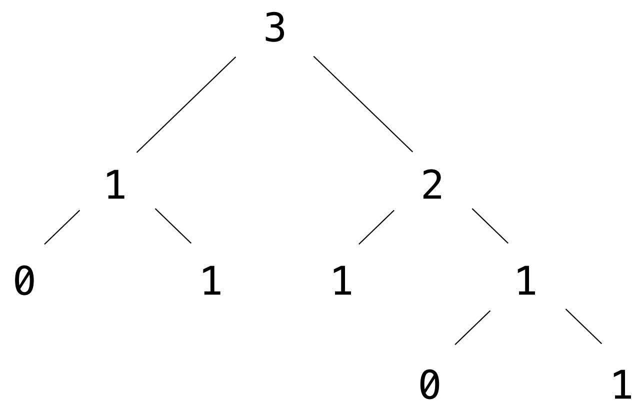
61A Lecture 19

Announcements

Tree Class

Tree Review



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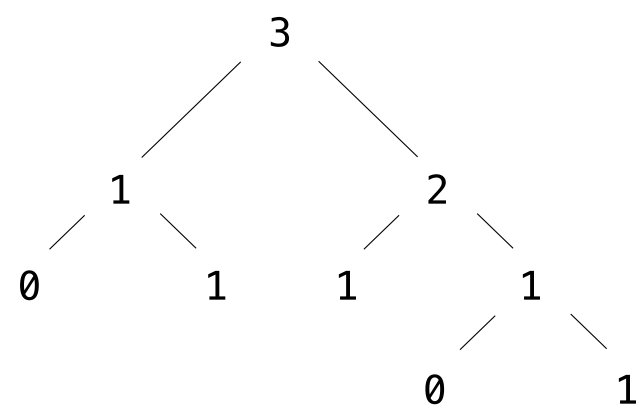

Recursive description (wooden trees):

Relative description (family trees):

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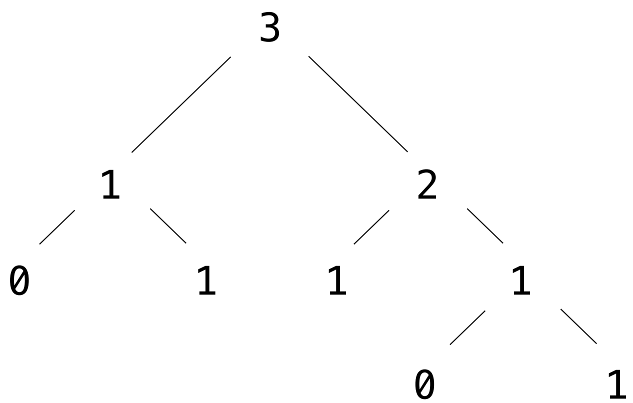

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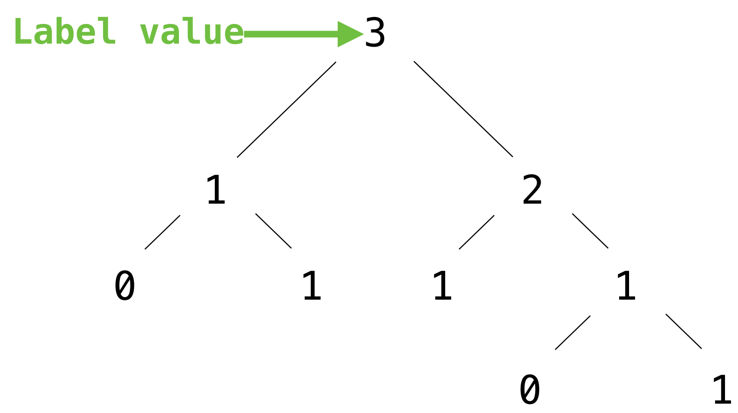

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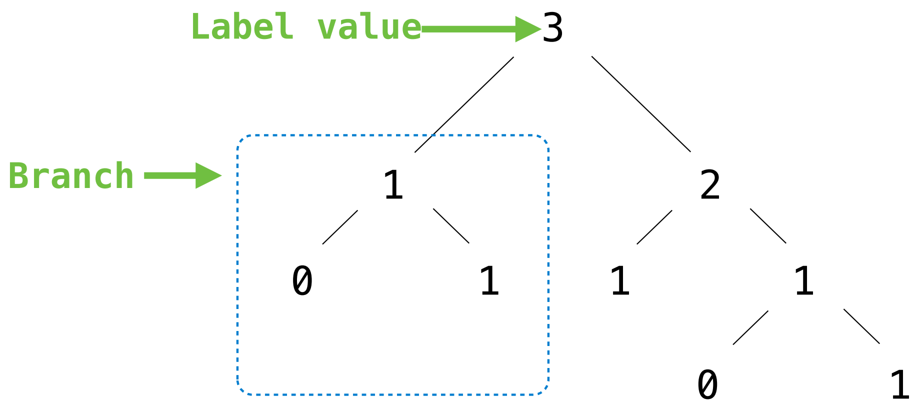

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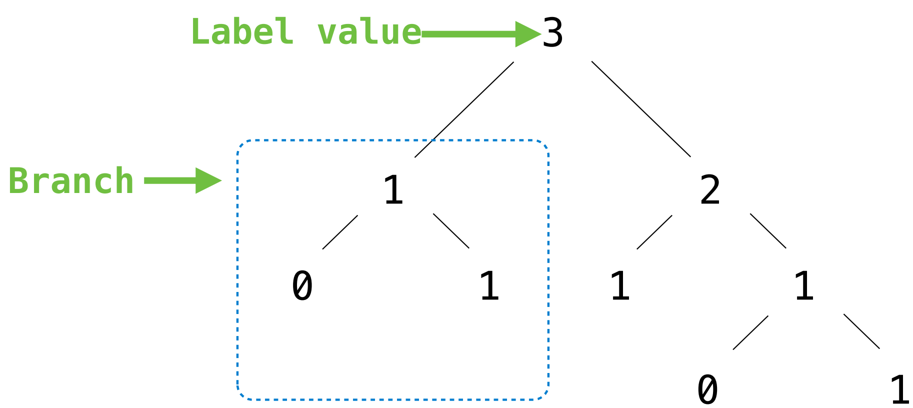

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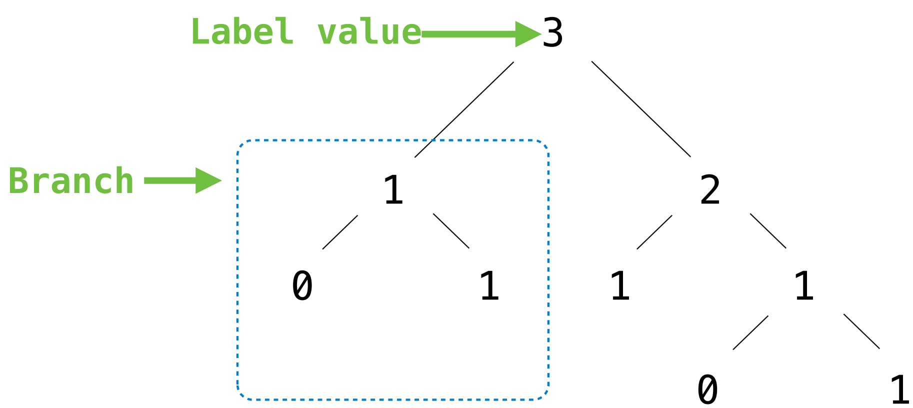

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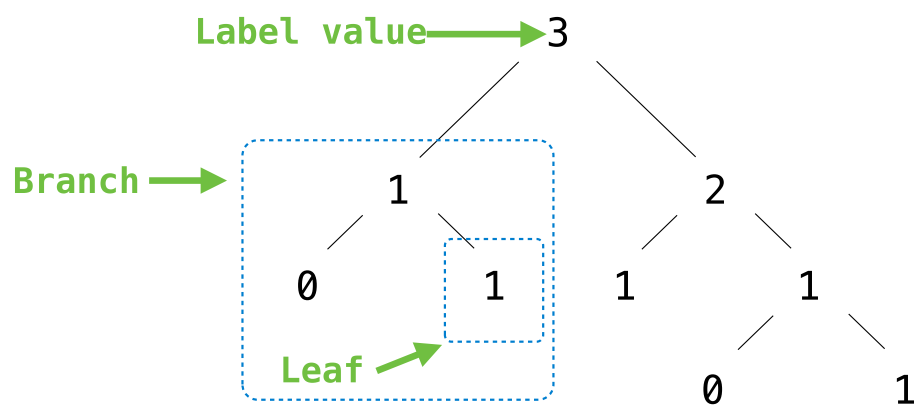

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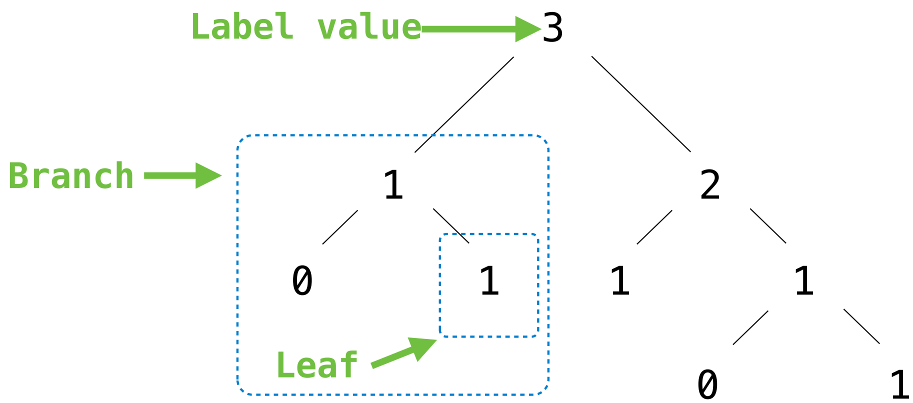

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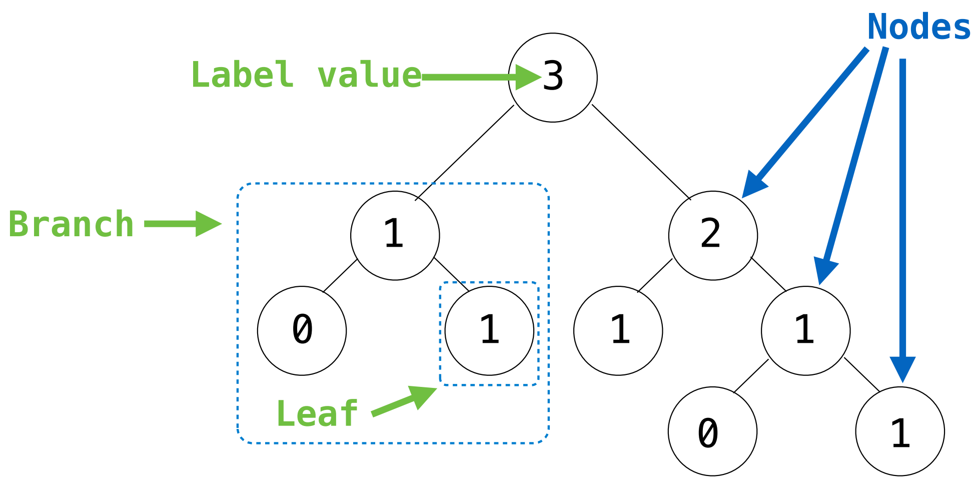

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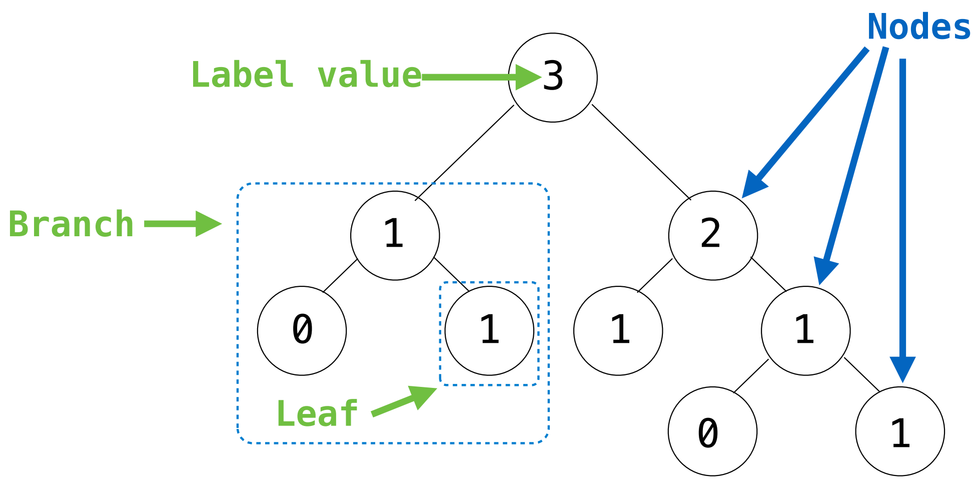

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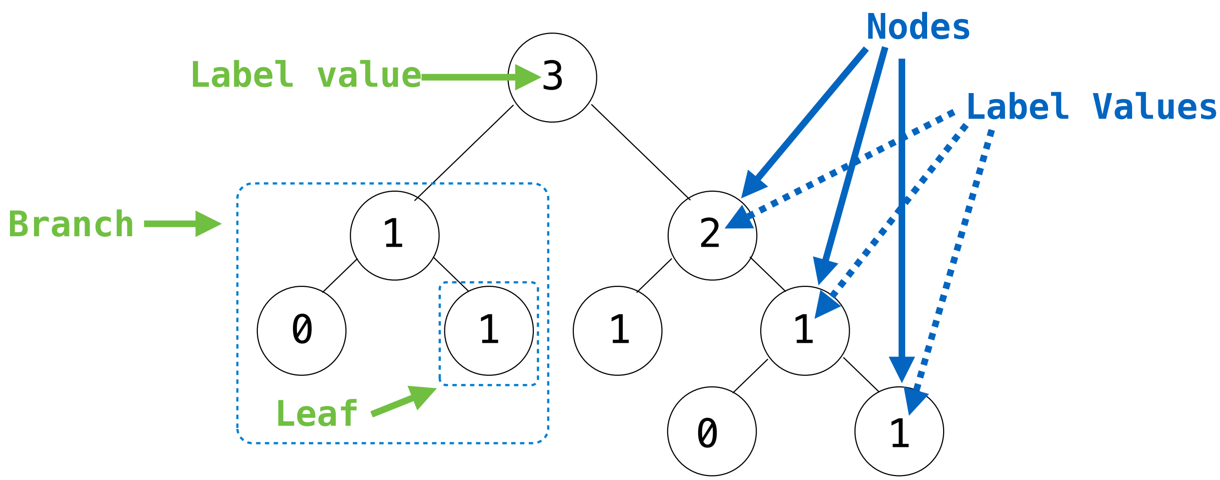

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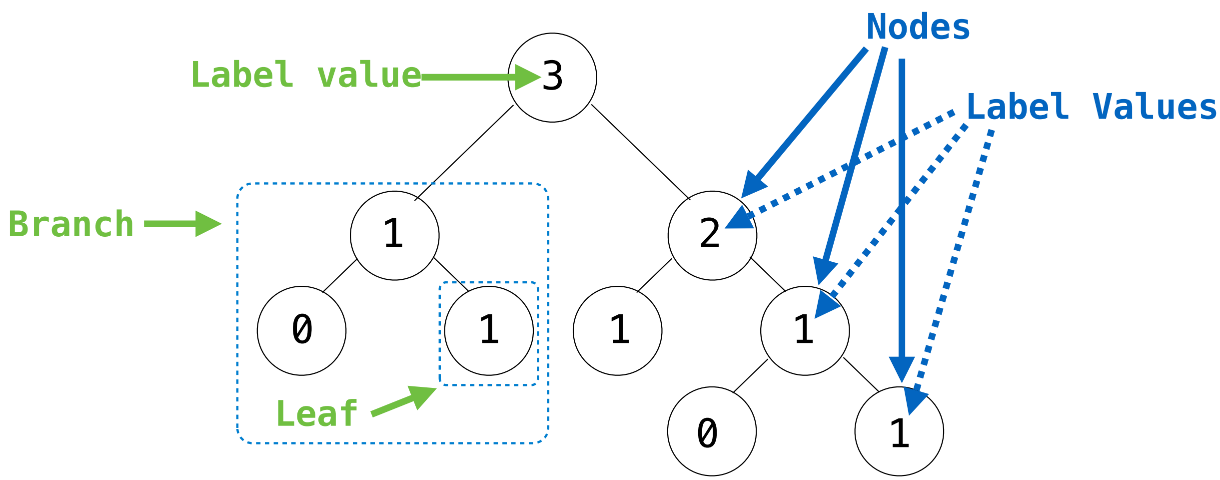

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One node can be the **parent/child** of another

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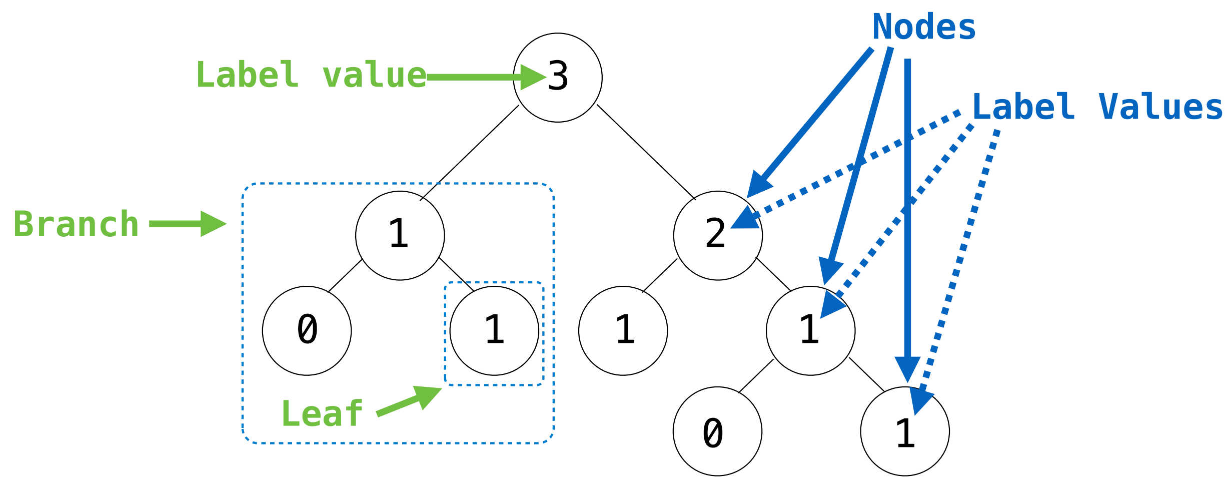

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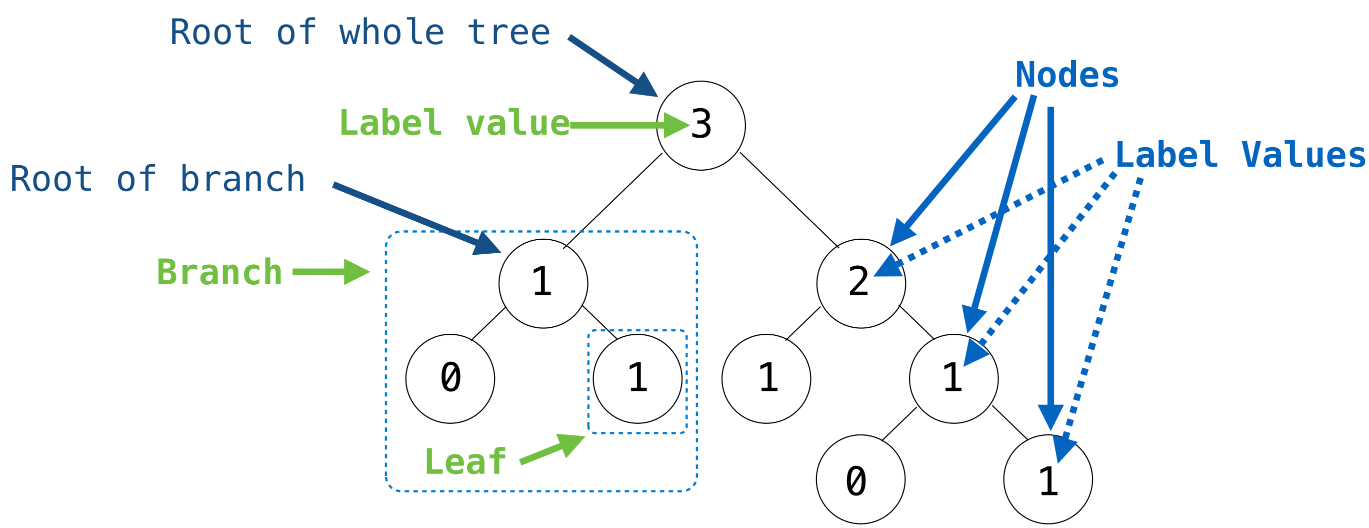

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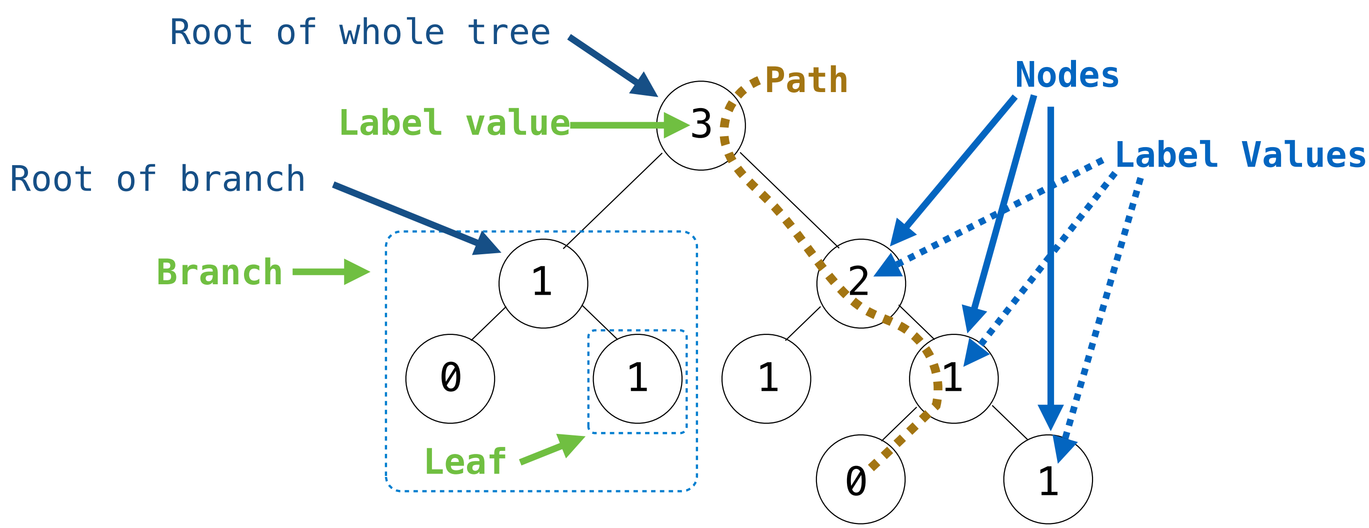

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```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)
def label(tree):
    return tree[0]
def branches(tree):
    return tree[1:]
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def fib_tree(n):
    if n == 0 or n == 1:
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        left = fib_tree(n-2)
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(Demo)

Side Excursion: Equality

If x and y are two objects, the equality test, x == y, does not automatically mean what you want it to mean.

For example, `Tree(4) != Tree(4)` but after performing x = Tree(4), we do have `x == x`

The reason for this is that in Python,

- All values (conceptually, at least) are in fact *pointers* to objects, and
- By default, == on pointers compares the pointers themselves ("are these pointing at exactly the same object?").
- That is, by default == and != are the same as the **is** and **is not** operators.
- That can be changed on a class-by-class basis. For example, == on numbers, lists, tuples, strings, sets, and dictionaries means what we expect: the *contents* are the same.

Tree Mutation

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Prune branches before recursive processing

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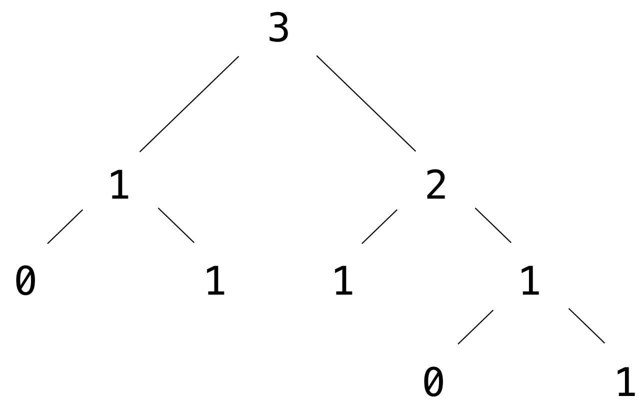

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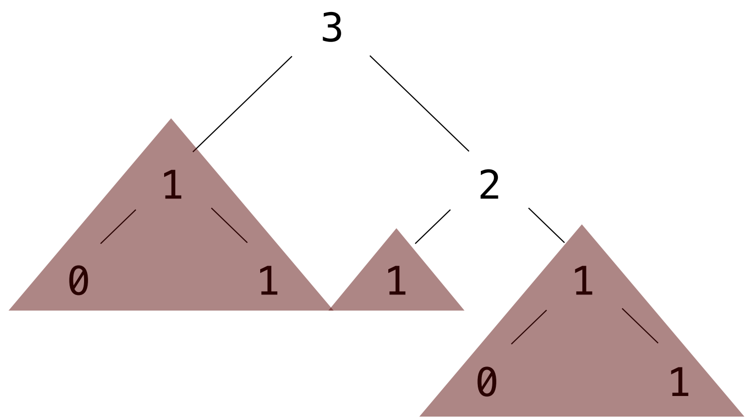

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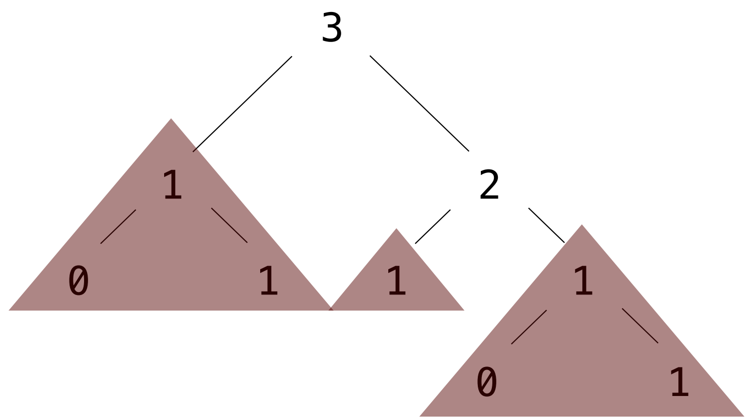


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def prune(t, n):
    """Prune sub-trees whose label value is n."""
    t.branches = [______________ for b in t.branches if ____________________]
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        prune(_______________________________, _______________________________)
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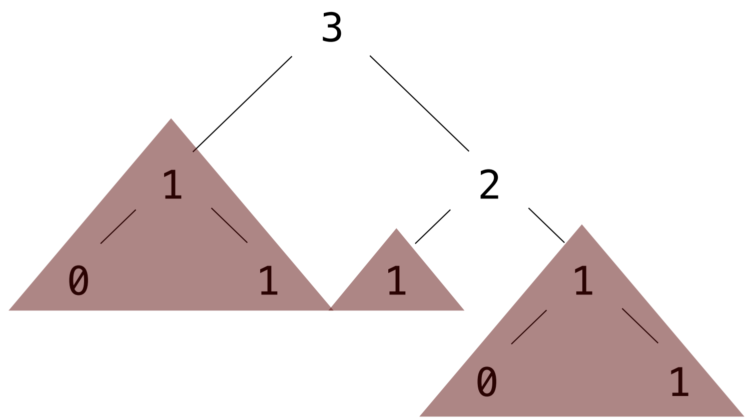


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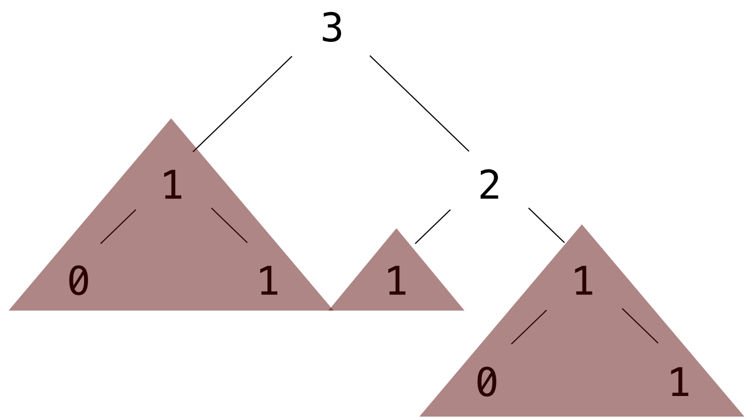


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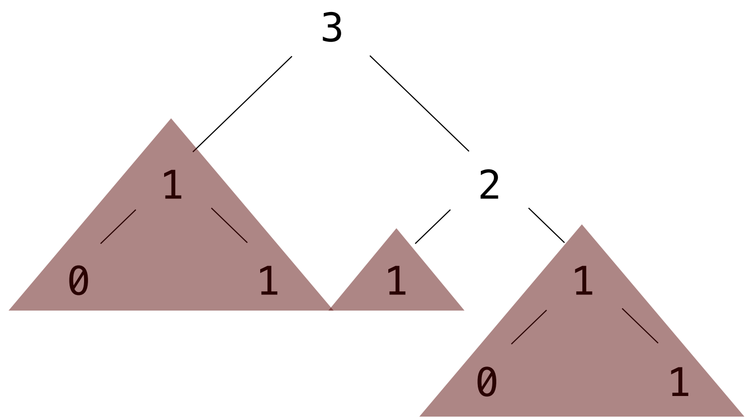


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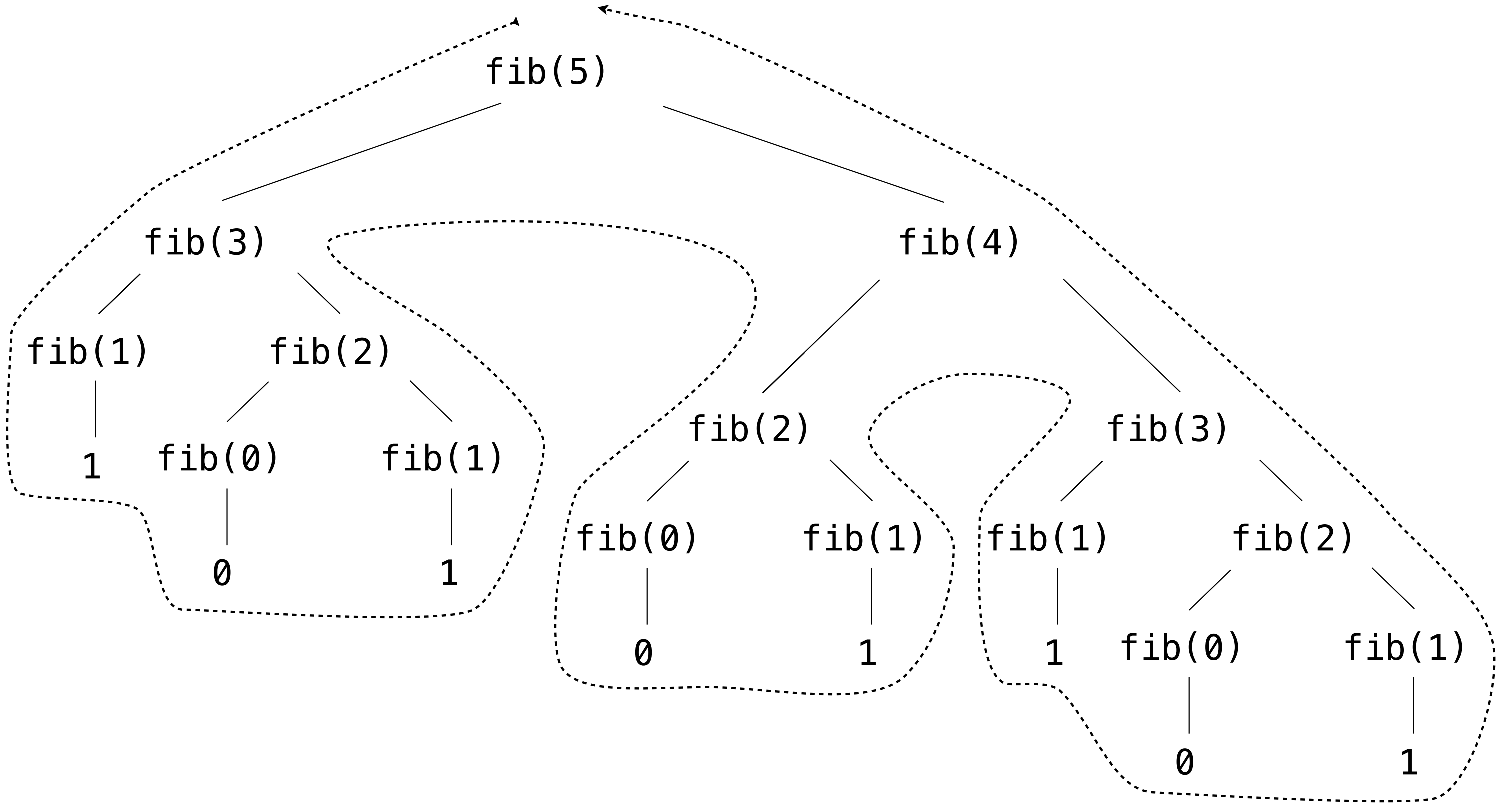

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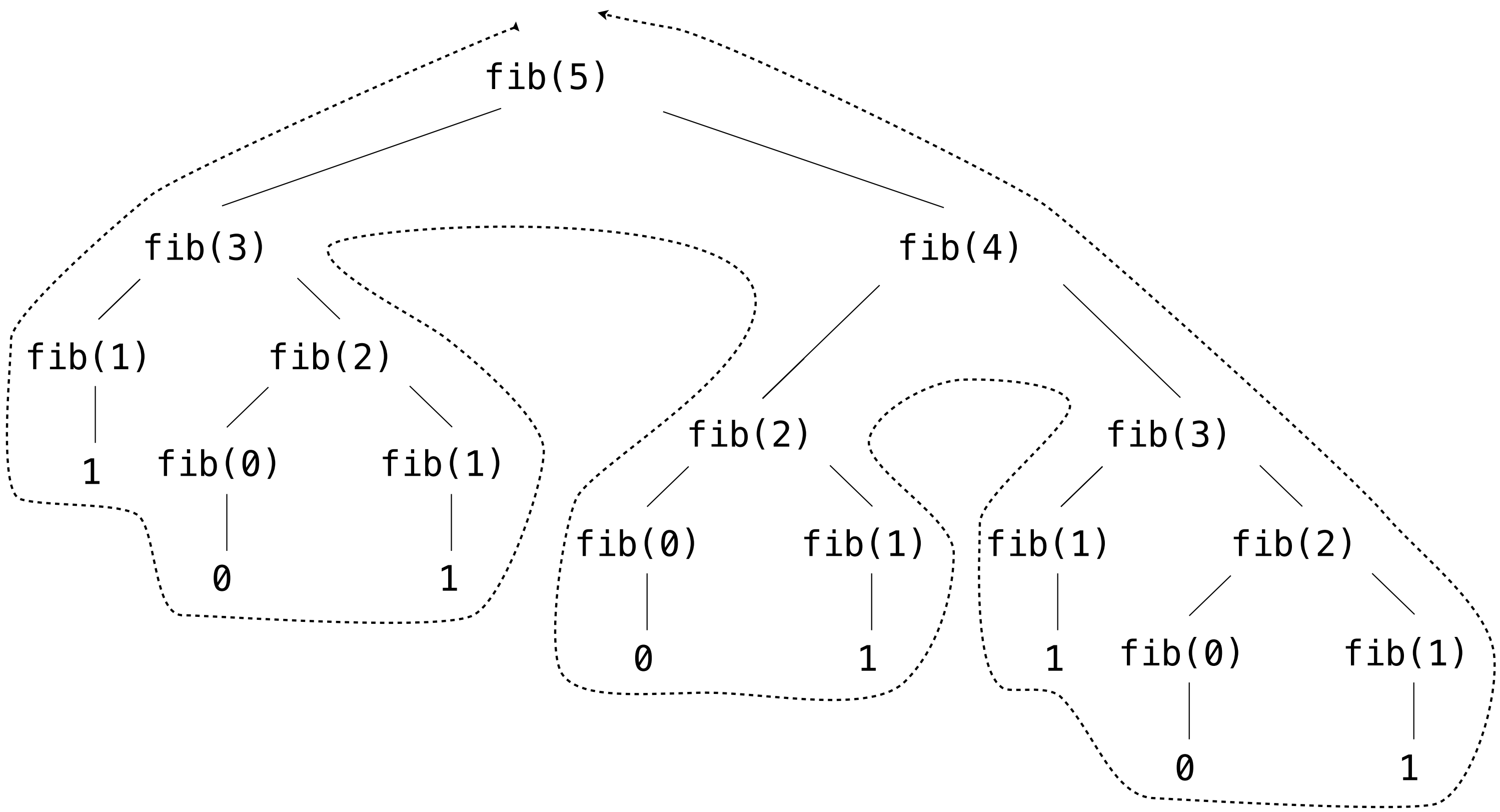

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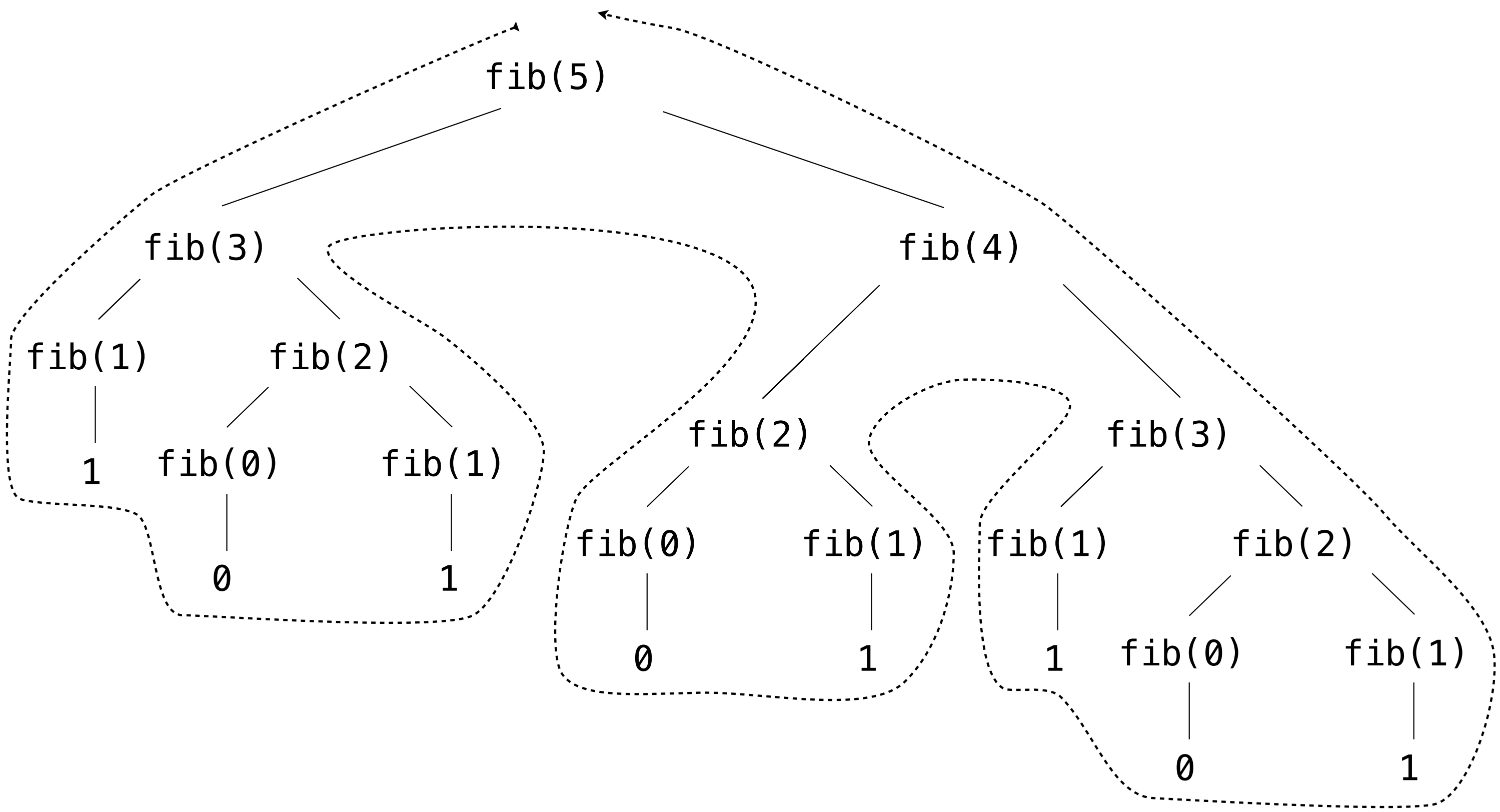

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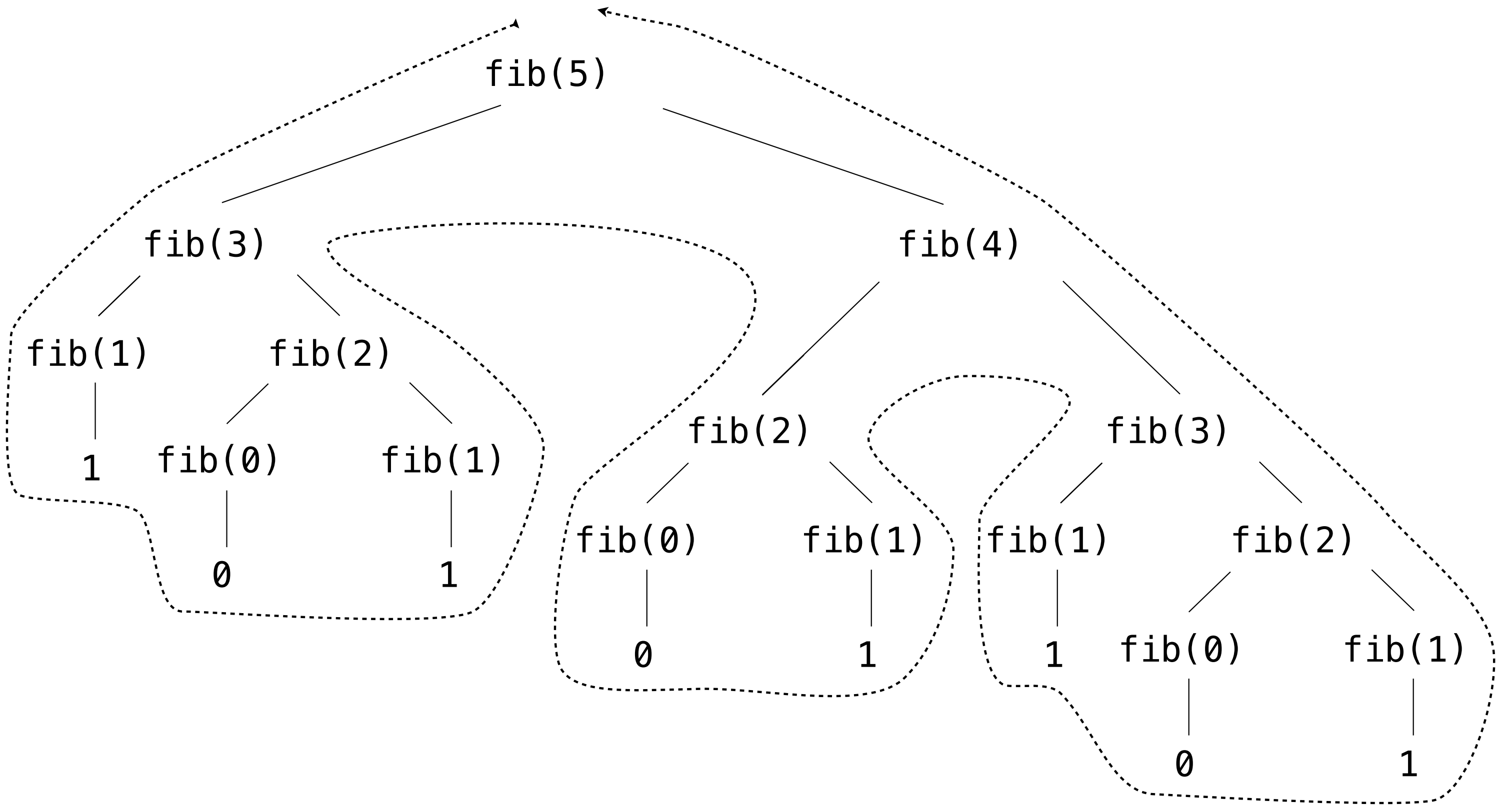

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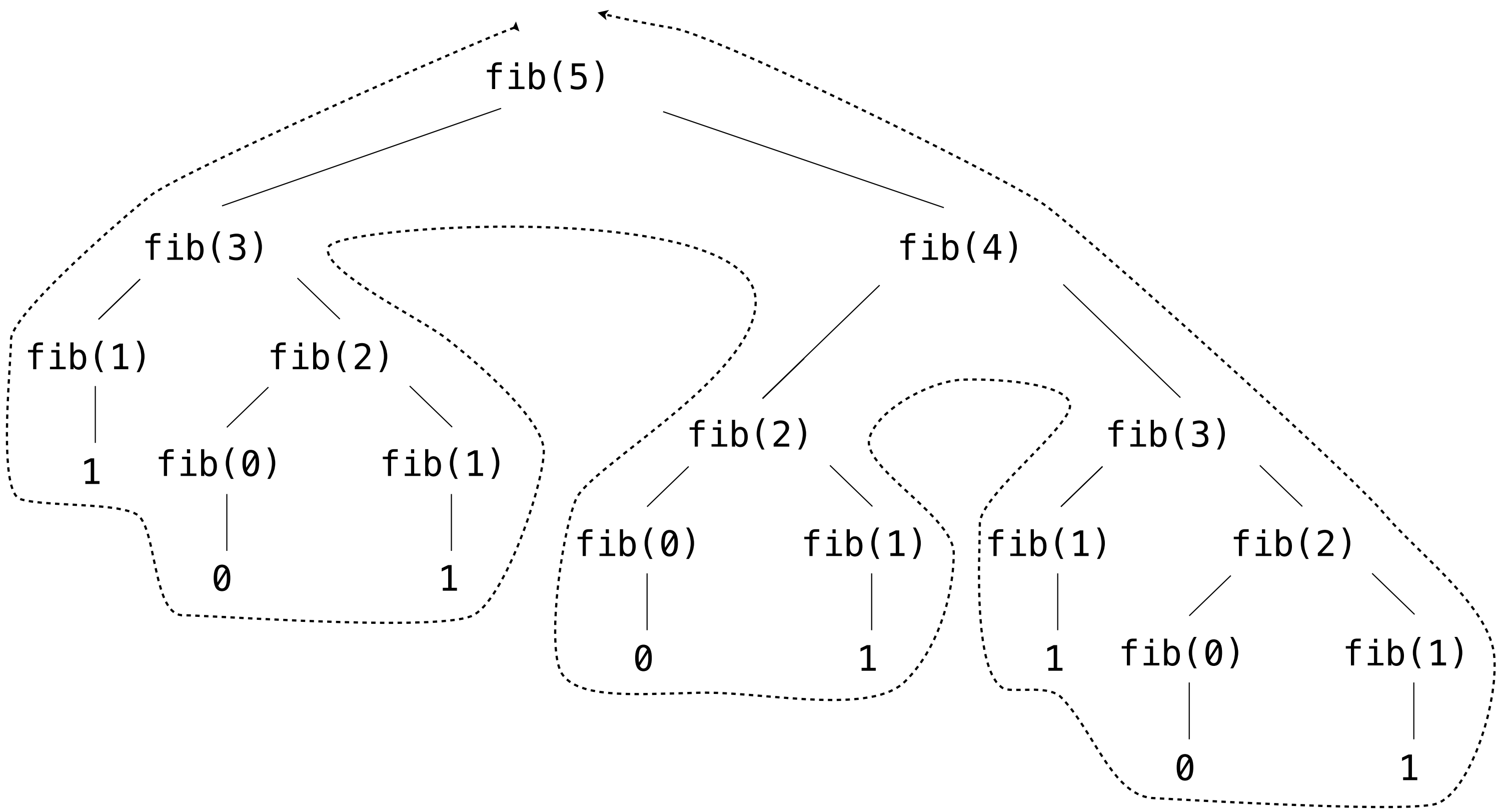

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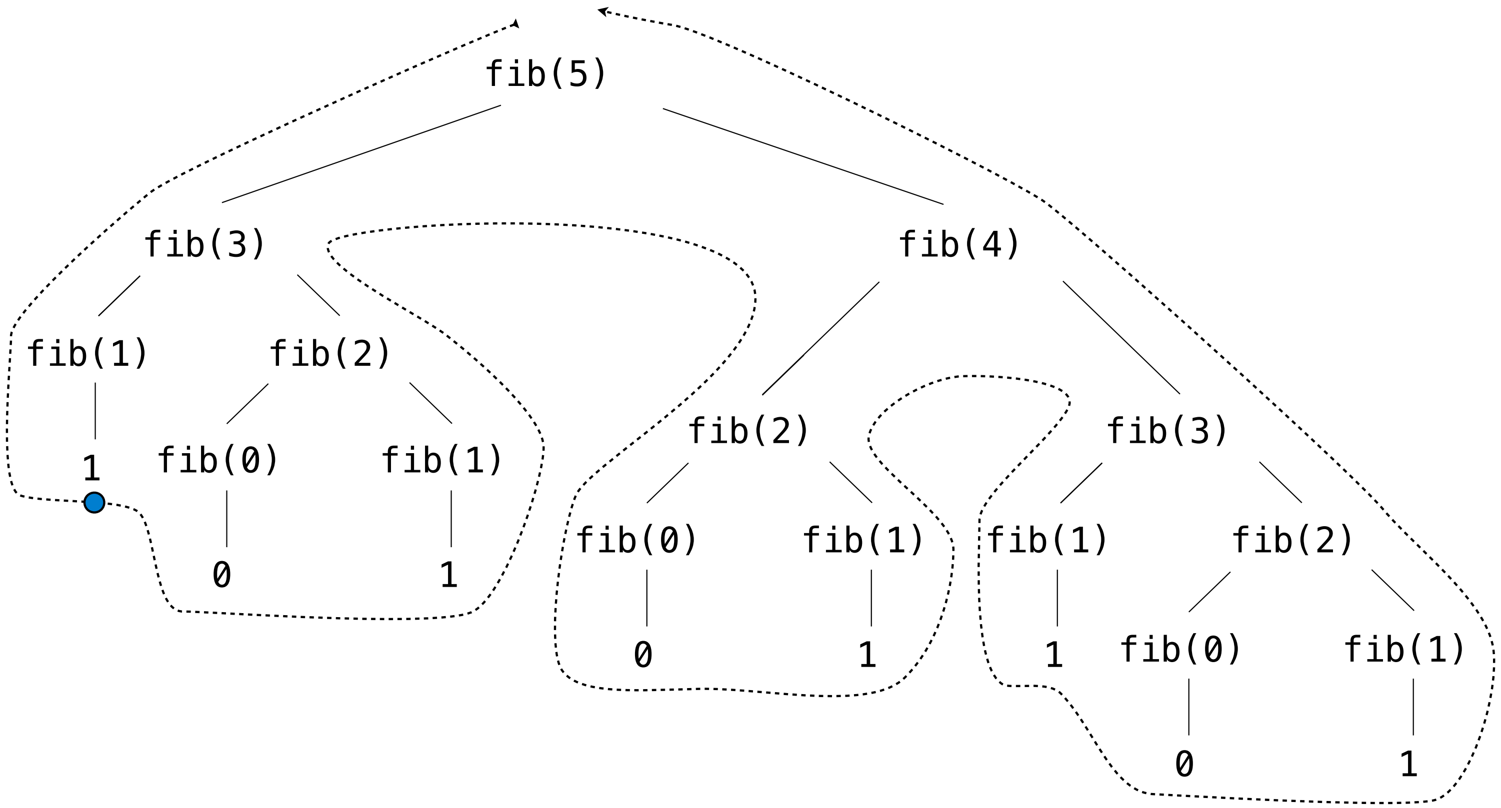

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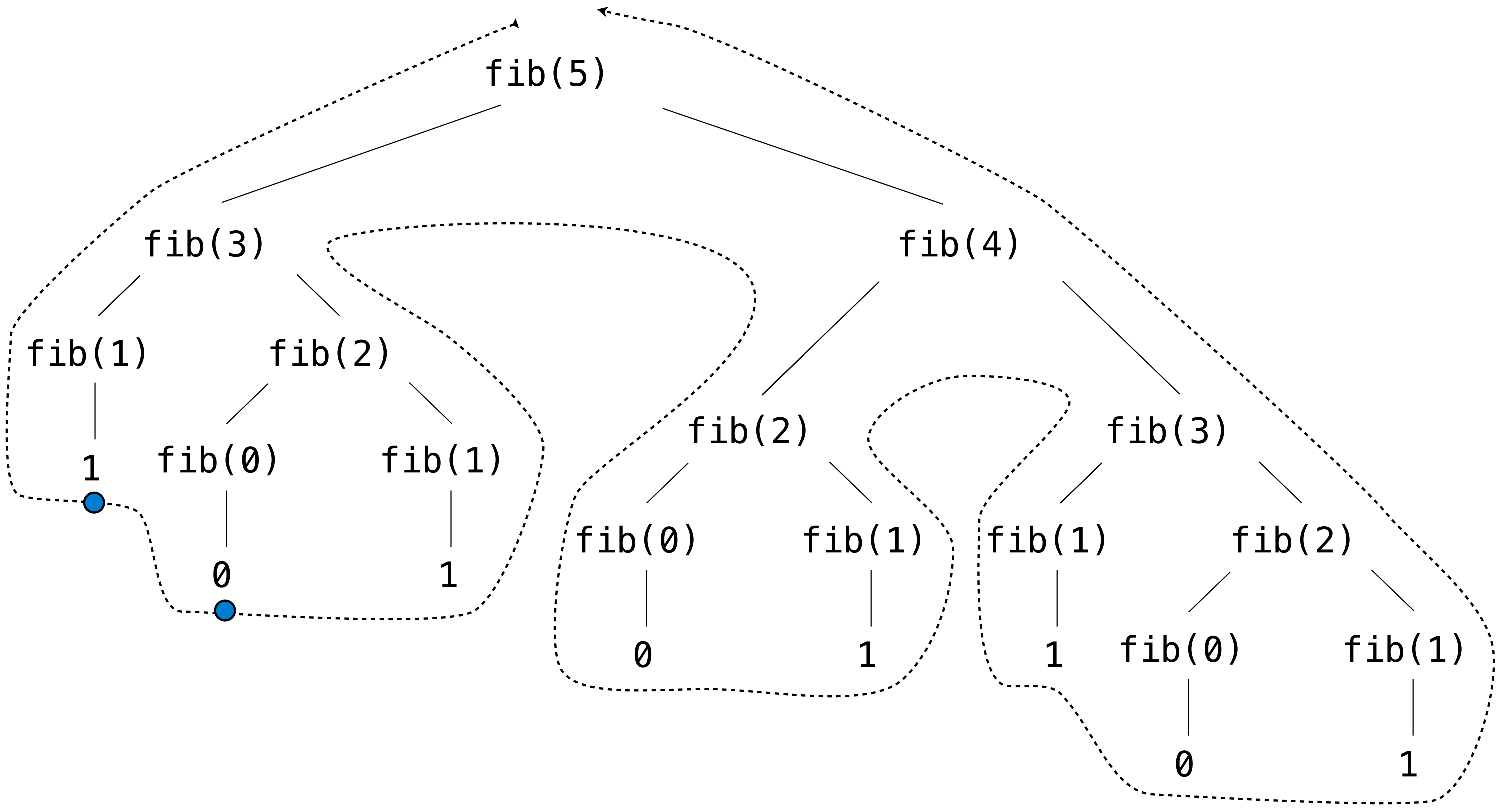

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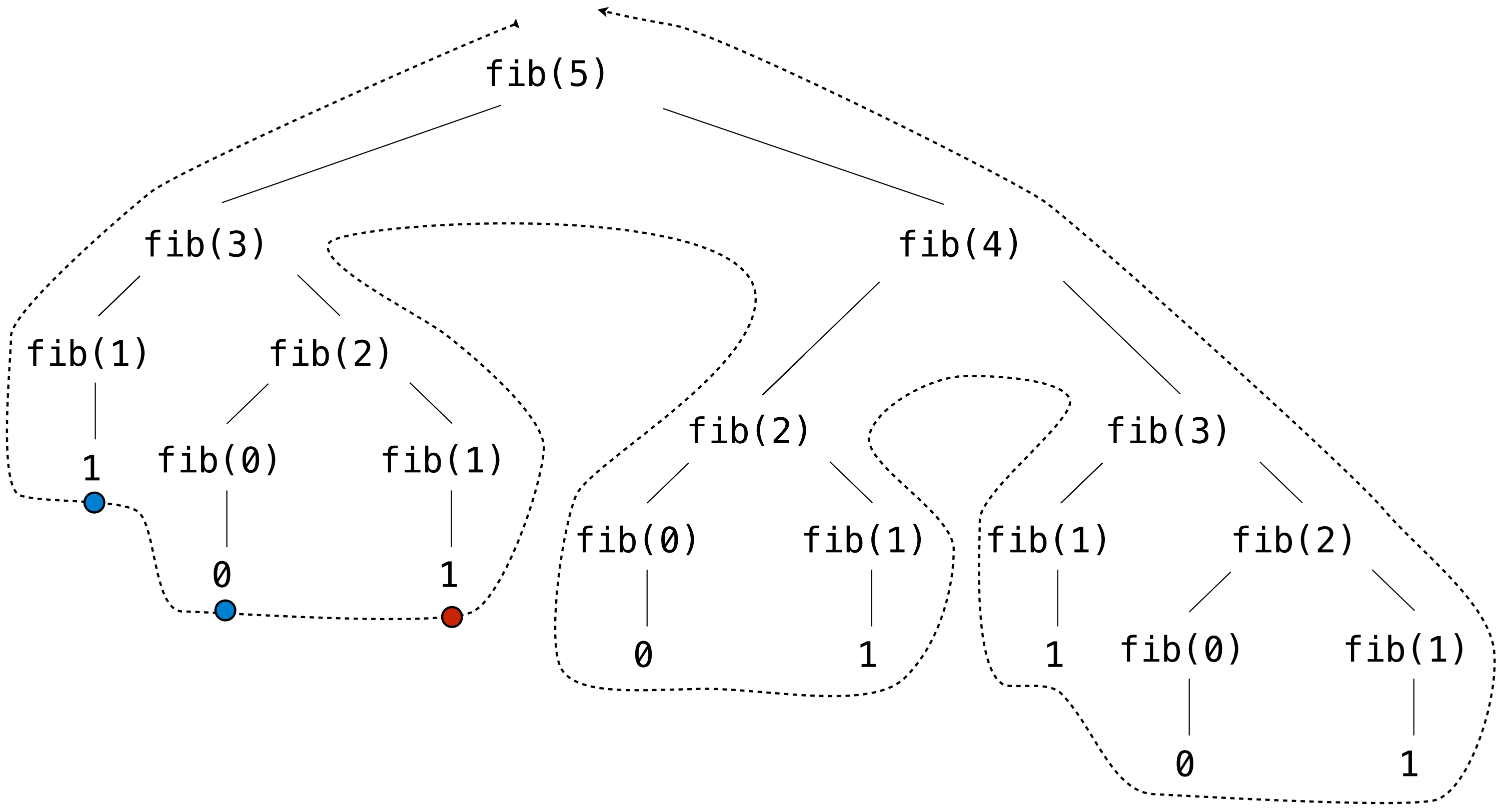

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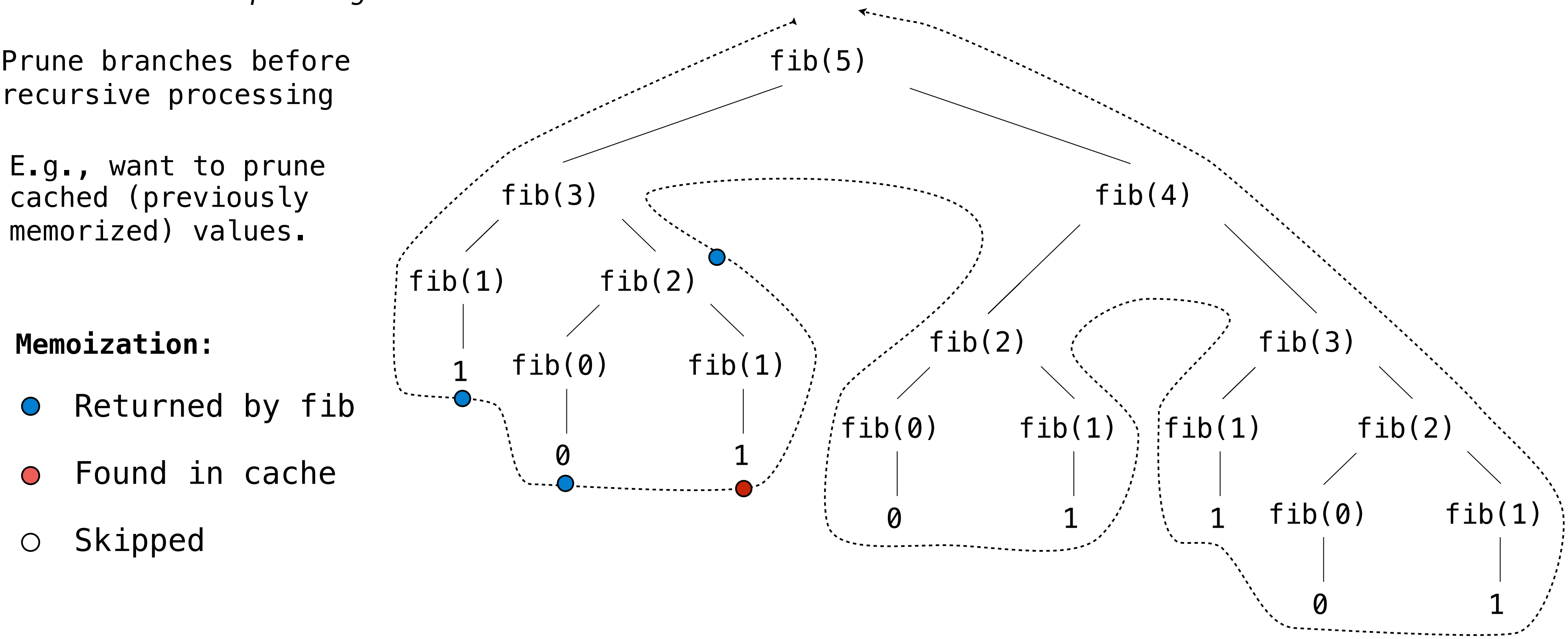

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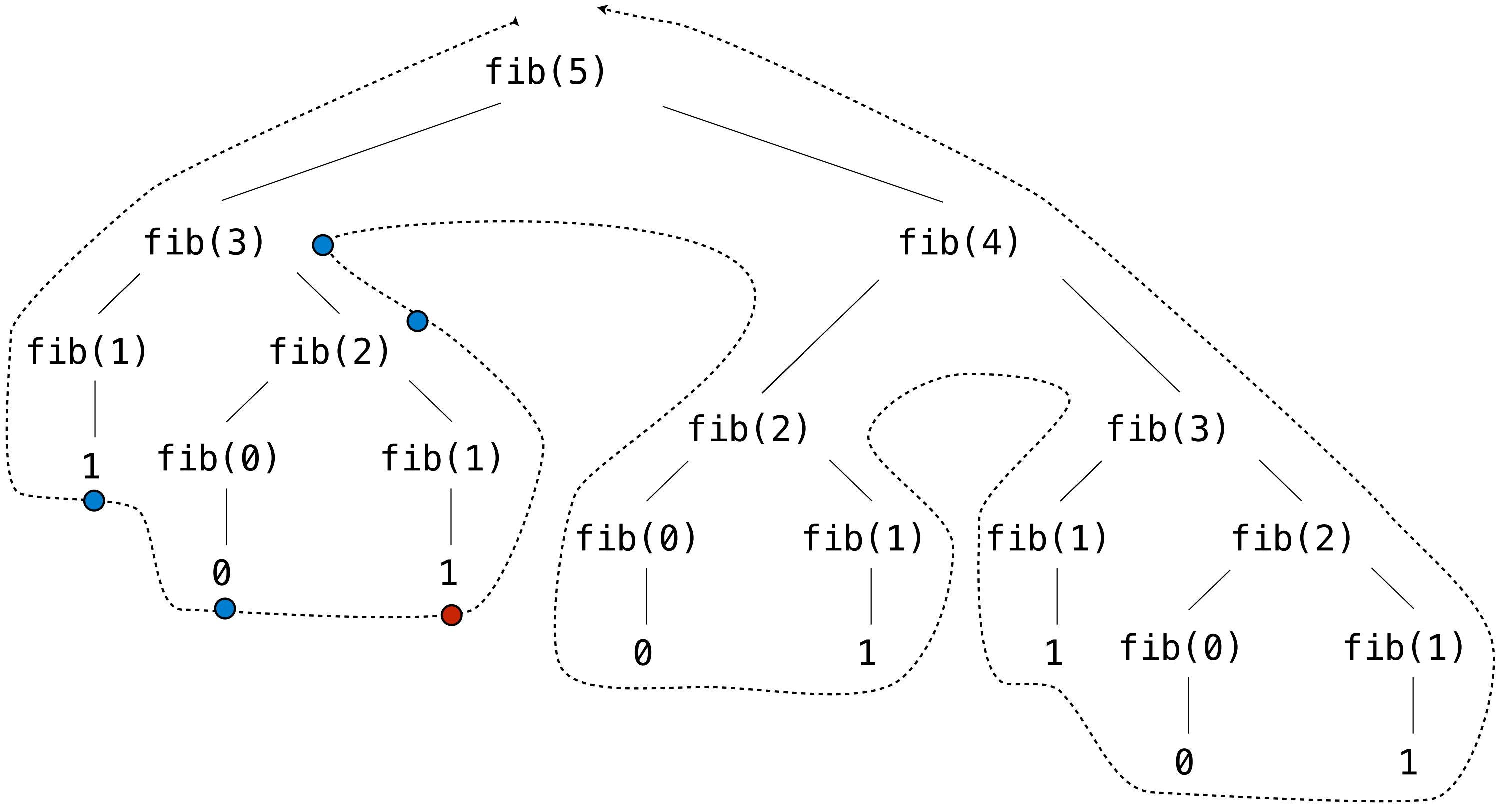

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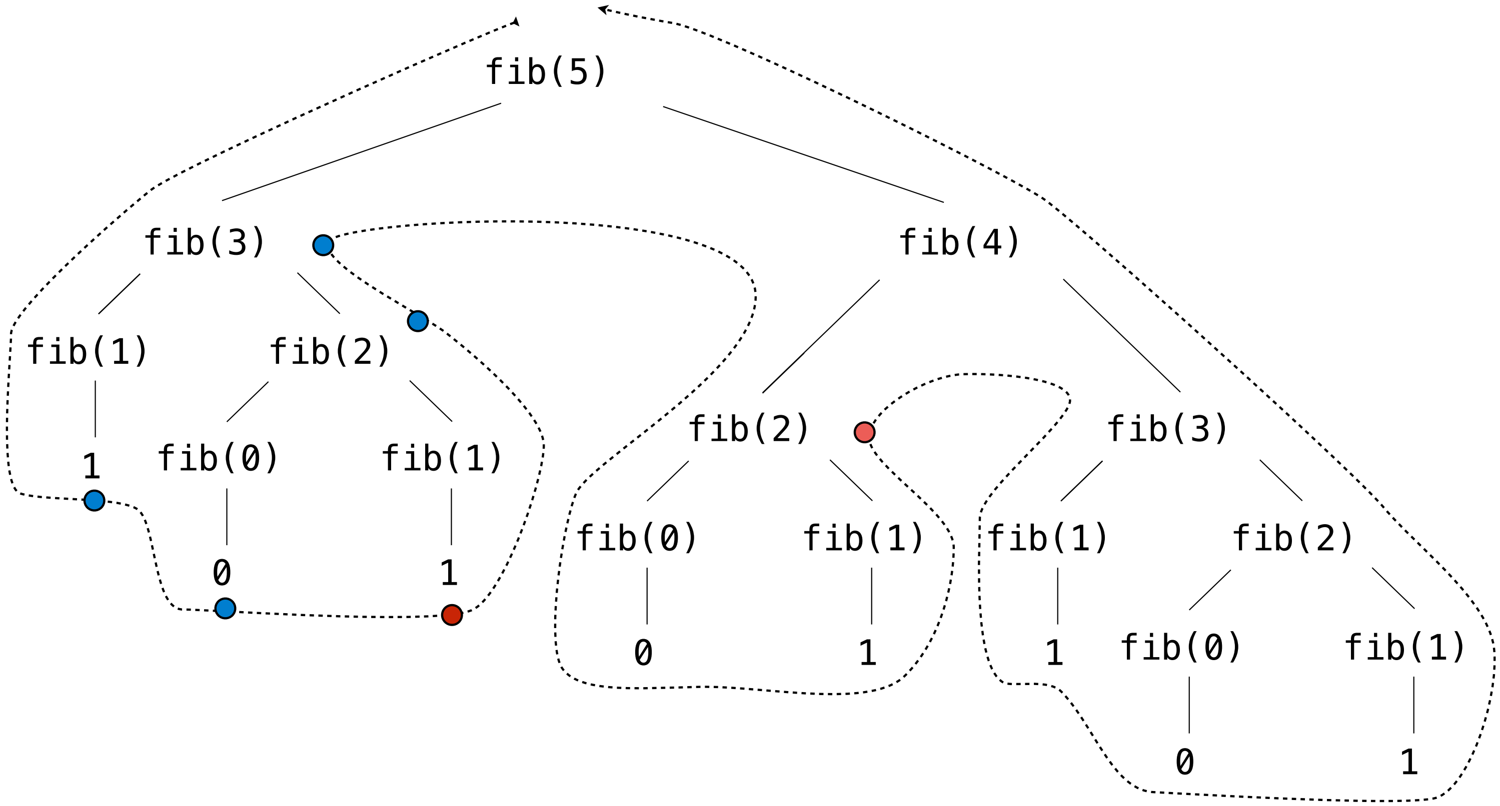

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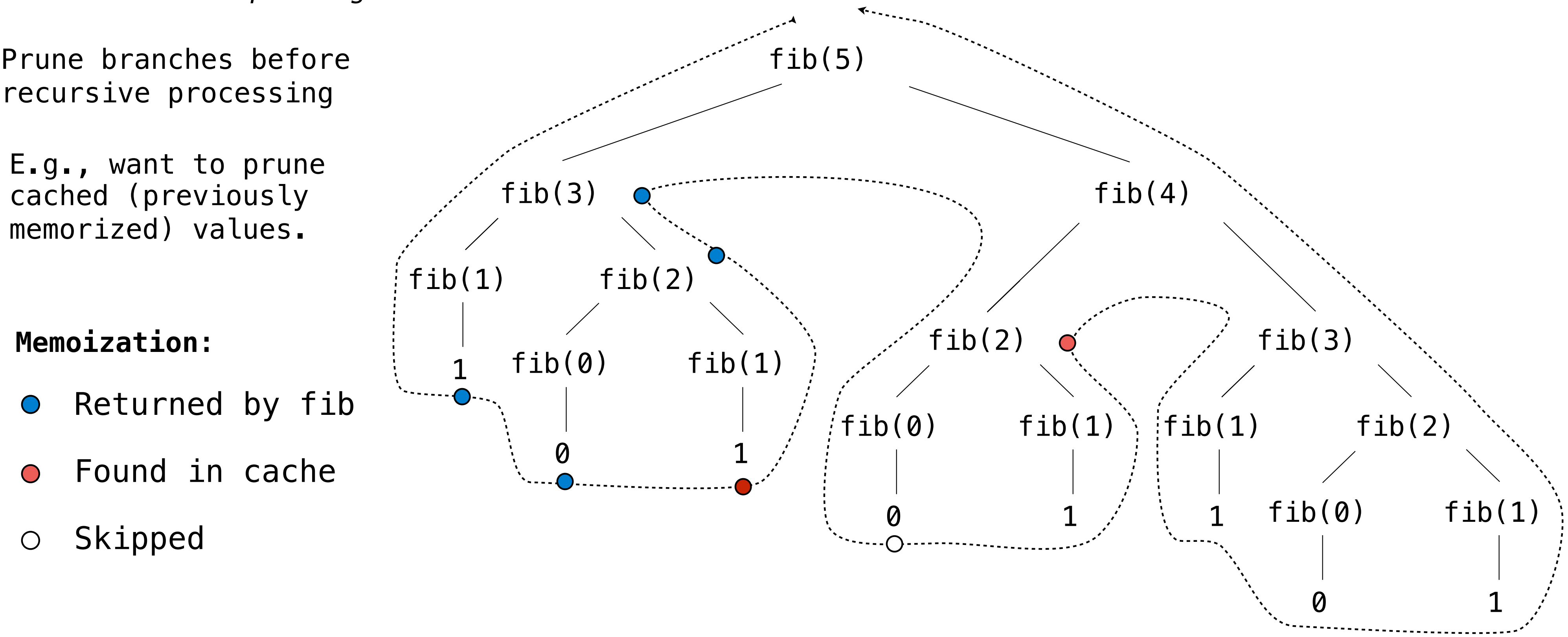

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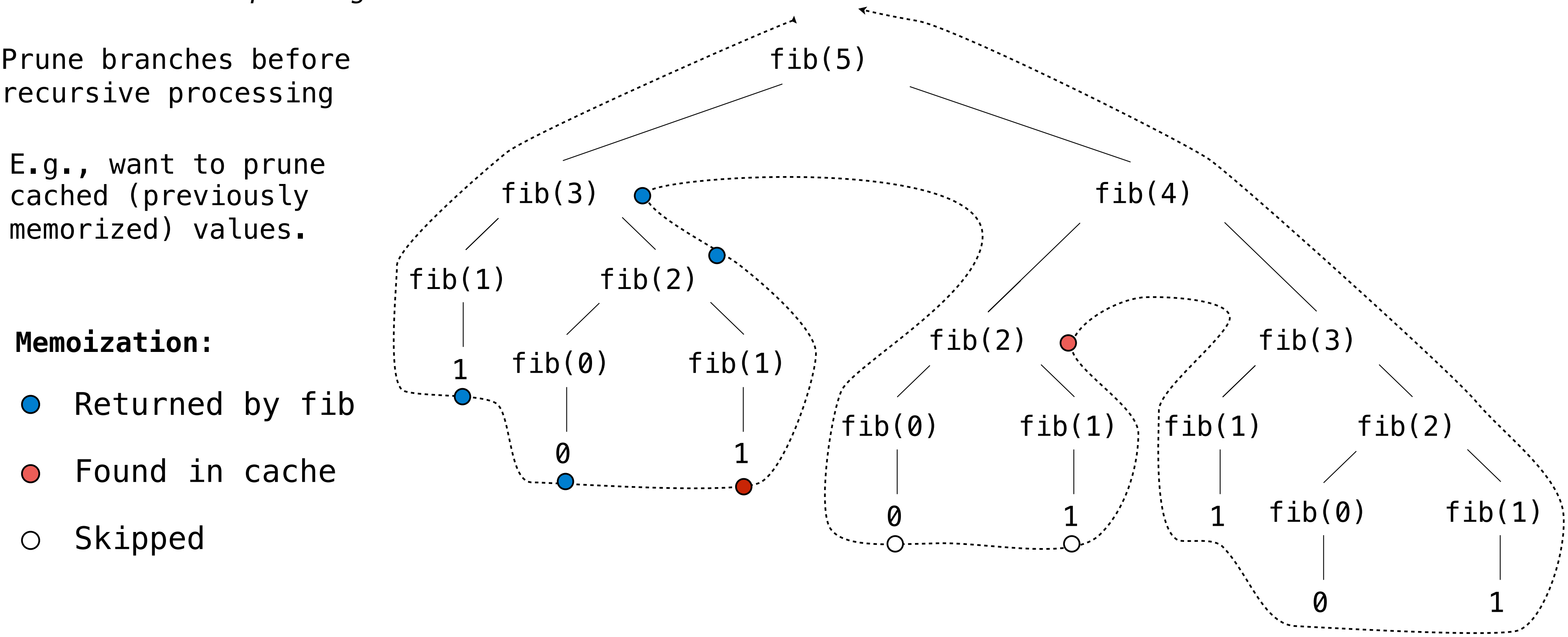

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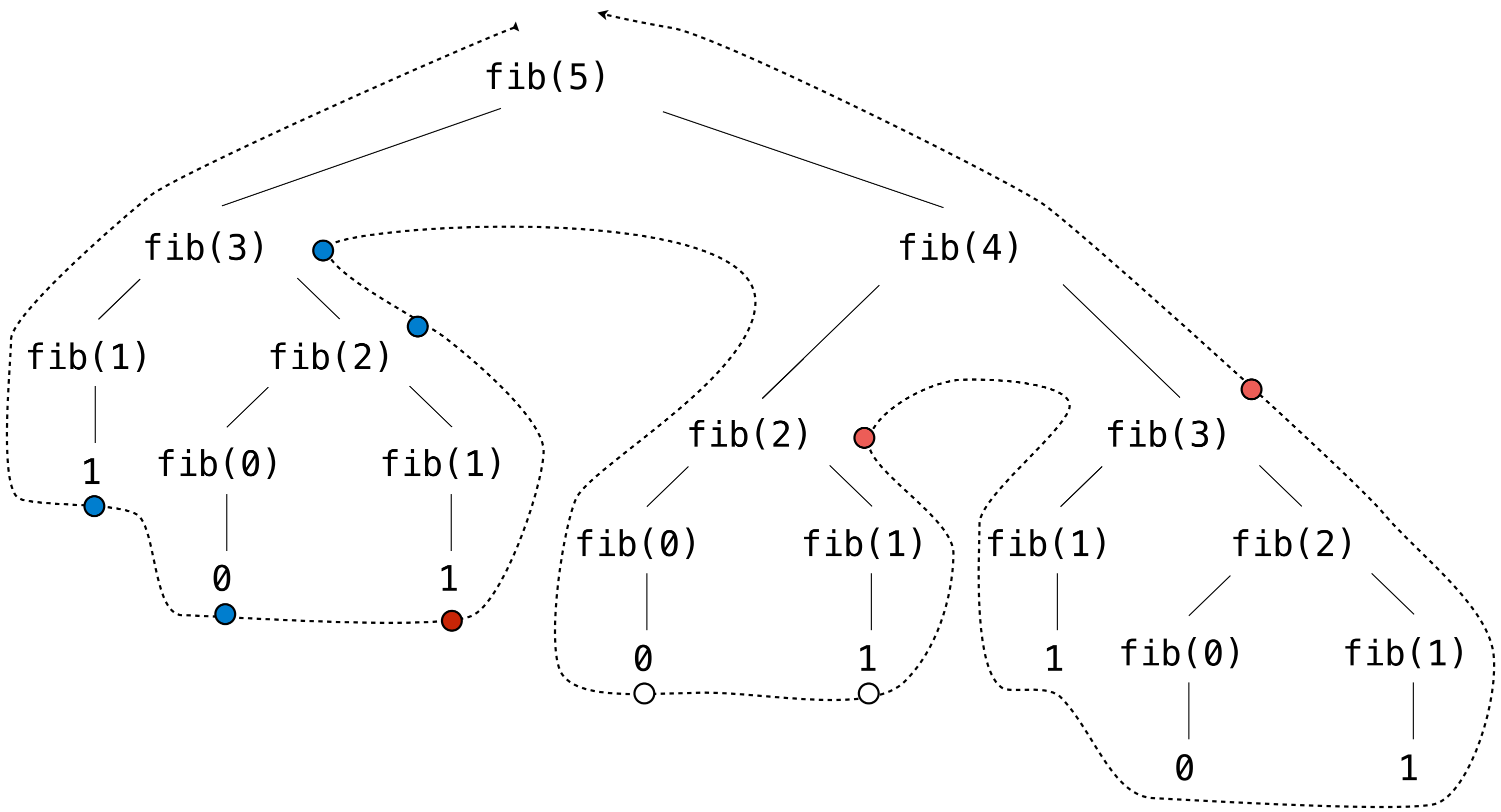

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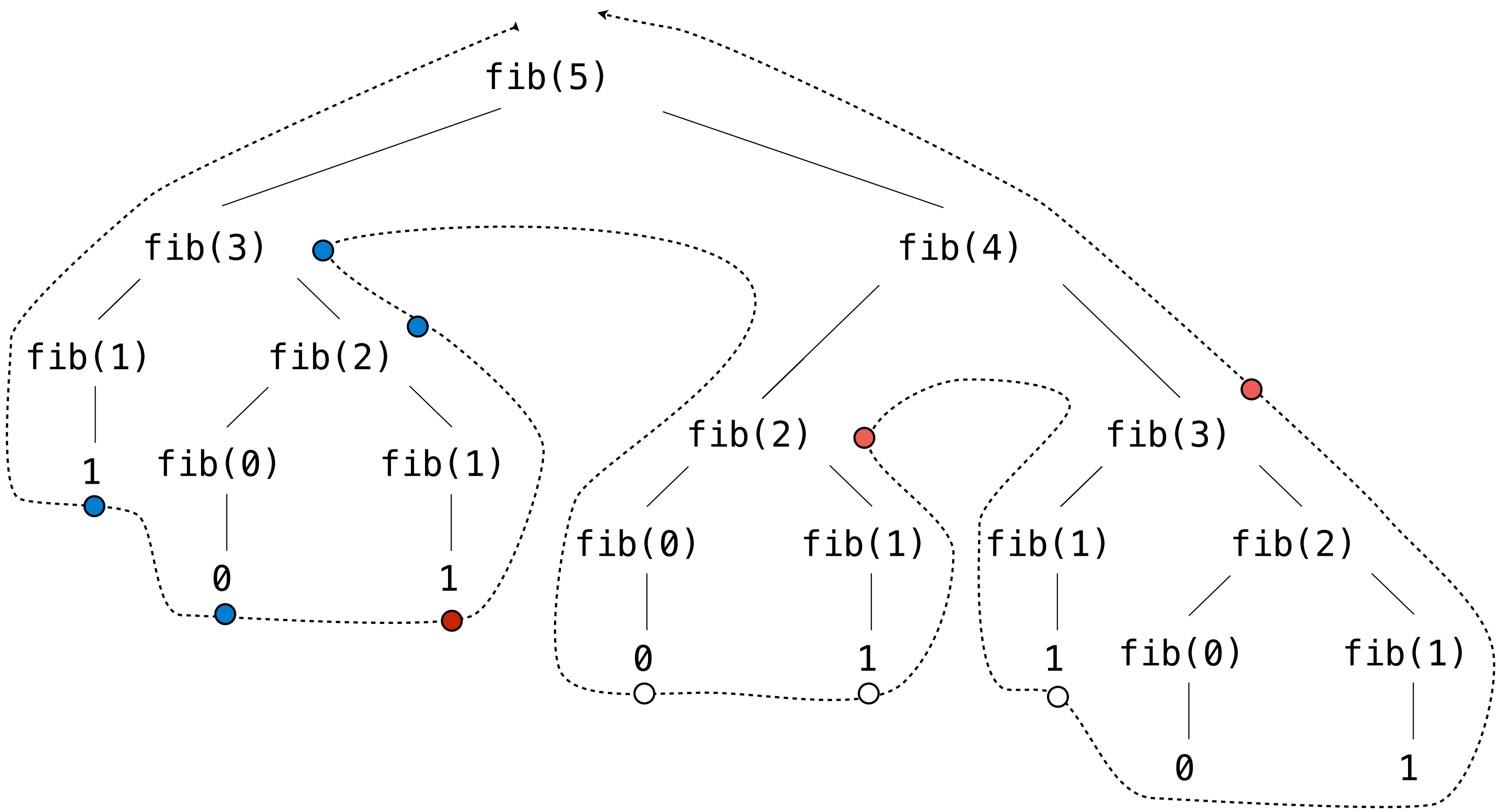
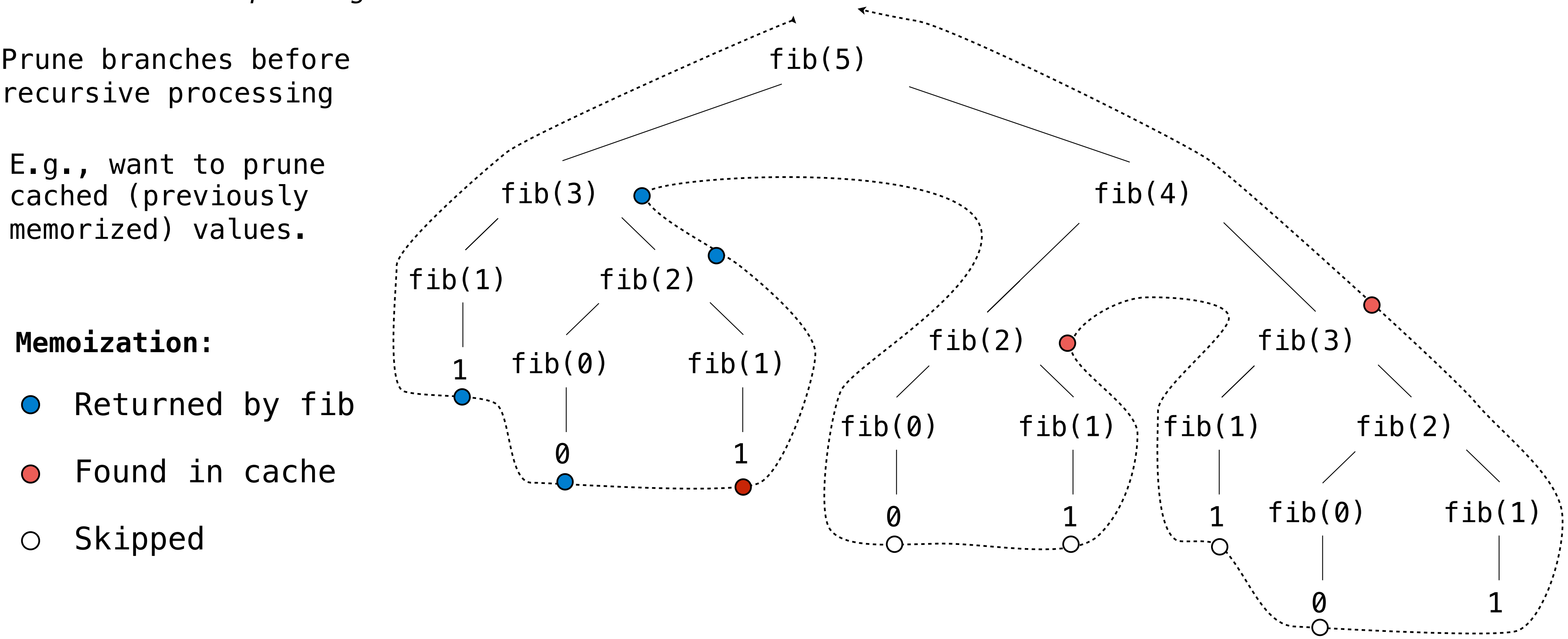


Example: Pruning Trees

Removing subtrees from a tree is called *pruning*

Prune branches before recursive processing

E.g., want to prune cached (previously memorized) values.



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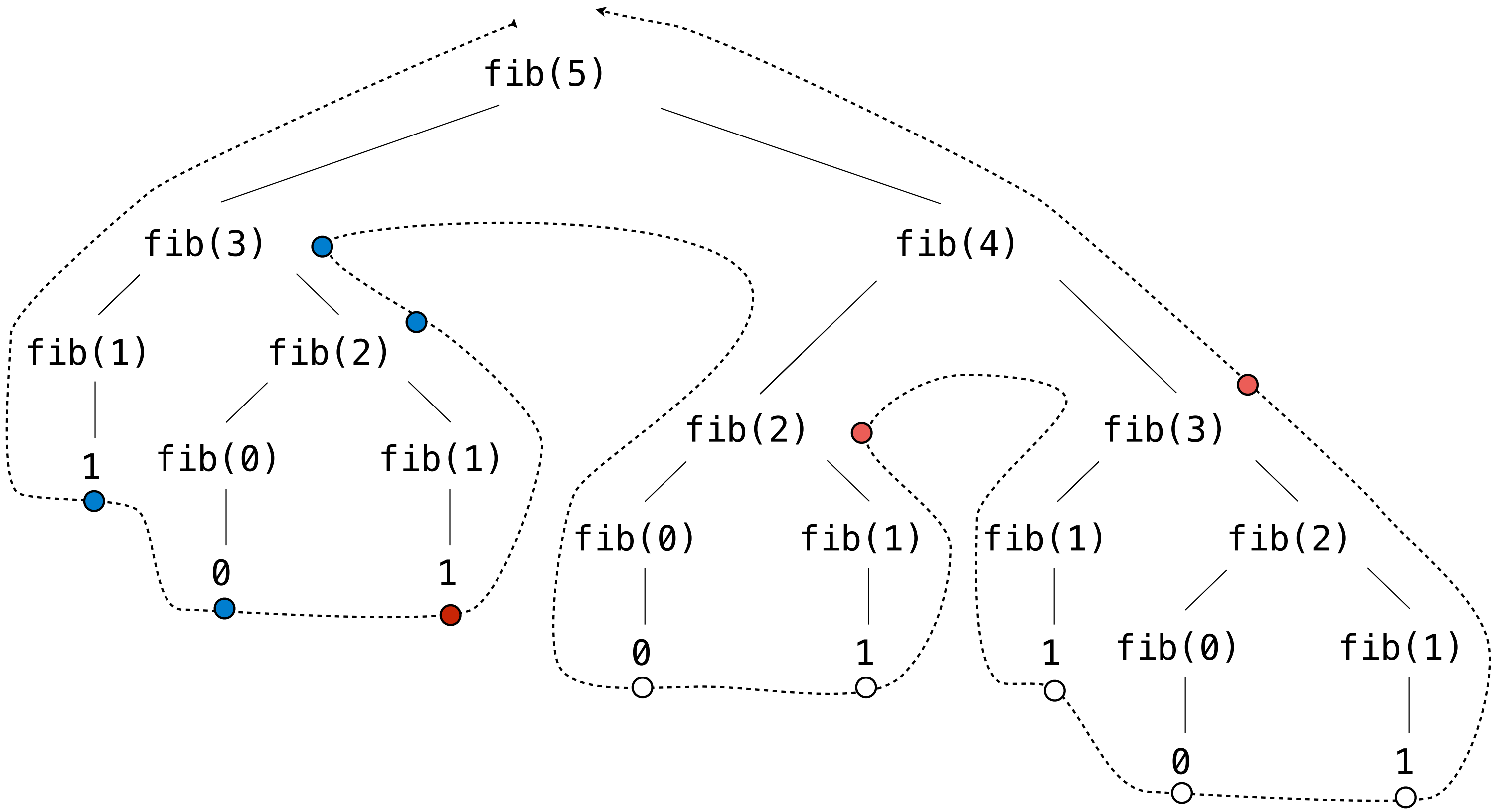
Memoization:

Returned by fib

Found in cache

Skipped



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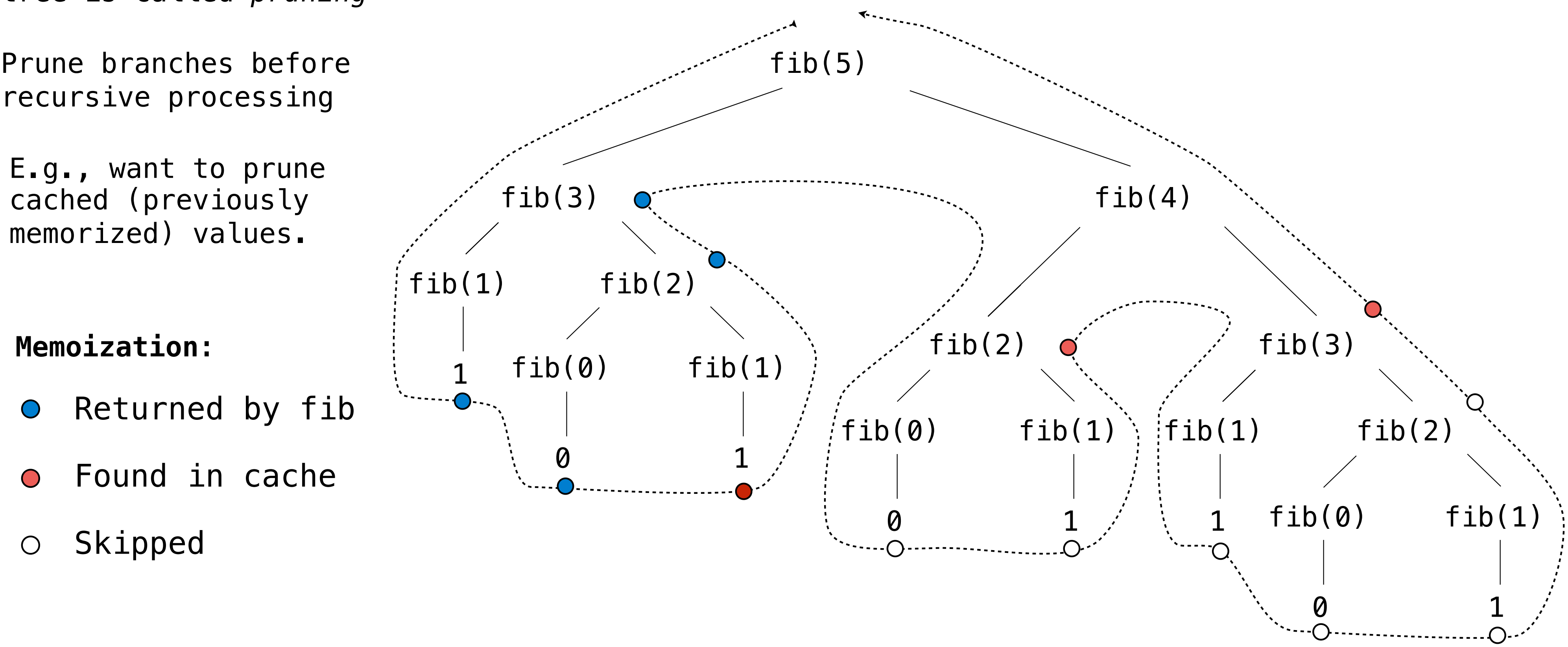

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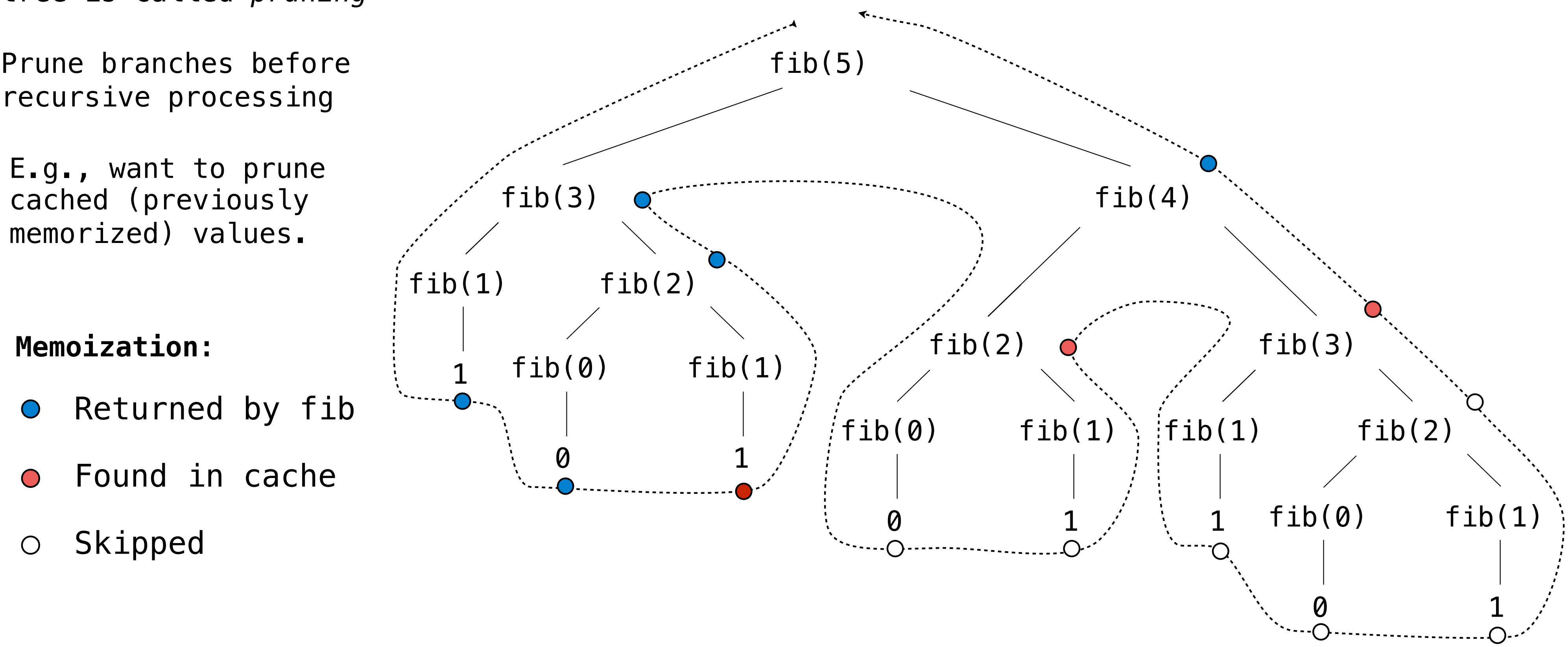

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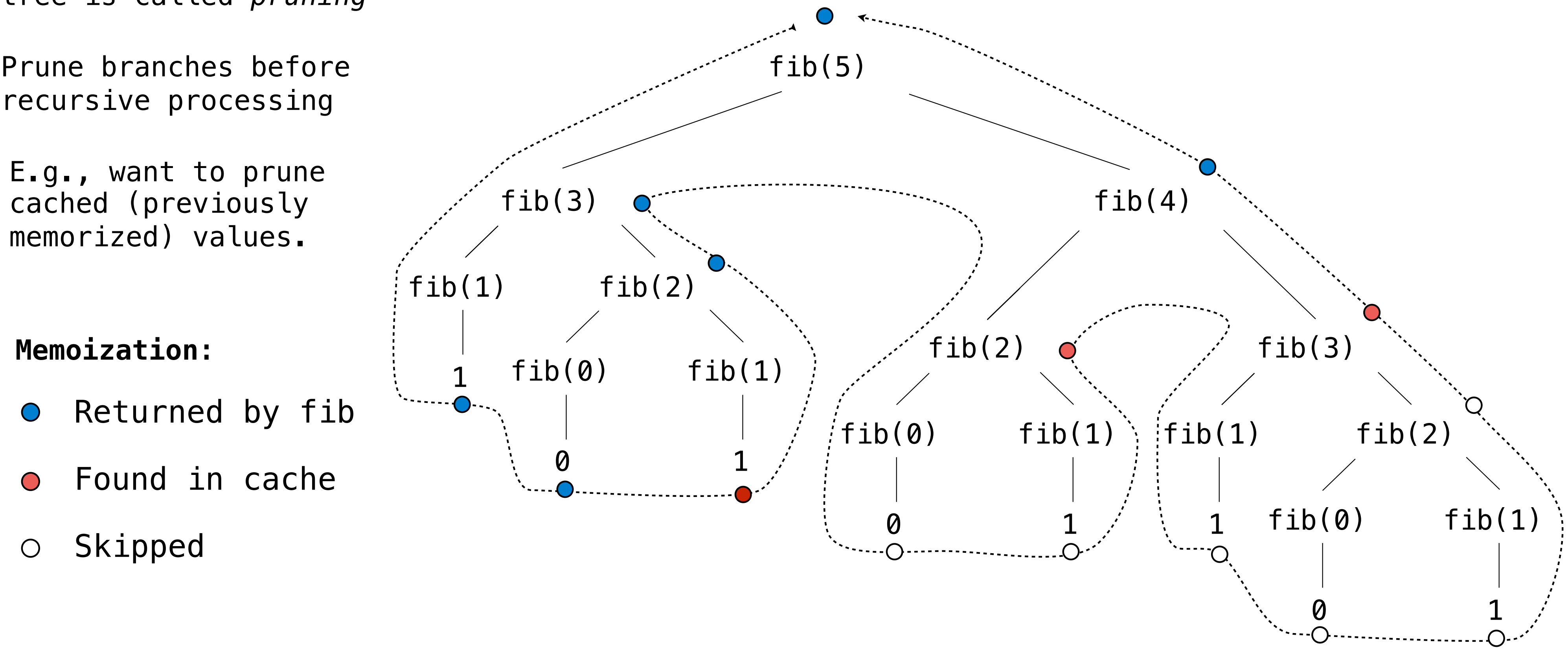

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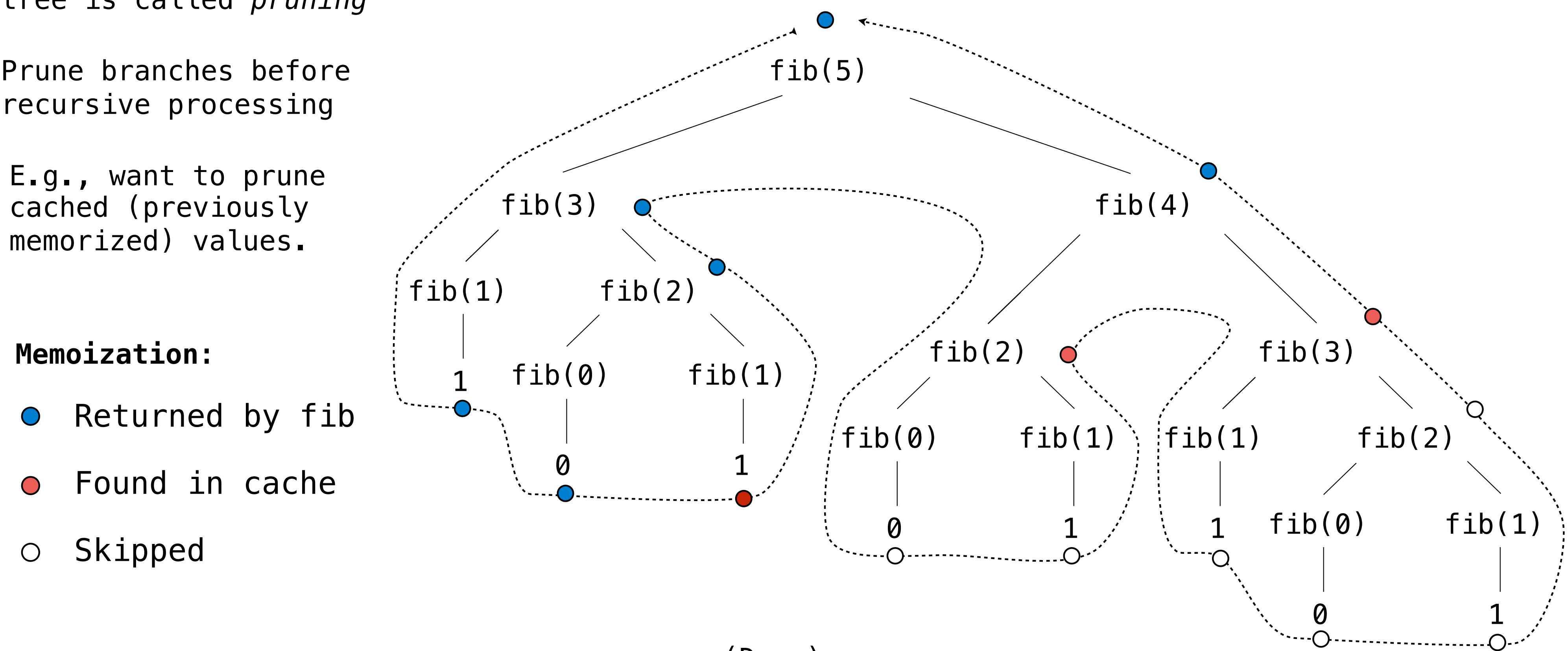

(Demo)

Hailstone Trees

Hailstone Trees

Hailstone Trees

```
Pick a positive integer n as the start
```

Hailstone Trees

```
Pick a positive integer n as the start

If n is even, divide it by 2
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Continue this process until n is 1
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(Demo)

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Pick a positive integer n as the start

If n is even, divide it by 2

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Continue this process until n is 1

(Demo)

1

2

Hailstone Trees

```
Pick a positive integer n as the start

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If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

              (Demo)
```

1

2

4

Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

(Demo)

1

2

4

8

Hailstone Trees

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Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

                (Demo)
```

1

2

4

8

16

Hailstone Trees

```
Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1
```

(Demo)

1

2

4

8

16

32

Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

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Continue this process until n is 1

(Demo)

1

2

4

8

16

32

64

Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

(Demo)

1

2

4

8

16

32

64

128

Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

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Continue this process until n is 1

(Demo)

```
  1
  |
  2
  |
  4
  |
  8
  |
 16
  |
 32
  |
 64
  |
128
```

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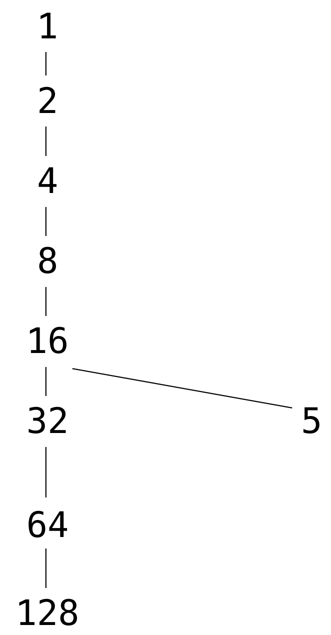
(Demo)



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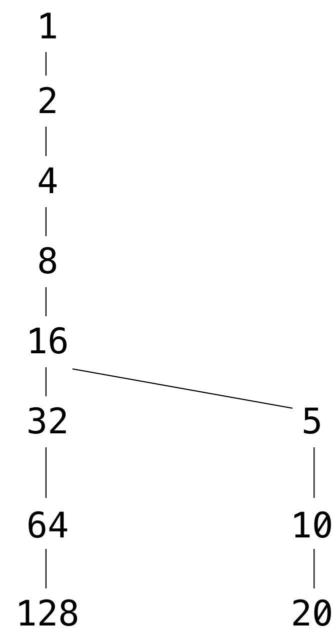

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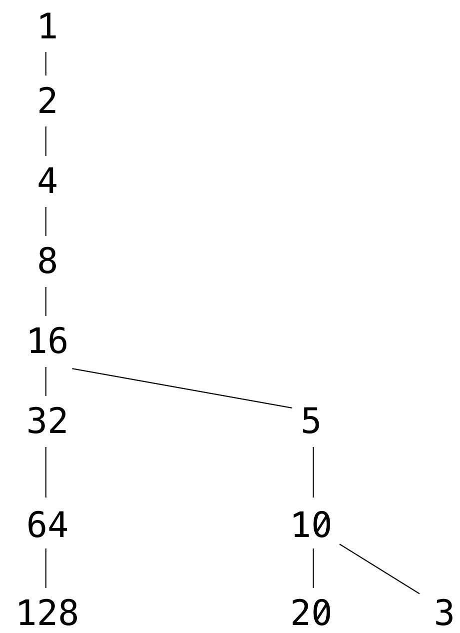

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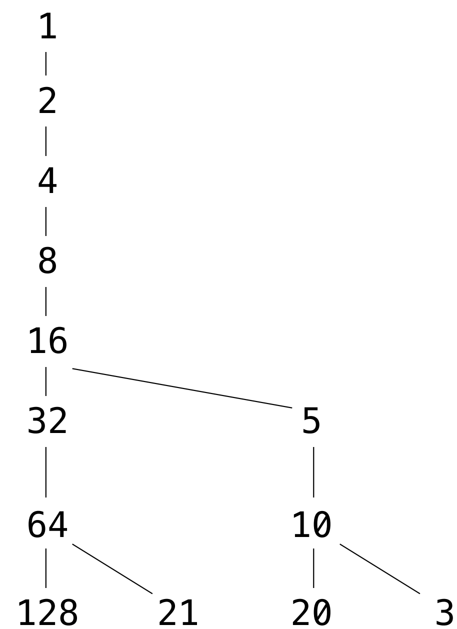

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              (Demo)
```
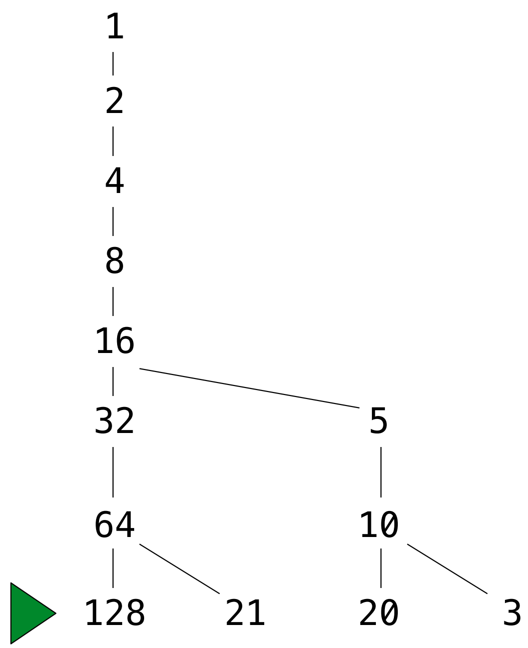

All possible n that start a length-8 hailstone sequence



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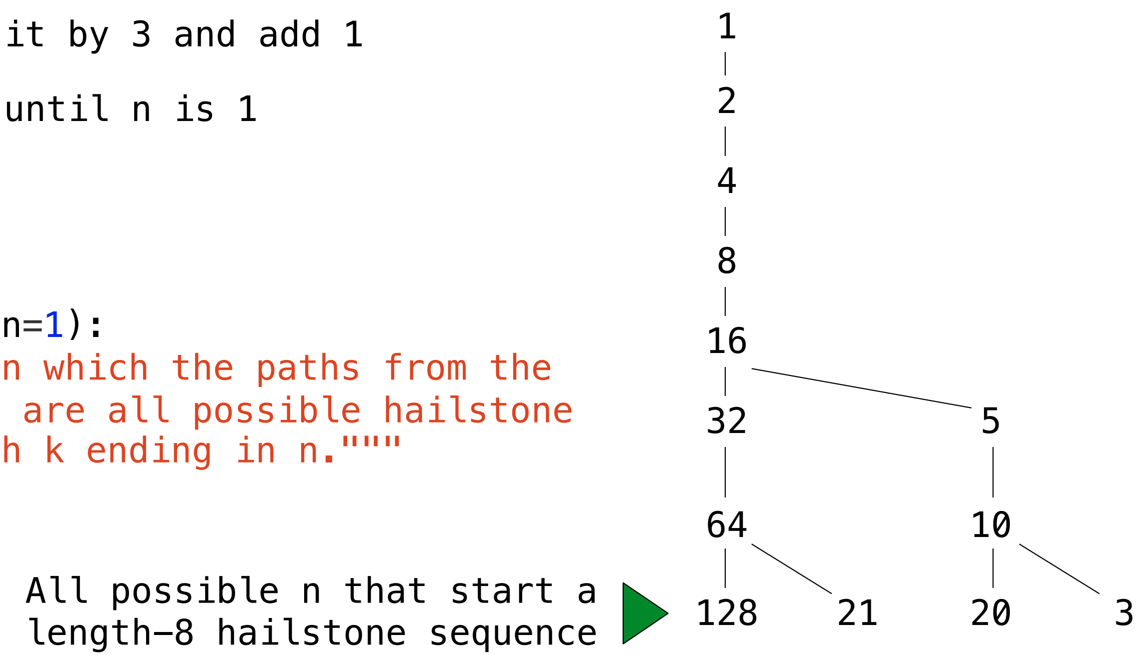
Continue this process until n is 1

(Demo)

```
def hailstone_tree(k, n=1):
    """Return a Tree in which the paths from the
    leaves to the root are all possible hailstone
    sequences of length k ending in n."""
```



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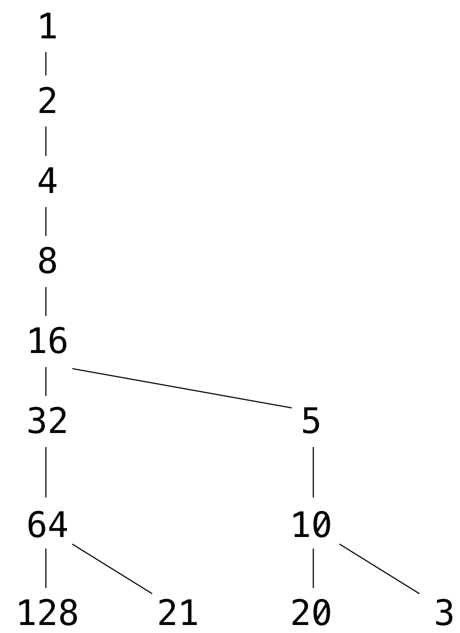

All possible n that start a length-8 hailstone sequence

(Demo)