

**Import statement**

```
1 from math import pi
2 tau = 2 * pi
```

**Assignment statement**

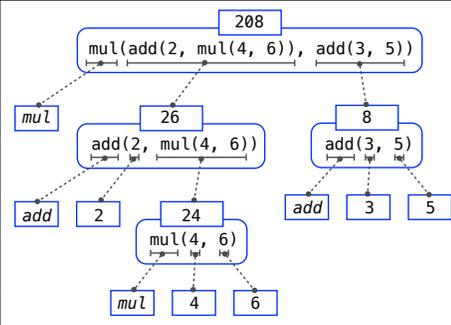
**Global frame**

| Name | Value  |
|------|--------|
| pi   | 3.1416 |

**Binding**

**Code (left):** Statements and expressions  
Red arrow points to next line. Gray arrow points to the line just executed

**Frames (right):** A name is bound to a value  
In a frame, there is at most one binding per name



**Pure Functions**

```
-2 abs(number): 2
2, 10 pow(x, y): 1024
```

**Non-Pure Functions**

```
-2 print(...): None
```

display "-2"

```
1 from operator import mul
2 def square(x):
3     return mul(x, x)
4 square(-2)
```

**Global frame**

**Intrinsic name of function called**

**Local frame**

**Formal parameter bound to argument**

**Return value is not a binding!**

**Built-in function**

**User-defined function**

**Return value**

**Defining:**

```
>>> def square(x):
    return mul(x, x)
```

**Def statement**

**Formal parameter**

**Return expression**

**Body (return statement)**

**Call expression:** square(2+2)

**operator:** square  
**function:** func square(x)

**operand:** 2+2  
**argument:** 4

**Compound statement**

**Clause**

```
<header>:
<statement>
<statement>
Suite
<separating header>:
<statement>
<statement>
...
def abs_value(x):
    1 statement,
    3 clauses,
    3 headers,
    3 suites,
    2 boolean contexts
    if x > 0:
        return x
    elif x == 0:
        return 0
    else:
        return -x
```

```
1 from operator import mul
2 def square(x):
3     return mul(x, x)
4 square(square(3))
```

A name evaluates to the value bound to that name in the earliest frame of the current environment in which that name is found.

**Global frame**

**Local frame**

**Return value**

**Calling/Applying:**

```
4 square(x):
    return mul(x, x)
```

**Argument**

**Intrinsic name**

**Return value**

**def abs\_value(x):**

1 statement, 3 clauses, 3 headers, 3 suites, 2 boolean contexts

```
if x > 0:
    return x
elif x == 0:
    return 0
else:
    return -x
```

**Evaluation rule for call expressions:**

- Evaluate the operator and operand subexpressions.
- Apply the function that is the value of the operator subexpression to the arguments that are the values of the operand subexpressions.

**Applying user-defined functions:**

- Create a new local frame with the same parent as the function that was applied.
- Bind the arguments to the function's formal parameter names in that frame.
- Execute the body of the function in the environment beginning at that frame.

```
1 def f(x, y):
2     return g(x)
3
4 def g(a):
5     return a + y
6
7 result = f(1, 2)
```

**Global frame**

**f1: f [parent=Global]**

**f2: g [parent=Global]**

**Error**

"y" is not found

- An environment is a sequence of frames
- An environment for a non-nested function (no def within def) consists of one local frame, followed by the global frame

**Execution rule for def statements:**

- Create a new function value with the specified name, formal parameters, and function body.
- Its parent is the first frame of the current environment.
- Bind the name of the function to the function value in the first frame of the current environment.

**Execution rule for assignment statements:**

- Evaluate the expression(s) on the right of the equal sign.
- Simultaneously bind the names on the left to those values, in the first frame of the current environment.

```
1 from operator import mul
2 def square(x):
3     return mul(x, x)
4 square(4)
```

**Global frame**

**f1: square [parent=Global]**

**Return value**

A call expression and the body of the function being called are evaluated in different environments

**Higher-order function:** A function that takes a function as an argument value or returns a function as a return value

**Nested def statements:** Functions defined within other function bodies are bound to names in the local frame

**Execution rule for conditional statements:**

Each clause is considered in order.

- Evaluate the header's expression.
- If it is a true value, execute the suite, then skip the remaining clauses in the statement.

**Evaluation rule for or expressions:**

- Evaluate the subexpression <left>.
- If the result is a true value v, then the expression evaluates to v.
- Otherwise, the expression evaluates to the value of the subexpression <right>.

**Evaluation rule for and expressions:**

- Evaluate the subexpression <left>.
- If the result is a false value v, then the expression evaluates to v.
- Otherwise, the expression evaluates to the value of the subexpression <right>.

**Evaluation rule for not expressions:**

- Evaluate <exp>; The value is True if the result is a false value, and False otherwise.

**Execution rule for while statements:**

- Evaluate the header's expression.
- If it is a true value, execute the (whole) suite, then return to step 1.

```
def fib(n):
    """Compute the nth Fibonacci number, for N >= 1."""
    pred, curr = 0, 1 # Zeroth and first Fibonacci numbers
    k = 1 # curr is the kth Fibonacci number
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

```
def cube(k):
    return pow(k, 3)
```

```
def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

0 + 1<sup>3</sup> + 2<sup>3</sup> + 3<sup>3</sup> + 4<sup>3</sup> + 5<sup>3</sup>

Function of a single argument (not called term)

A formal parameter that will be bound to a function

The cube function is passed as an argument value

The function bound to term gets called here

```
square = lambda x,y: x * y
```

Evaluates to a function. No "return" keyword!

A function with formal parameters  $x$  and  $y$  that returns the value of " $x * y$ "

Must be a single expression

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n."""
    def adder(k):
        return k + n
    return adder
```

A function that returns a function

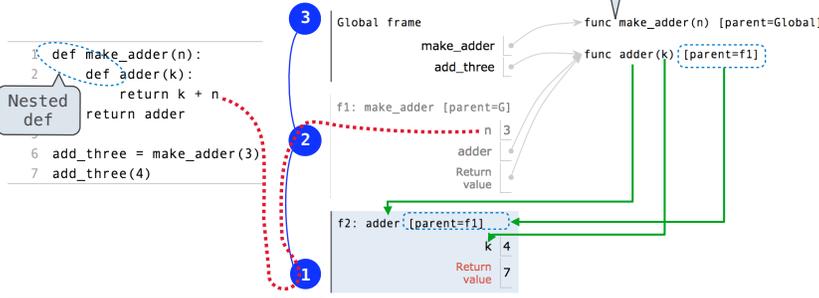
The name `add_three` is bound to a function

A local def statement

Can refer to names in the enclosing function

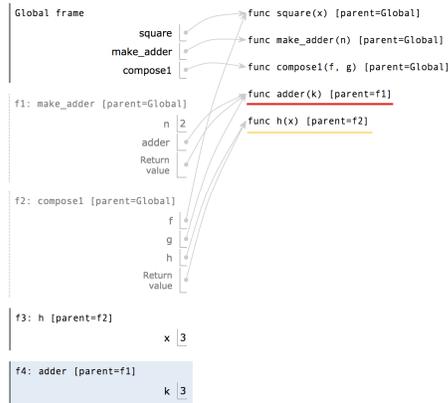
- Every user-defined function has a **parent frame** (often global)
- The parent of a **function** is the frame in which it was **defined**
- Every local **frame** has a **parent frame** (often global)
- The parent of a **frame** is the parent of the function **called**

A function's signature has all the information to create a local frame

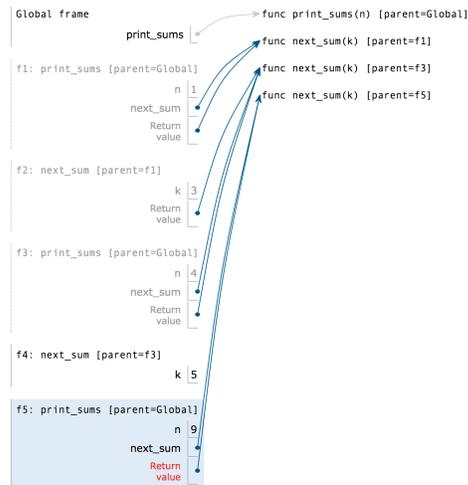


```
1 def square(x):
2   return x * x
3
4 def make_adder(n):
5   def adder(k):
6     return k + n
7   return adder
8
9 def compose1(f, g):
10  def h(x):
11    return f(g(x))
12  return h
13
14 compose1(square, make_adder(2))(3)
```

Return value of `make_adder` is an argument to `compose1`



```
1 def print_sums(n):
2   print(n)
3   def next_sum(k):
4     return print_sums(n+k)
5   return next_sum
6
7 print_sums(1)(3)(5)
```



VS

```
def square(x):
    return x * x
```

- Both create a function with the same domain, range, and behavior.
- Both functions have as their parent the environment in which they were defined.
- Both bind that function to the name `square`.
- Only the `def` statement gives the function an intrinsic name.

When a function is defined:

1. Create a **function value**: `func <name>(<formal parameters>)`
2. Its parent is the current frame.

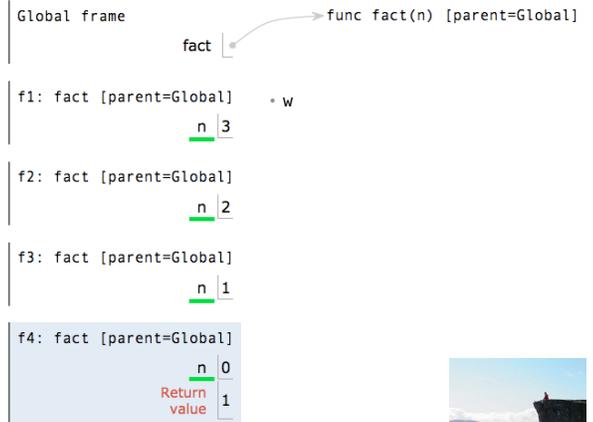
```
f1: make_adder      func adder(k) [parent=f1]
```

3. Bind `<name>` to the **function value** in the current frame (which is the first frame of the current environment).

When a function is called:

1. Add a **local frame**, titled with the `<name>` of the function being called.
2. Copy the parent of the function to the **local frame**: `[parent=<label>]`
3. Bind the `<formal parameters>` to the arguments in the **local frame**.
4. Execute the **body** of the function in the environment that starts with the **local frame**.

```
1 def fact(n):
2   if n == 0:
3     return 1
4   else:
5     return n * fact(n-1)
6
7 fact(3)
```



Is `fact` implemented correctly?

1. Verify the base case.
2. Treat `fact` as a functional abstraction!
3. Assume that `fact(n-1)` is correct.
4. Verify that `fact(n)` is correct, assuming that `fact(n-1)` correct.



Anatomy of a recursive function:

- The **def statement header** is similar to other functions
- Conditional statements check for **base cases**
- Base cases are evaluated **without recursive calls**
- Recursive cases are evaluated **with recursive calls**

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = n // 10, n % 10
        return sum_digits(all_but_last) + last
```

```
from operator import floordiv, mod
def divide_exact(n, d):
    """Return the quotient and remainder of dividing N by D.
    """
    >>> q, r = divide_exact(2012, 10)
    >>> q
    201
    >>> r
    2
    """
    return floordiv(n, d), mod(n, d)
```

Multiple assignment to two names

Two return values, separated by commas