61A Lecture 9

Announcements

Data Abstraction

Data Abstraction

Data Abstraction

- Compound values combine other values together

Data Abstraction

- Compound values combine other values together
  - A date: a year, a month, and a day

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  - A geographic position: latitude and longitude

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All Programmers

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All Programmers

Great Programmers

Rational Numbers

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$$\frac{\texttt{numerator}}{\texttt{denominator}}$$

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Exact representation of fractions

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A pair of integers

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As soon as division occurs, the exact representation may be lost! (Demo)

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- `rational(n, d)` returns a rational number x

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Assume we can compose and decompose rational numbers:

- rational(n, d) returns a rational number x
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Rational Numbers

numerator
―――――――――
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Assume we can compose and decompose rational numbers:

Constructor

- rational(n, d) returns a rational number x

Selectors

- numer(x) returns the numerator of x
- denom(x) returns the denominator of x

Rational Number Arithmetic

Example

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5}$$

Example

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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Example

$$\frac{nx}{dx} * \frac{ny}{dy}$$

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Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

Example

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

$$\frac{3}{2} + \frac{3}{5}$$

Example

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

General Form

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$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

General Form

Rational Number Arithmetic Implementation

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

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- `rational(n, d)` returns a rational number `x`
- `numer(x)` returns the numerator of `x`
- `denom(x)` returns the denominator of `x`

Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```

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These functions implement an abstract representation for rational numbers

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```

Constructor

Selectors

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

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Selectors

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

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def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

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    return rational(nx * dy + ny * dx, dx * dy)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

```
def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

- rational(n, d) returns a rational number x
- numer(x) returns the numerator of x
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These functions implement an abstract representation for rational numbers

Pairs

Representing Pairs Using Lists

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```
>>> pair = [1, 2]
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```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
```

```
A list literal:
Comma-separated expressions in brackets
```

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2
```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
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```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]
```

A list literal:
Comma-separated expressions in brackets

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>>> x, y = pair
>>> x
1
>>> y
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"Unpacking" a list

```
>>> pair[0]
1
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Representing Pairs Using Lists

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A list literal:
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"Unpacking" a list

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>>> pair[0]
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```

Element selection using the selection operator

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>>> y
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```
>>> pair[0]
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Element selection using the selection operator

```
>>> from operator import getitem
```

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>>> pair
[1, 2]
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A list literal:
Comma-separated expressions in brackets

```
>>> x, y = pair
>>> x
1
>>> y
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```

"Unpacking" a list

```
>>> pair[0]
1
>>> pair[1]
2
```

Element selection using the selection operator

```
>>> from operator import getitem
>>> getitem(pair, 0)
1
```

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>>> pair = [1, 2]
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Element selection function

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Element selection using the selection operator

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Element selection function

More lists next lecture

Representing Rational Numbers

```
def rational(n, d):
    “""A representation of the rational number N/D."""
    return [n, d]
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Representing Rational Numbers

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Construct a list

Representing Rational Numbers

```
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    return [n, d]
```

Construct a list

```
def numer(x):
    """Return the numerator of rational number X."""
    return x[0]
```

Representing Rational Numbers

```
def rational(n, d):
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Construct a list

```
def numer(x):
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def denom(x):
    """Return the denominator of rational number X."""
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Select item from a list

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Select item from a list

(Demo)

A Problem of Specification

Our specification at the moment is ambiguous:

- "Numerator" refers to a particular way of writing a certain rational.
- For example, what is the numerator of 6/8?
    - Could say it is 6, but 6/8 = 3/4, so why not 3?

- Let's be more precise:

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- Let's be more precise:

```
def numer(x):
    """Return the numerator of rational number X in lowest terms and having
    the same sign as X."""
```

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- "Numerator" refers to a particular way of writing a certain rational.
- For example, what is the numerator of 6/8?
  - Could say it is 6, but 6/8 = 3/4, so why not 3?

- Let's be more precise:

```
def numer(x):
    """Return the numerator of rational number X in lowest terms and having
    the same sign as X."""

def denom(x):
    """Return the denominator of rational number X in lowest terms and positive."""
```

Reducing to Lowest Terms

Example:

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \boxed{\frac{5}{2}}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \boxed{\frac{5}{2}} \qquad \frac{2}{5} + \frac{1}{10}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}$$

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Example:

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```
from fractions import gcd
```

Reducing to Lowest Terms

Example:

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def rational(n, d):
```

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from fractions import gcd

def rational(n, d):
    “""A representation of the rational number N/D."""
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from fractions import gcd

def rational(n, d):
    """A representation of the rational number N/D."""
    g = gcd(n, d)          # Always has the sign of d
```

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \boxed{\frac{5}{2}}$$

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from fractions import gcd

def rational(n, d):
    """A representation of the rational number N/D."""
    g = gcd(n, d)          # Always has the sign of d
    return [n//g, d//g]
```

Reducing to Lowest Terms

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Greatest common divisor

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(Demo)

Abstraction Barriers

Abstraction Barriers

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation		

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	add_rational, mul_rational rationals_are_equal, print_rational

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	add_rational, mul_rational rationals_are_equal, print_rational
Create rationals or implement rational operations		

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	add_rational, mul_rational rationals_are_equal, print_rational
Create rationals or implement rational operations	numerators and denominators	

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	add_rational, mul_rational rationals_are_equal, print_rational
Create rationals or implement rational operations	numerators and denominators	rational, numer, denom

Abstraction Barriers

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Implementation of lists

Violating Abstraction Barriers

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add_rational( [1, 2], [1, 4] )

def divide_rational(x, y):
    return [ x[0] * y[1], x[1] * y[0] ]
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Data Representations

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(Demo)

Rationals Implemented as Functions

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def rational(n, d):
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        if name == 'n':
            return n
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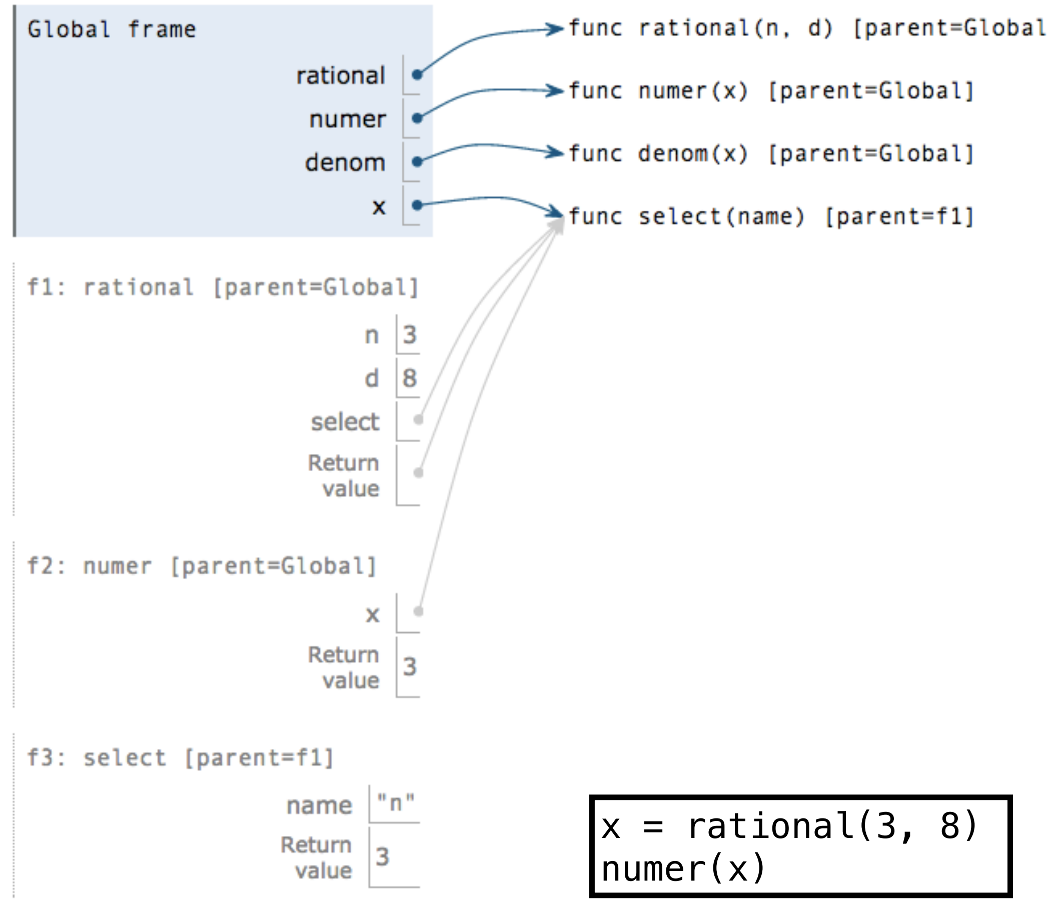


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Interactive Diagram