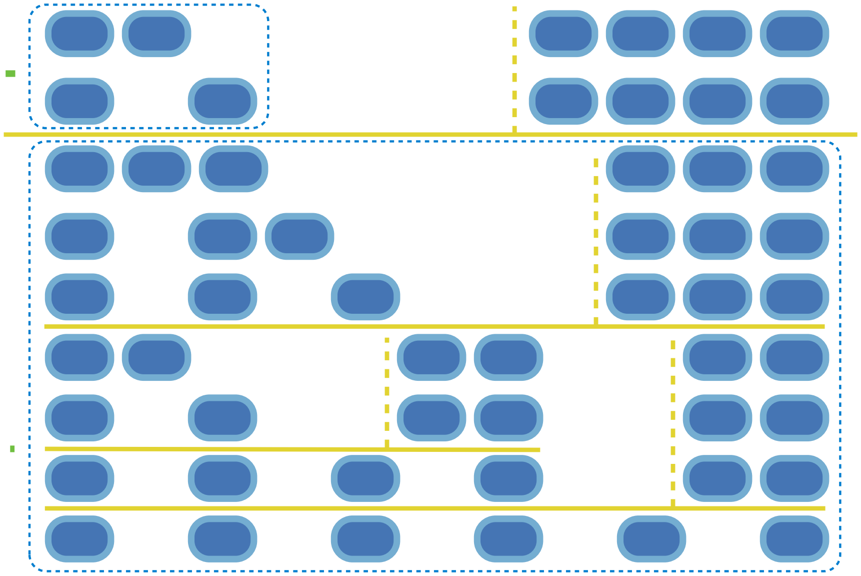

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    else:
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def count_partitions(n, m):




    else:
        with_m = count_partitions(n-m, m)
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        return with_m + without_m
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```
def count_partitions(n, m):
    if n == 0:





    else:
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        return with_m + without_m
```

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    else:
        with_m = count_partitions(n-m, m)
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Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
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(Demo)

Interactive Diagram