61A Lecture 14

Announcements

Mutable Functions

A Function with Behavior That Varies Over Time

```
Let's model a bank account that has a balance of $100
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Second withdrawal of the same amount

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Different return value!

A Function with Behavior That Varies Over Time

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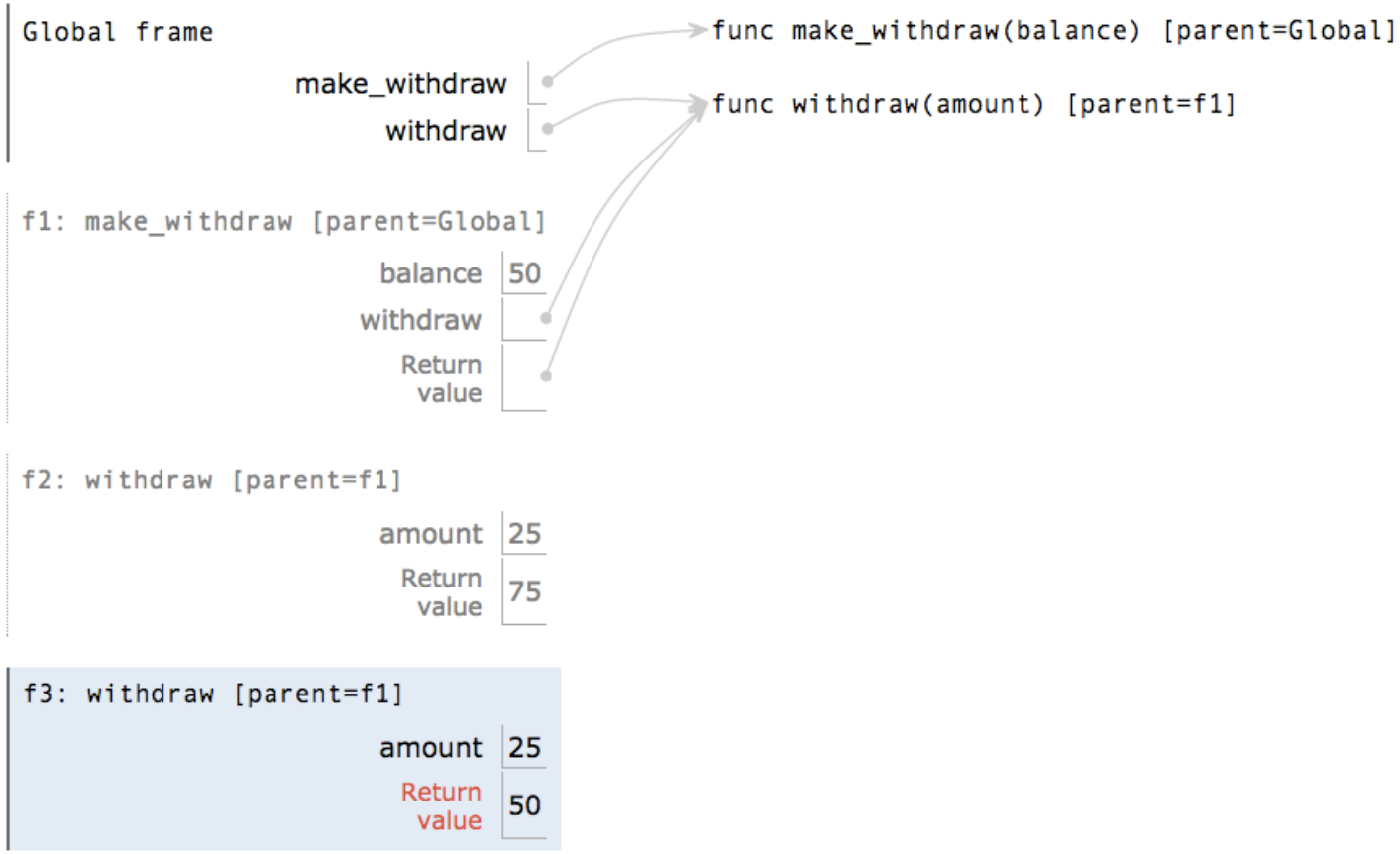
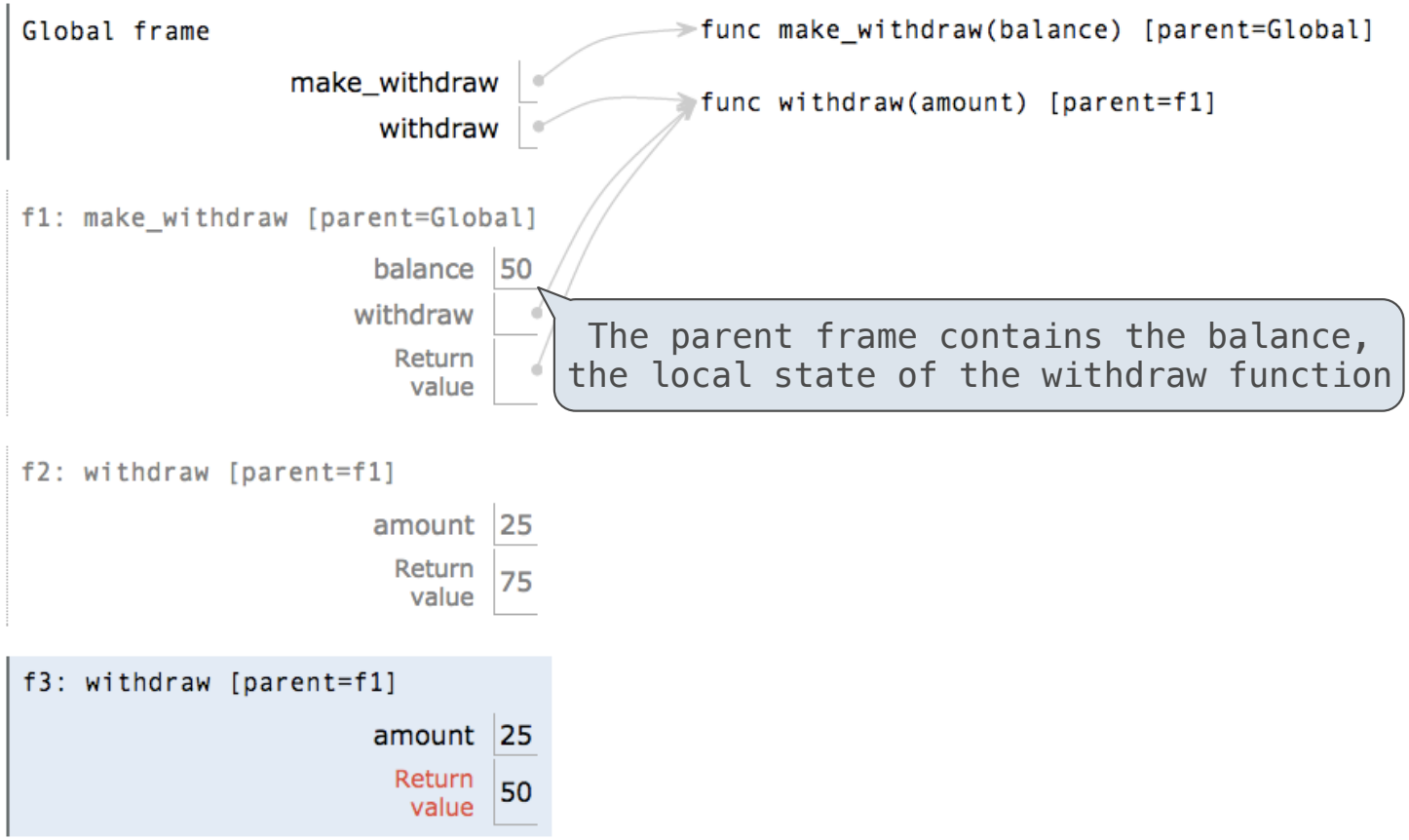
A function has a body and a parent environment

Persistent Local State Using Environments



Interactive Diagram

Persistent Local State Using Environments



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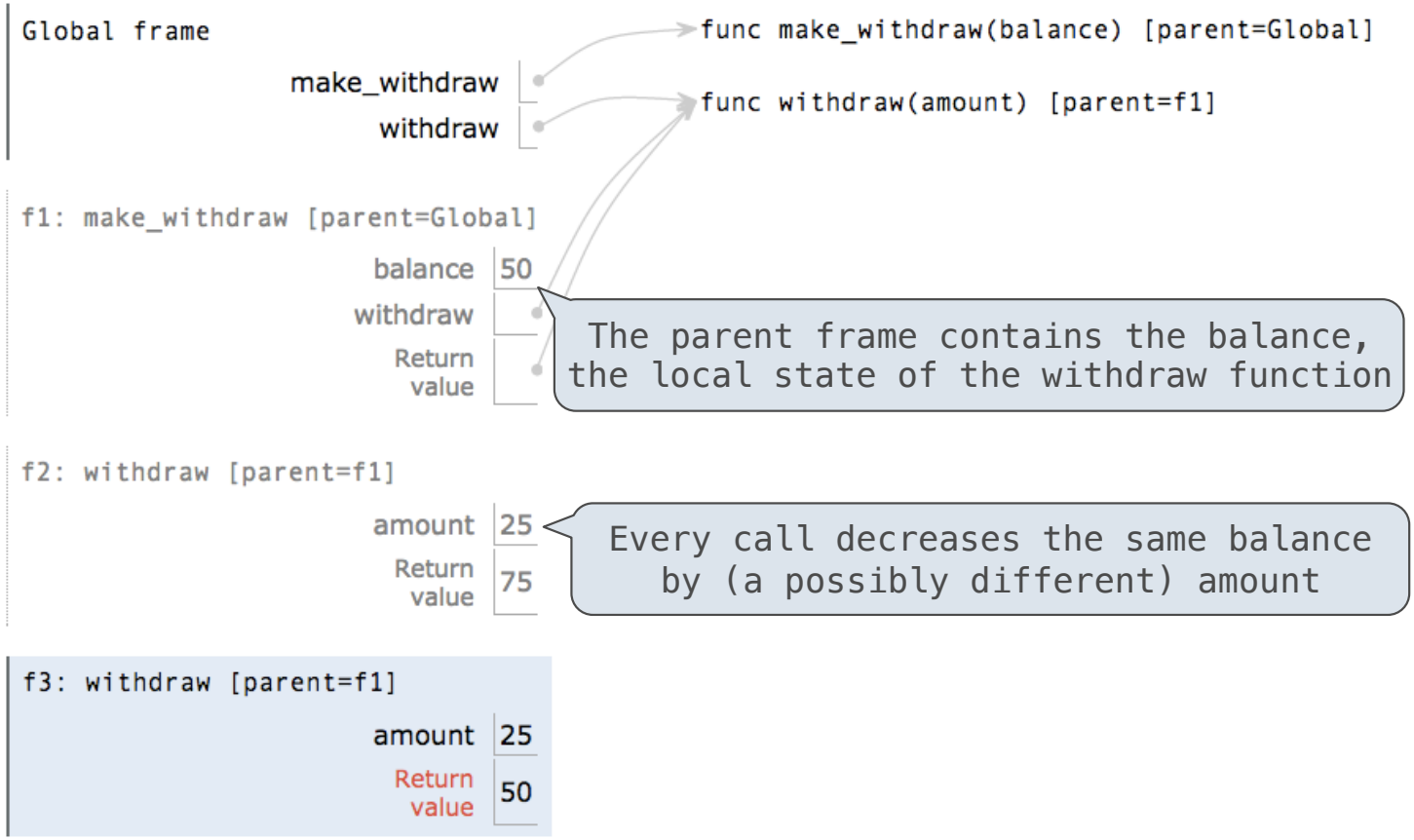

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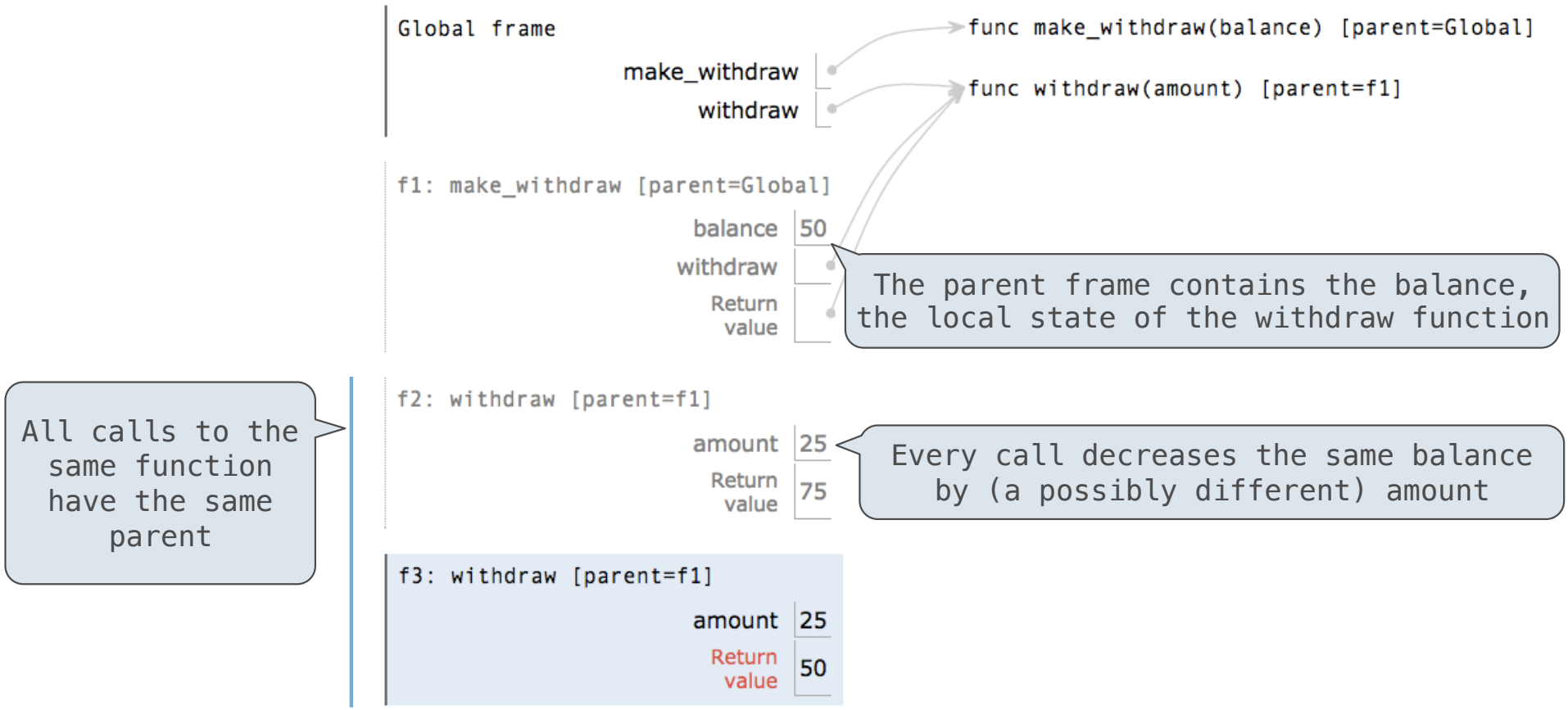
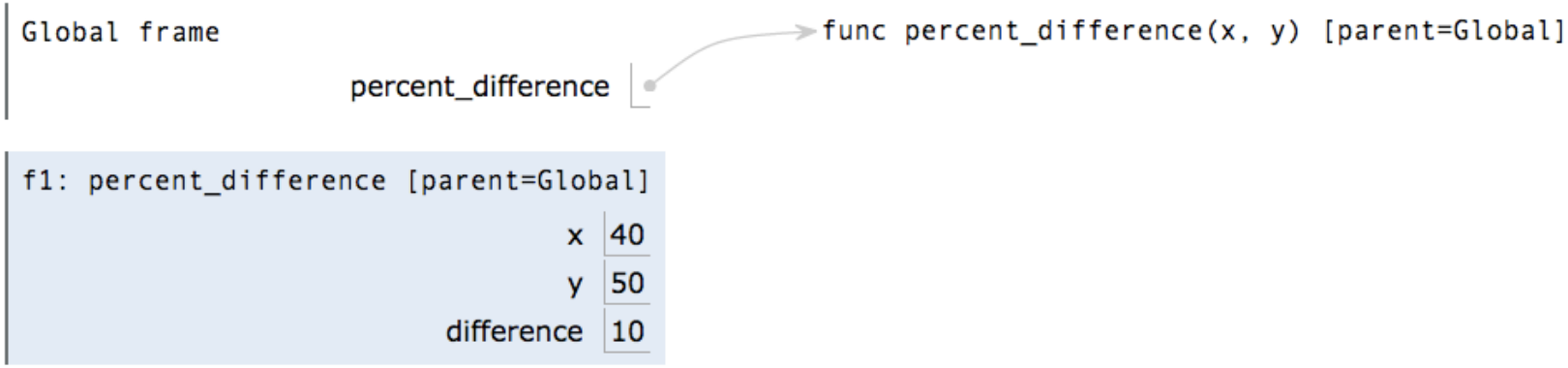


Interactive Diagram

Reminder: Local Assignment

```
def percent_difference(x, y):
        difference = abs(x-y)
        return 100 * difference / x
diff = percent_difference(40, 50)
```



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Assignment binds name(s) to value(s) in the first frame of the current environment

Global frame

percent_difference → func percent_difference(x, y) [parent=Global]

f1: percent_difference [parent=Global]

x	40
y	50
difference	10

Interactive Diagram

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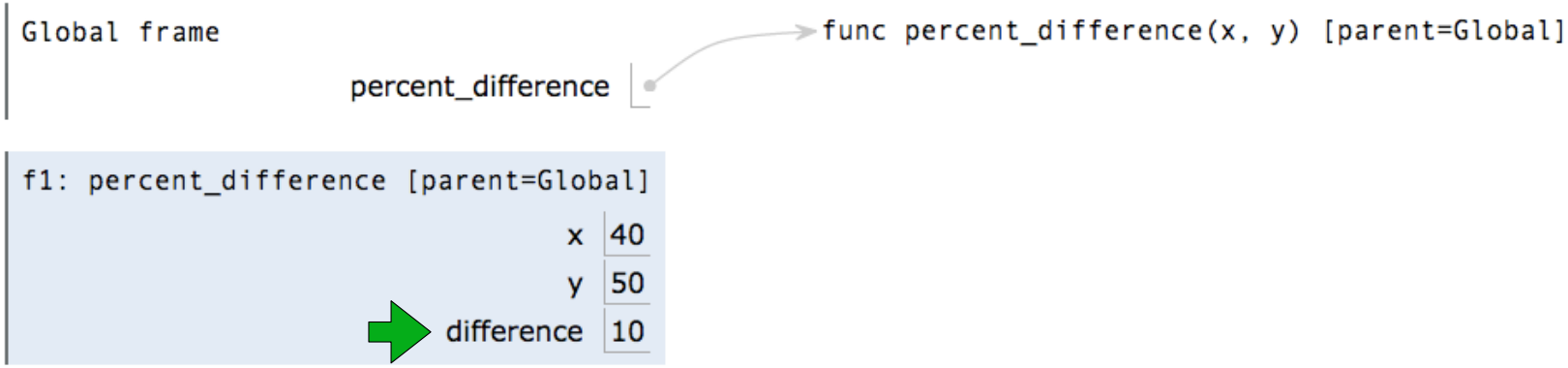

Execution rule for assignment statements:

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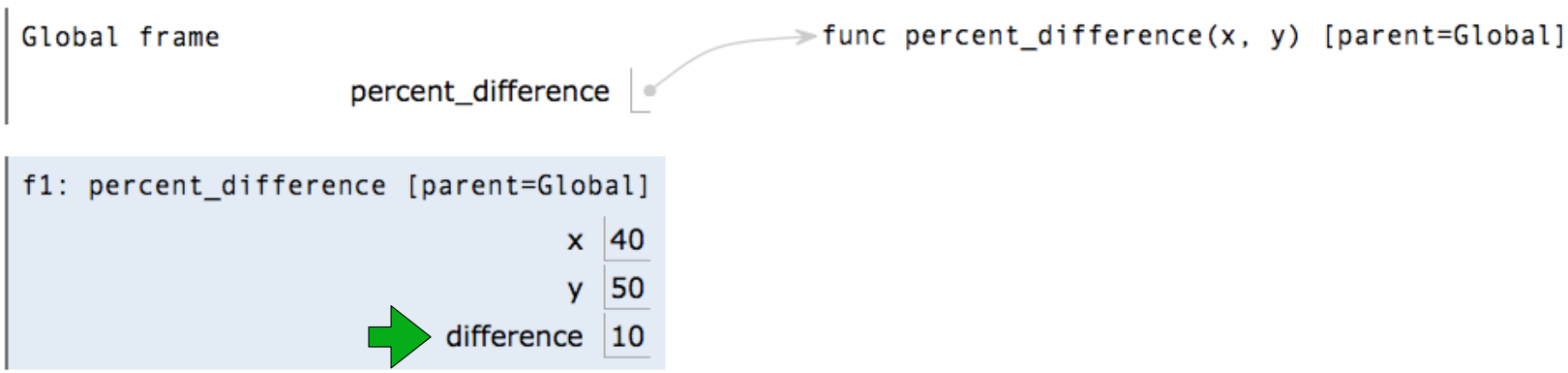

Execution rule for assignment statements:

1. Evaluate all expressions right of =, from left to right

2. Bind the names on the left to the resulting values in the **current frame**

Interactive Diagram

Non-Local Assignment & Persistent Local State

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def make_withdraw(balance):
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(Demo)

Non-Local Assignment

The Effect of Nonlocal Statements

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http://www.python.org/dev/peps/pep-3104/

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Status **Effect**

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Interactive Diagram

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Local assignment

UnboundLocalError: local variable 'balance' referenced before assignment

Interactive Diagram

Mutable Values & Persistent Local State

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def make_withdraw_list(balance):
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withdraw = make_withdraw_list(100)
withdraw(25)
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Name bound outside of withdraw def

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Element assignment changes a list

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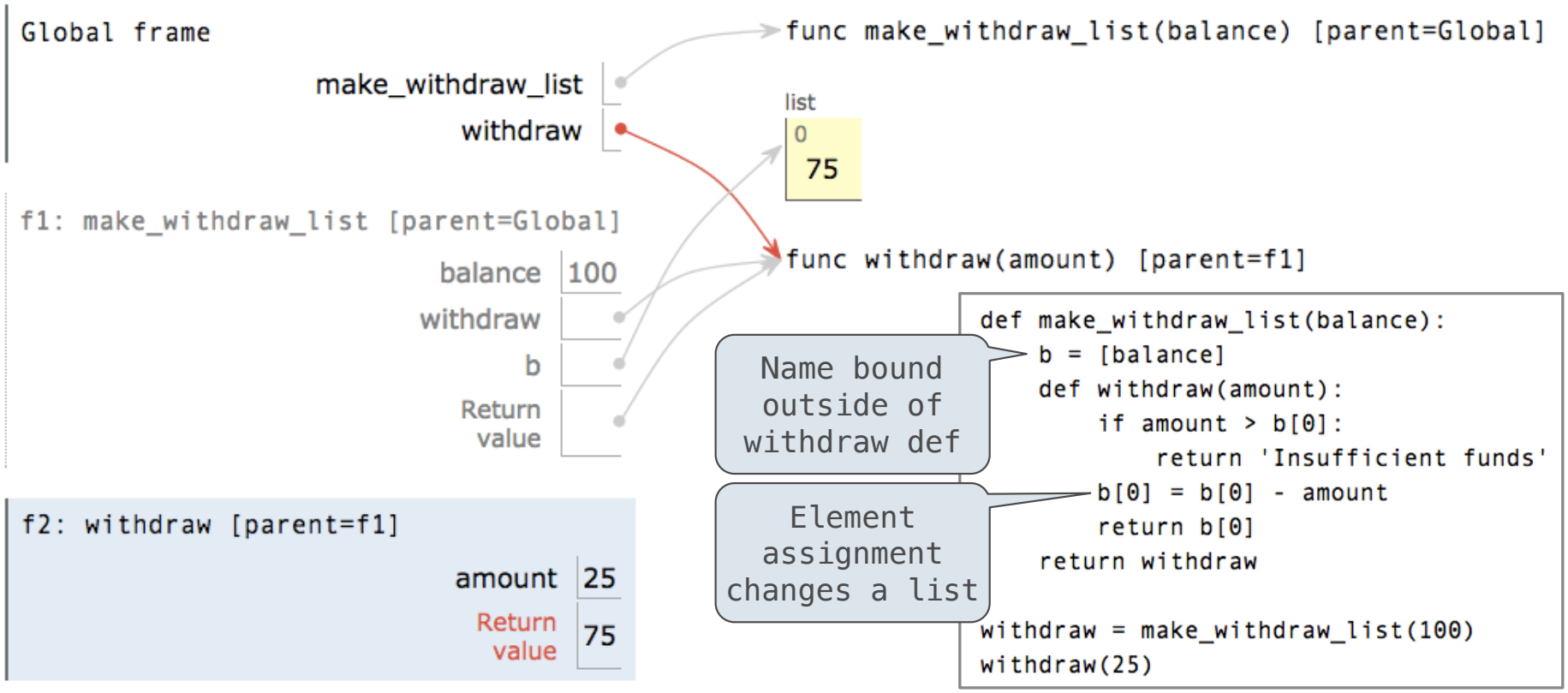

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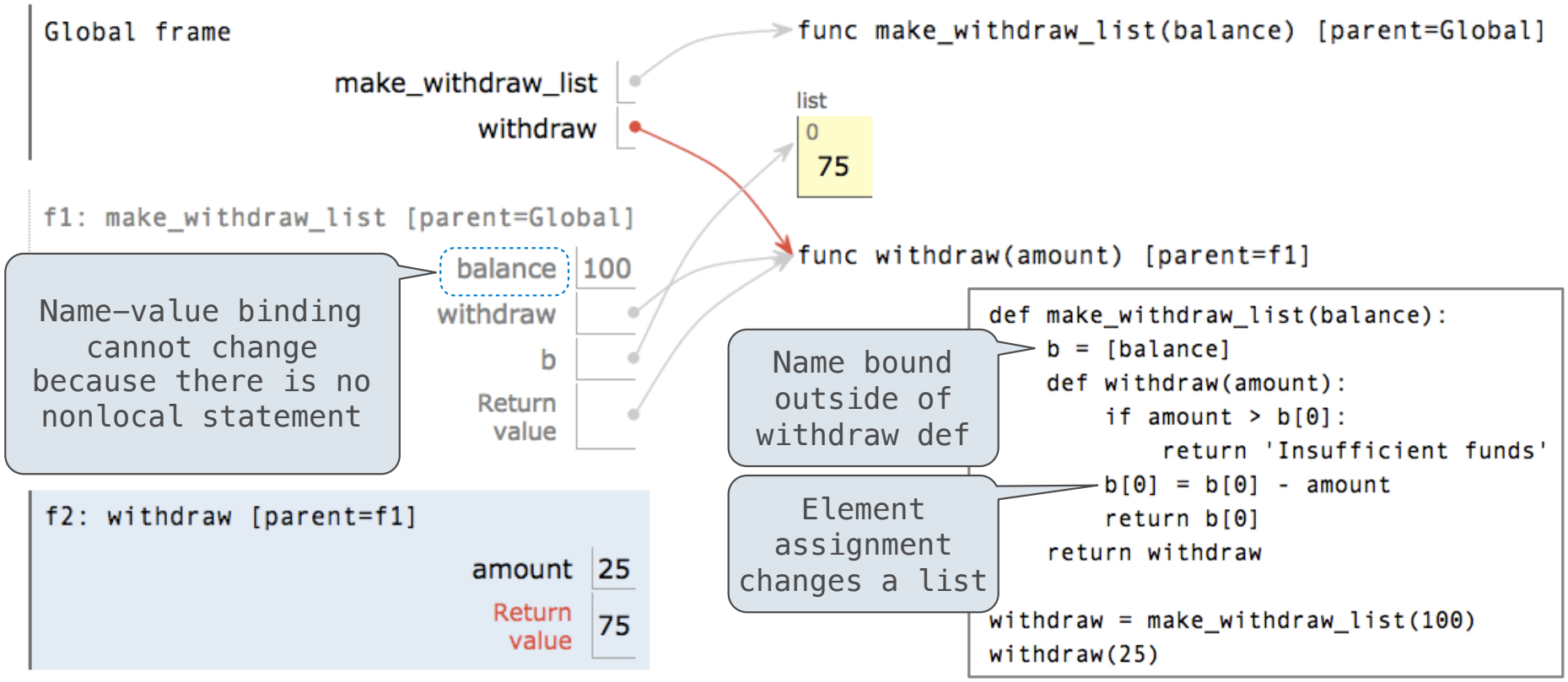

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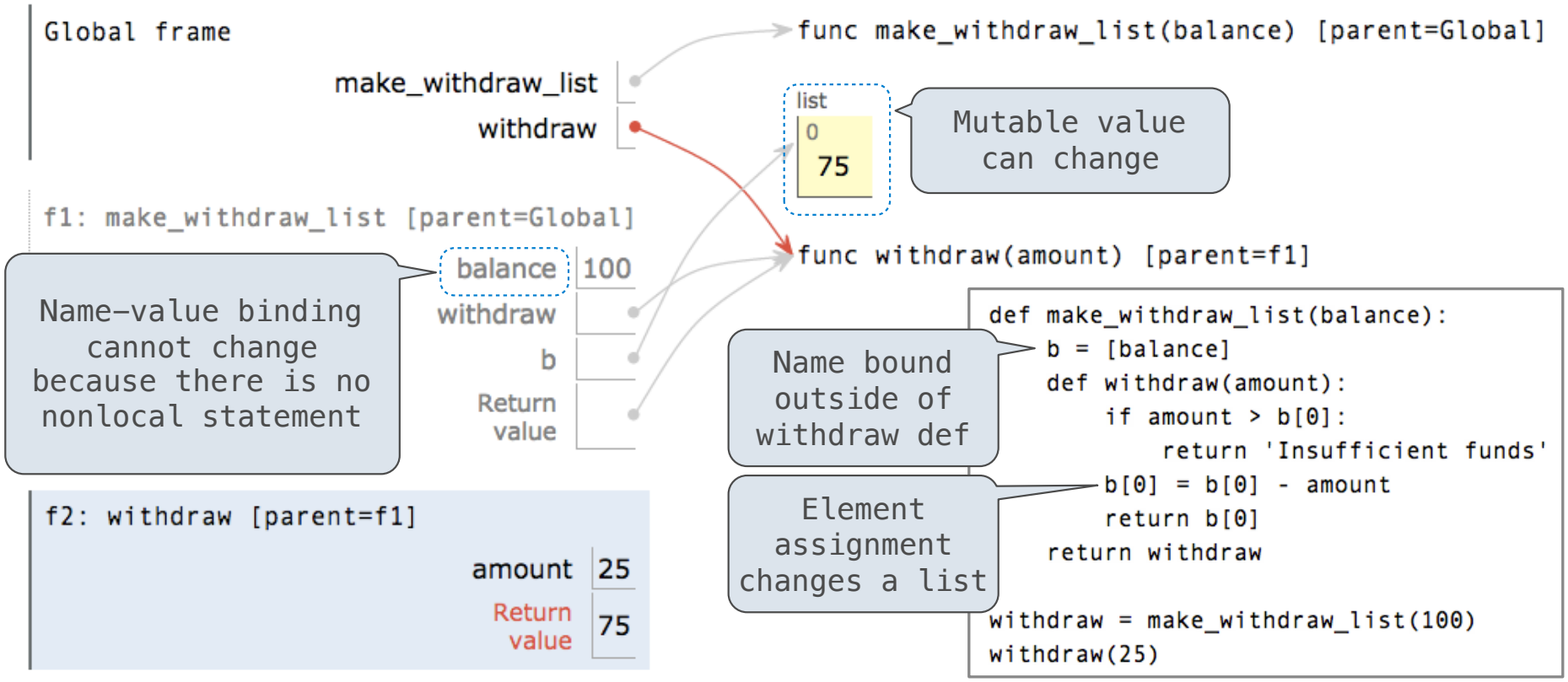

Interactive Diagram

Multiple Mutable Functions

(Demo)

Referential Transparency, Lost

Interactive Diagram

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