61A Lecture 21

Announcements

Binary Trees

Binary Tree Class

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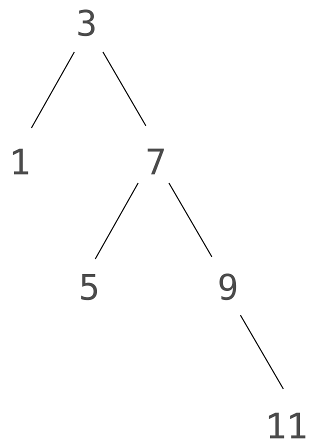

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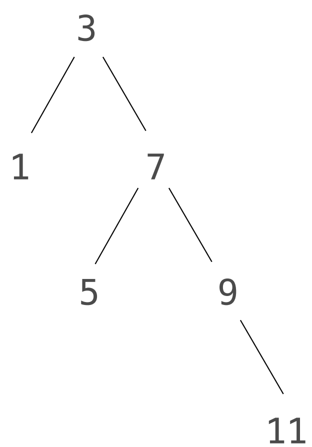


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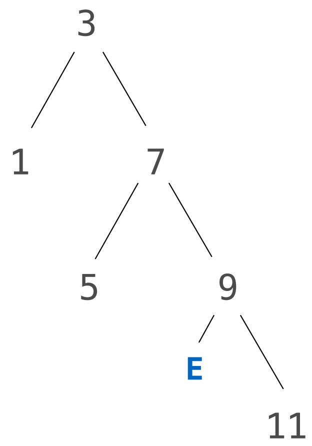


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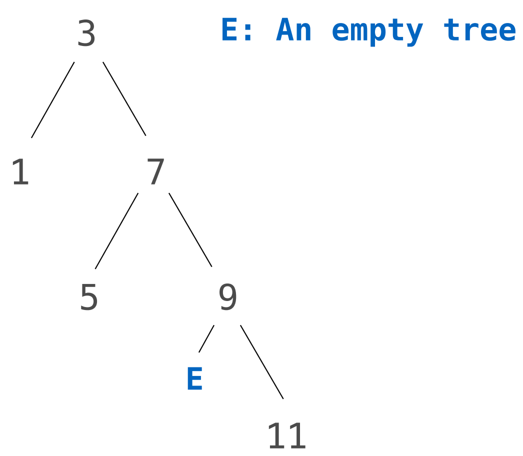

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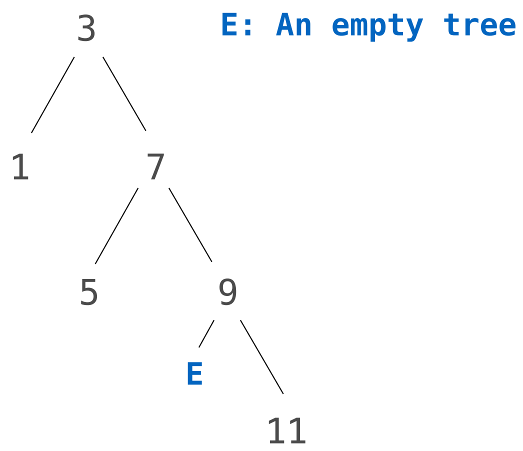

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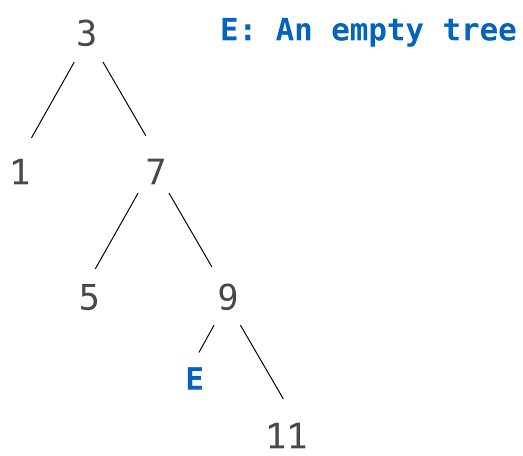


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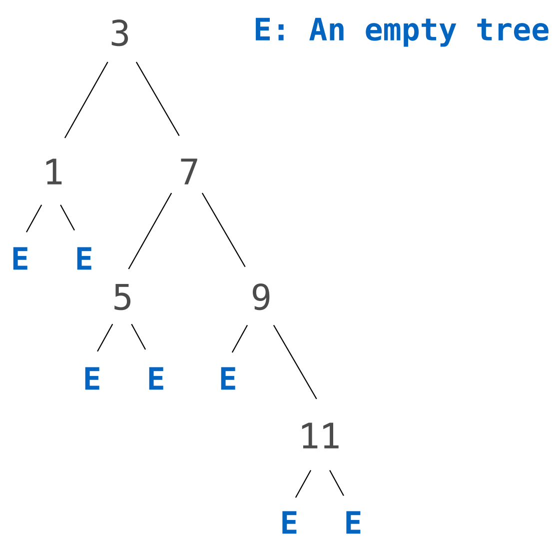


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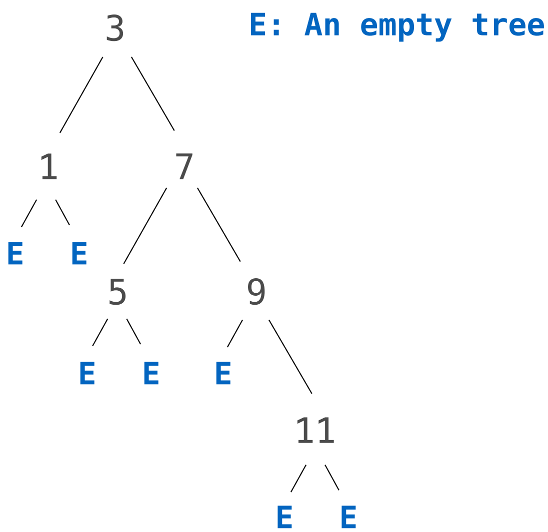


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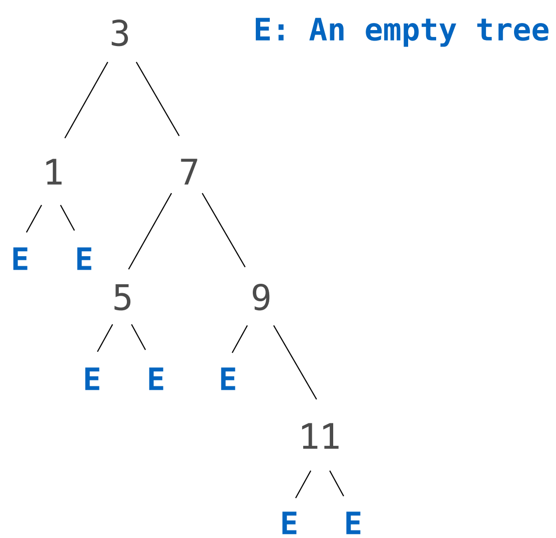


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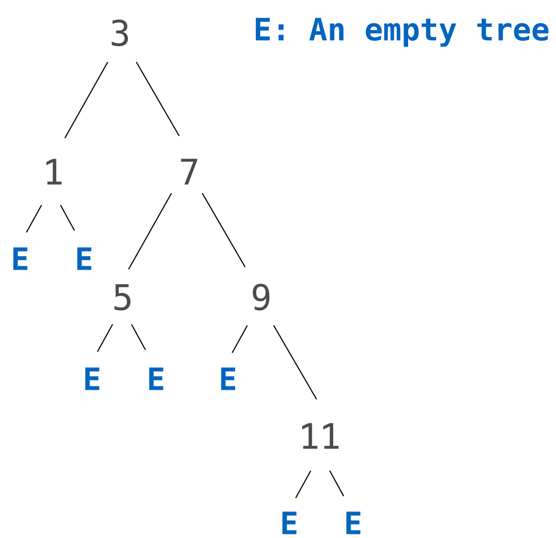


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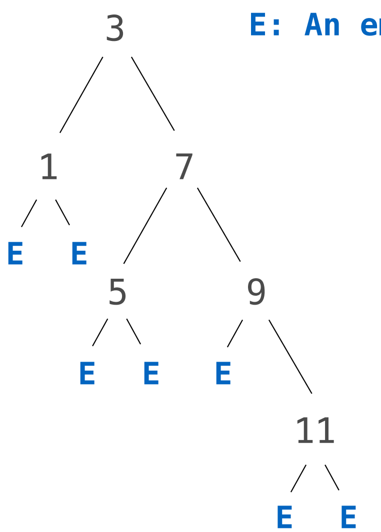

E: An empty tree

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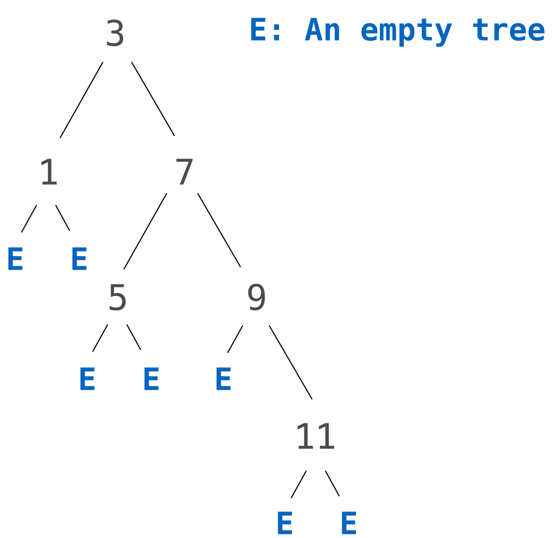


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(Demo)

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A strategy for finding a value in a sorted list: check the middle and eliminate half
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For a sorted list of length n, what Theta expression describes the time required?

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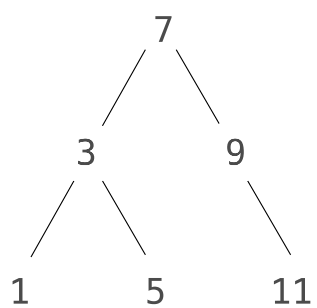

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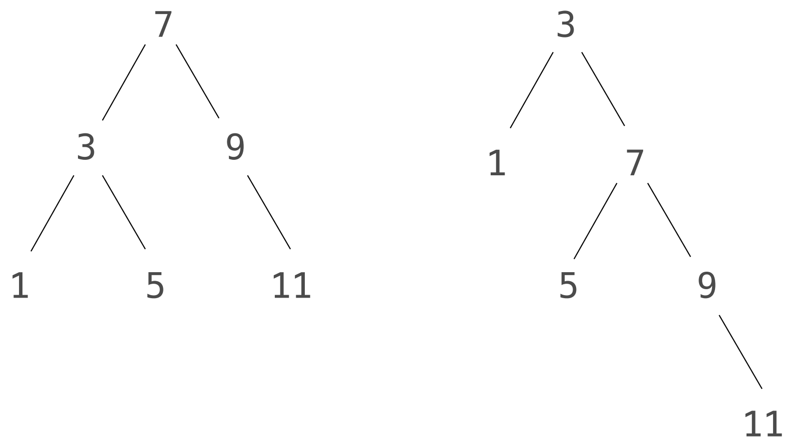

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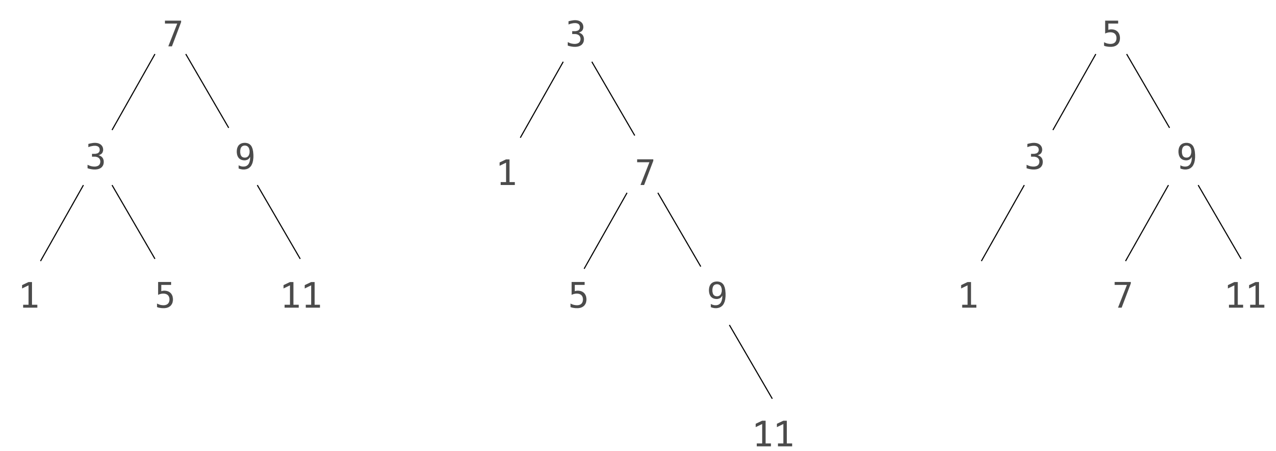

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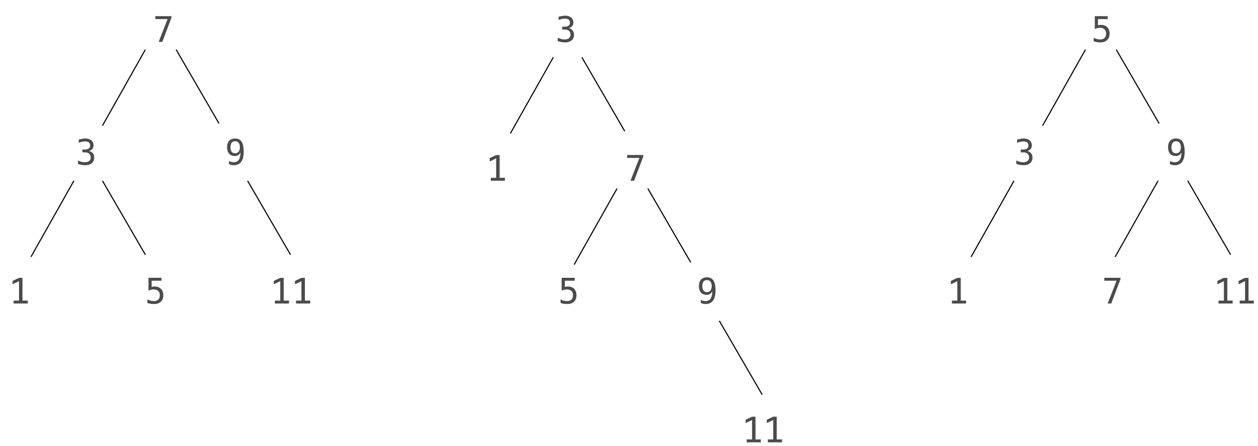

(Demo)

Discussion Questions

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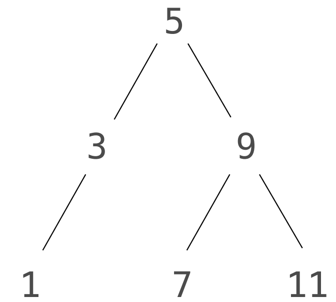

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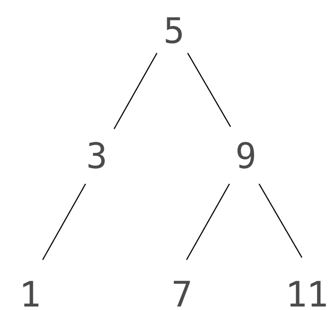

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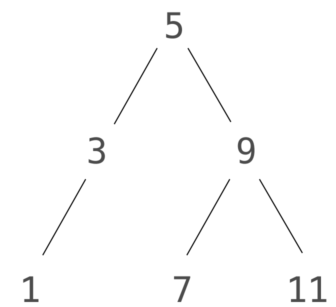

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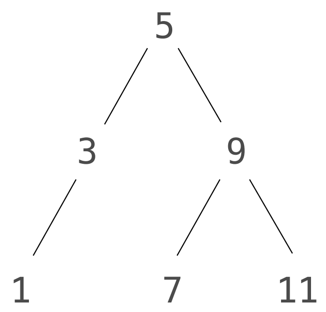

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def second(t):
    if t.is_leaf():
        return None
    elif ________________________:
        return __________________
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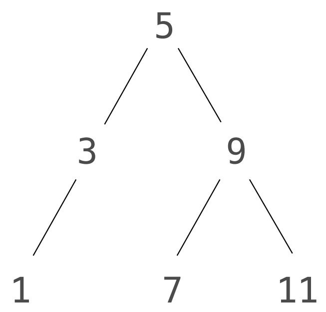
        return t.label
    else:
        return __________________
```



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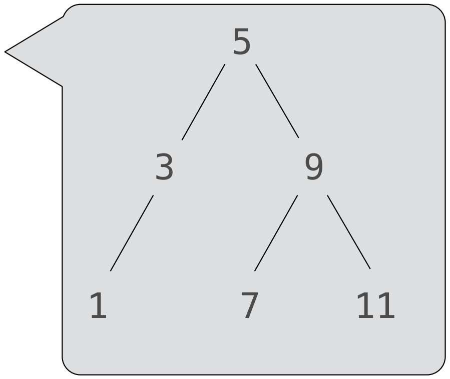

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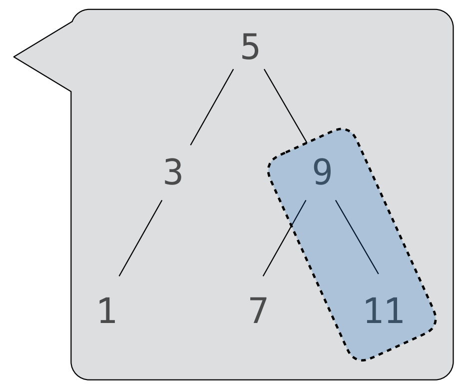

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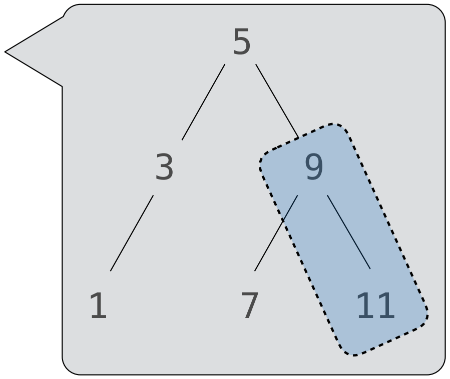

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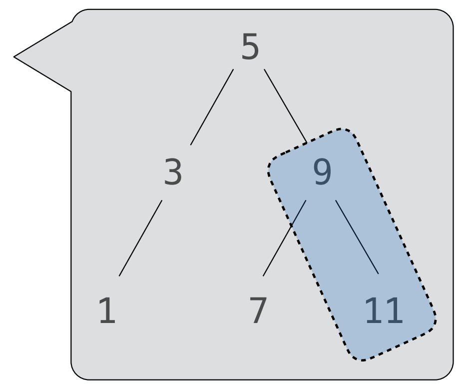

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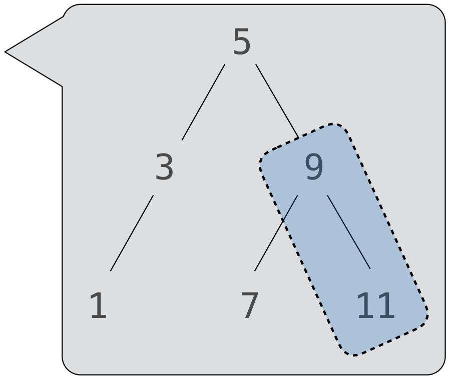

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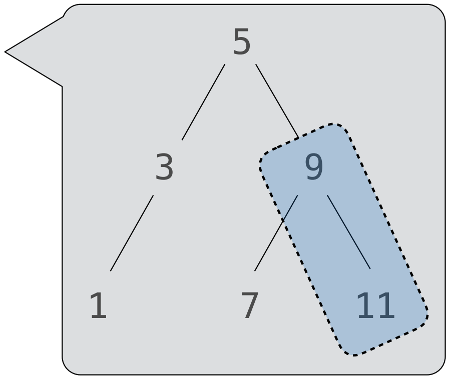

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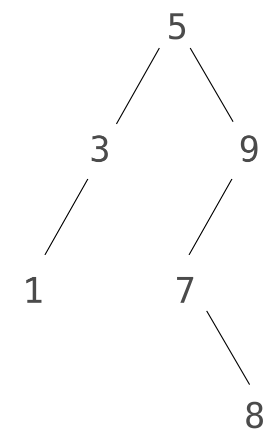

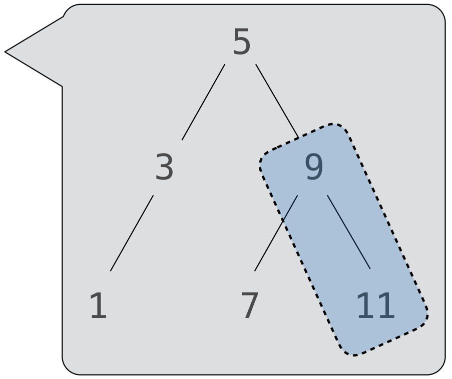

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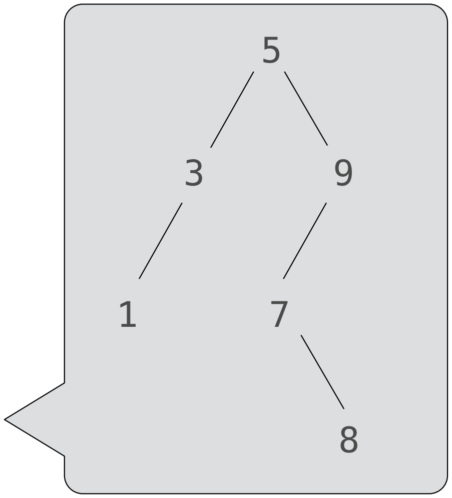
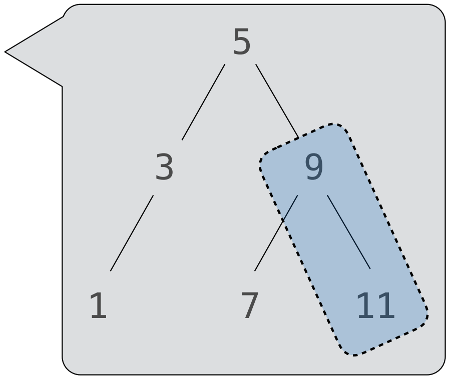
    elif  t.right is BTree.empty :
        return  largest(t.left)
    elif  t.right.is_leaf() :
        return t.label
    else:
        return  second(t.right)
```





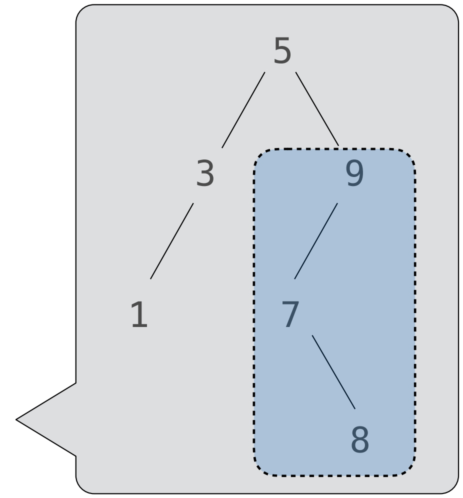
Discussion Questions

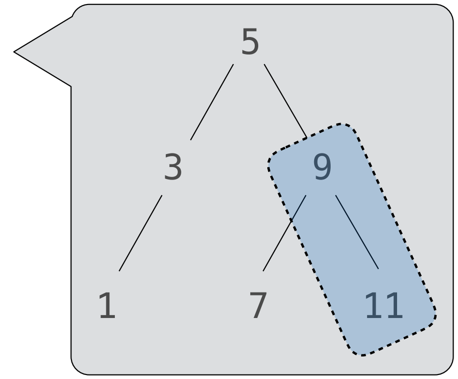
What's the largest element in a binary search tree?

```
def largest(t):
    if _t.right is BTree.empty_:
        return ____t.label____
    else:
        return _largest(t.right)_
```

What's the second largest element in a binary search tree?

```
def second(t):
    if t.is_leaf():
        return None
    elif _t.right is BTree.empty_:
        return _largest(t.left)_
    elif __t.right.is_leaf()__:
        return t.label
    else:
        return _second(t.right)_
```





Sets as Binary Search Trees

Membership in Binary Search Trees

Membership in Binary Search Trees

`contains` traverses the tree

Membership in Binary Search Trees

`contains` `traverses the tree`

- `If the element is not at the root, it can only be in either the left or right branch`

Membership in Binary Search Trees

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- If the element is not at the root, it can only be in either the left or right branch
- By focusing on one branch, we reduce the set by the size of the other branch

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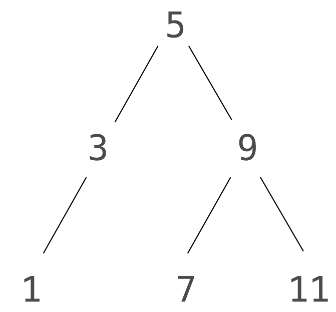

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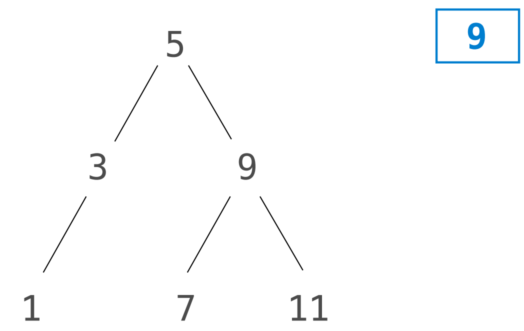

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contains traverses the tree

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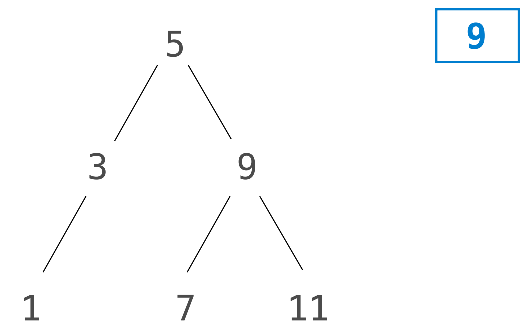
```
def contains(s, v):
```



Membership in Binary Search Trees

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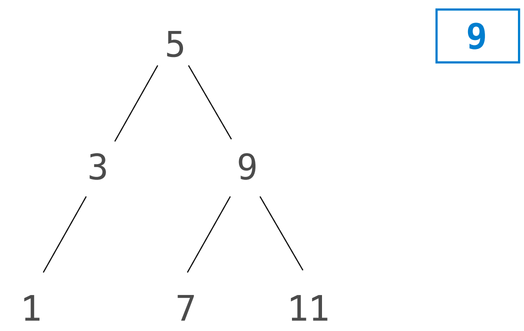
```
def contains(s, v):
    if s is BTree.empty:
        return False
```



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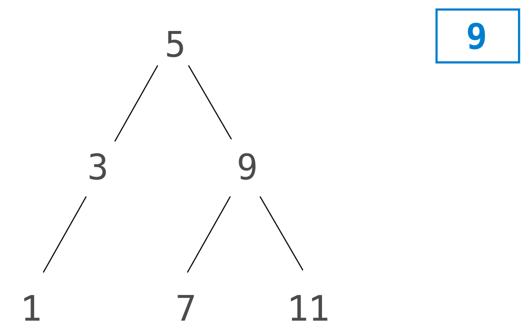

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```
def contains(s, v):
    if s is BTree.empty:
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    elif s.label == v:
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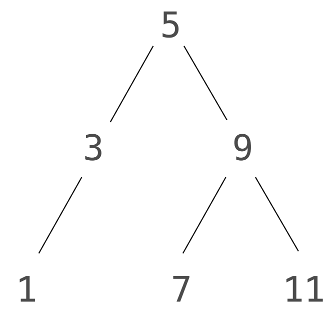
    elif s.label < v:
        return contains(s.right, v)
```



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Membership in Binary Search Trees

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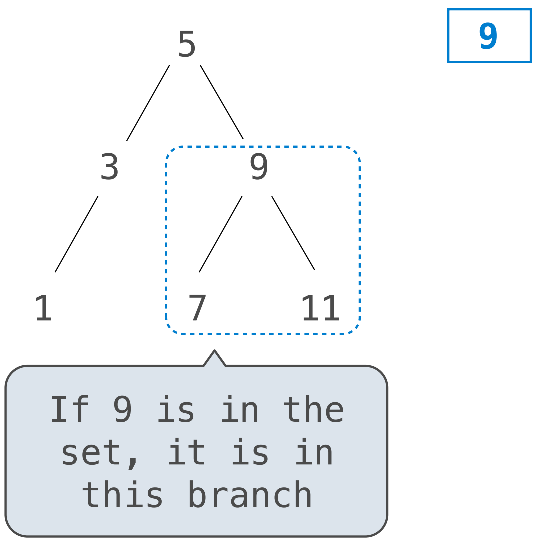

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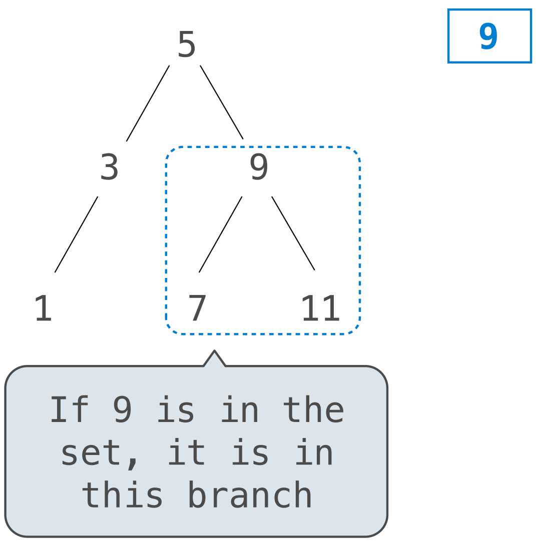
    elif s.label > v:
        return contains(s.left, v)
```



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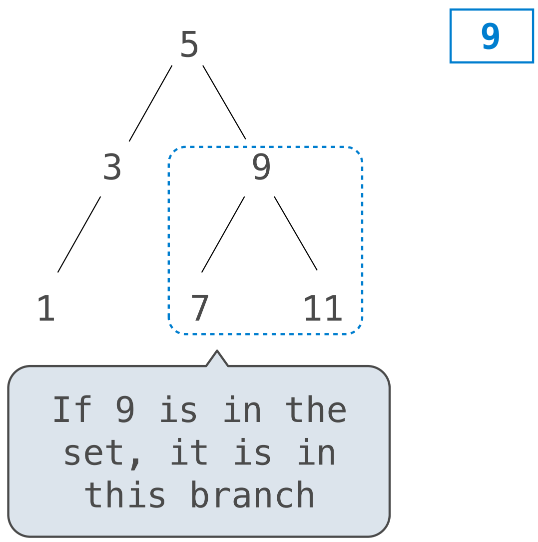

Order of growth?

Membership in Binary Search Trees

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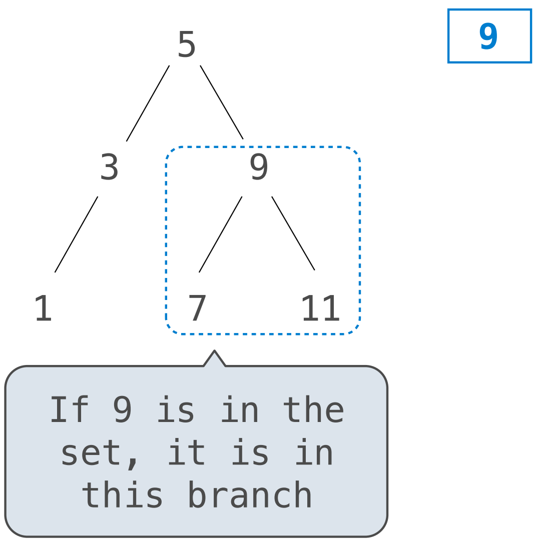
        return contains(s.left, v)
```



Order of growth? $\Theta(h)$ on average

Membership in Binary Search Trees

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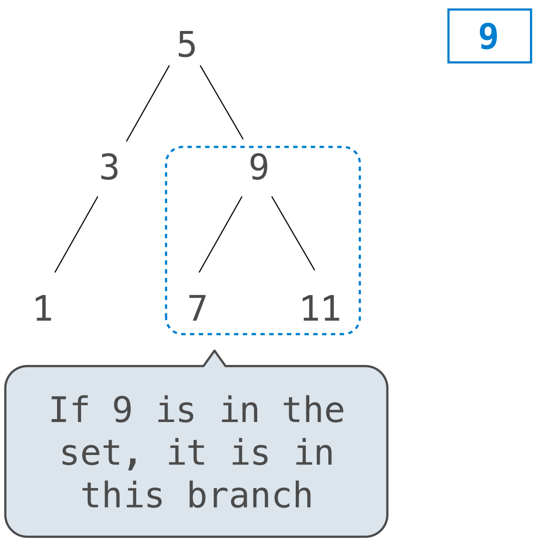
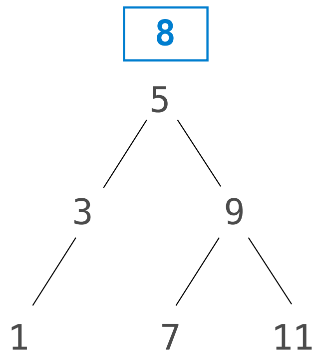
```



Order of growth? $\Theta(h)$ on average $\Theta(\log n)$ on average for a balanced tree

Adjoining to a Tree Set

Adjoining to a Tree Set



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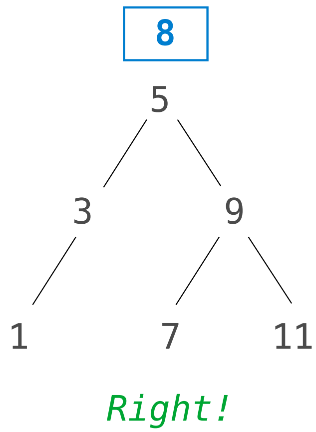

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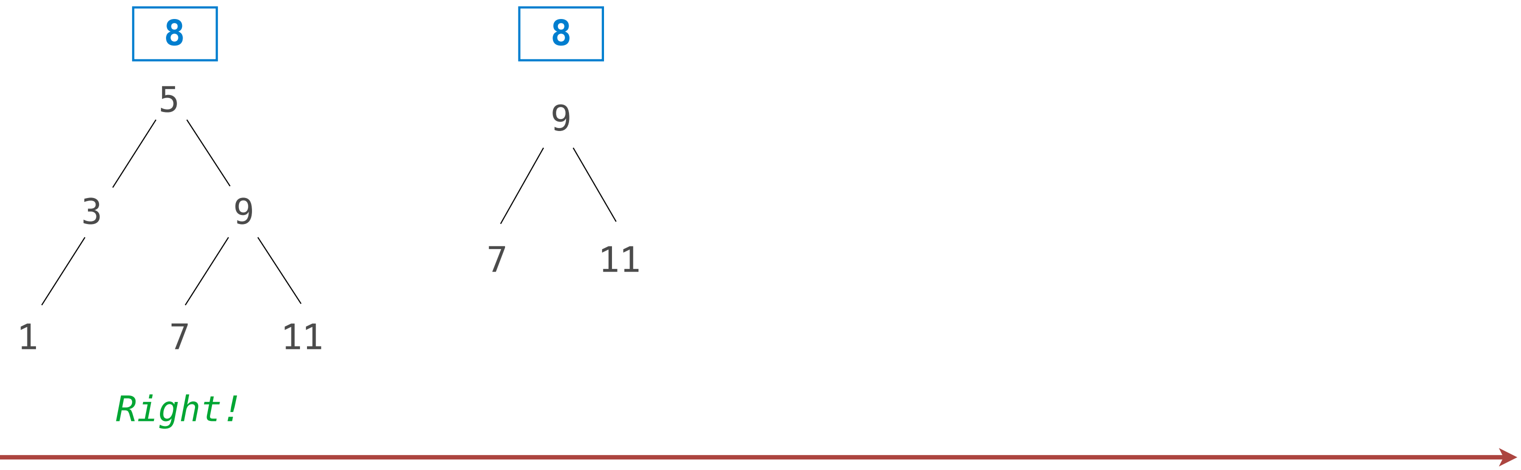

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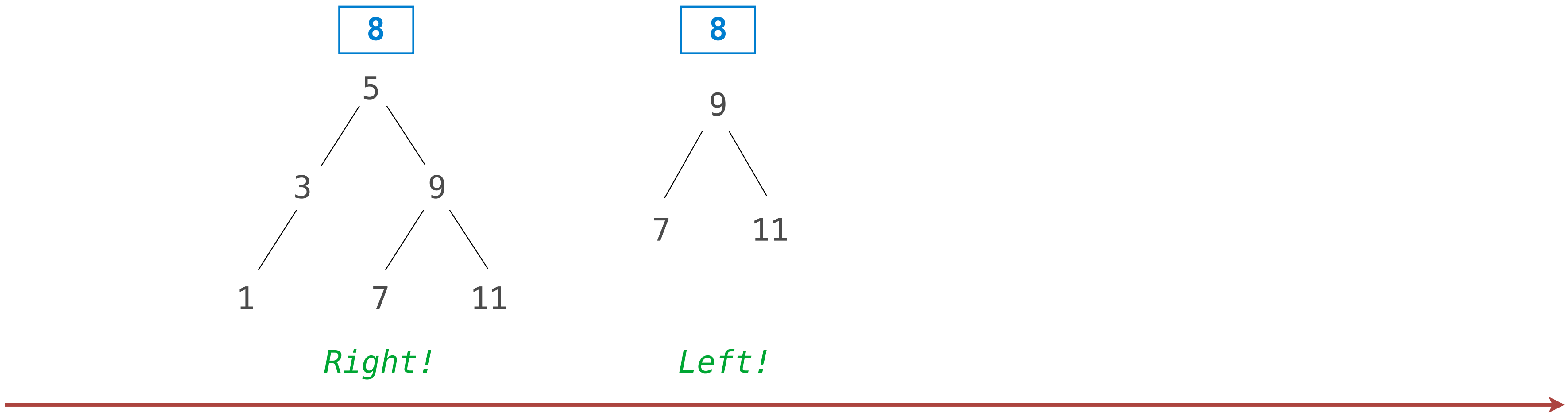

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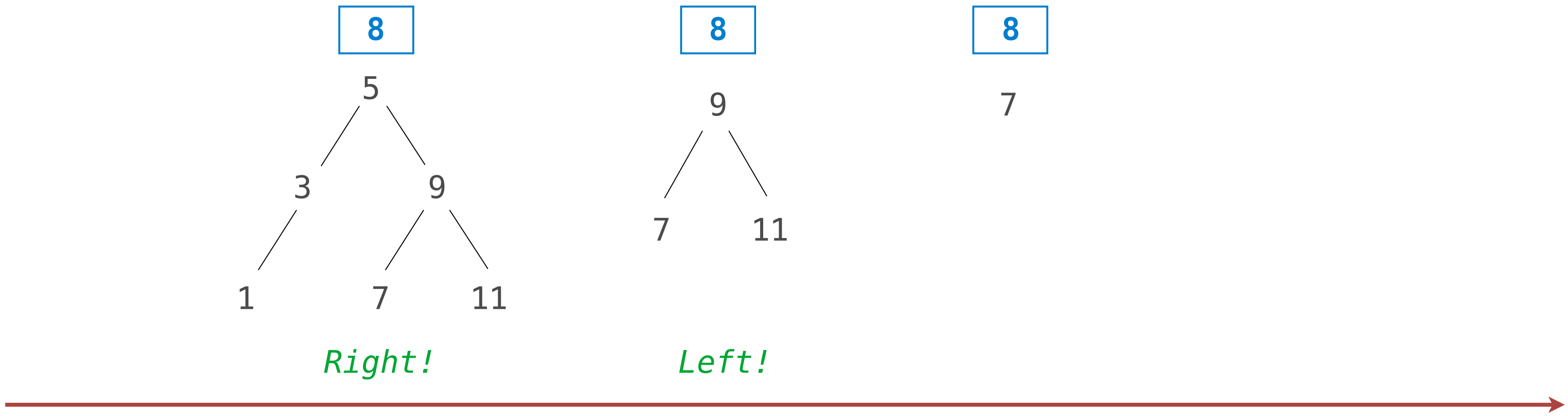

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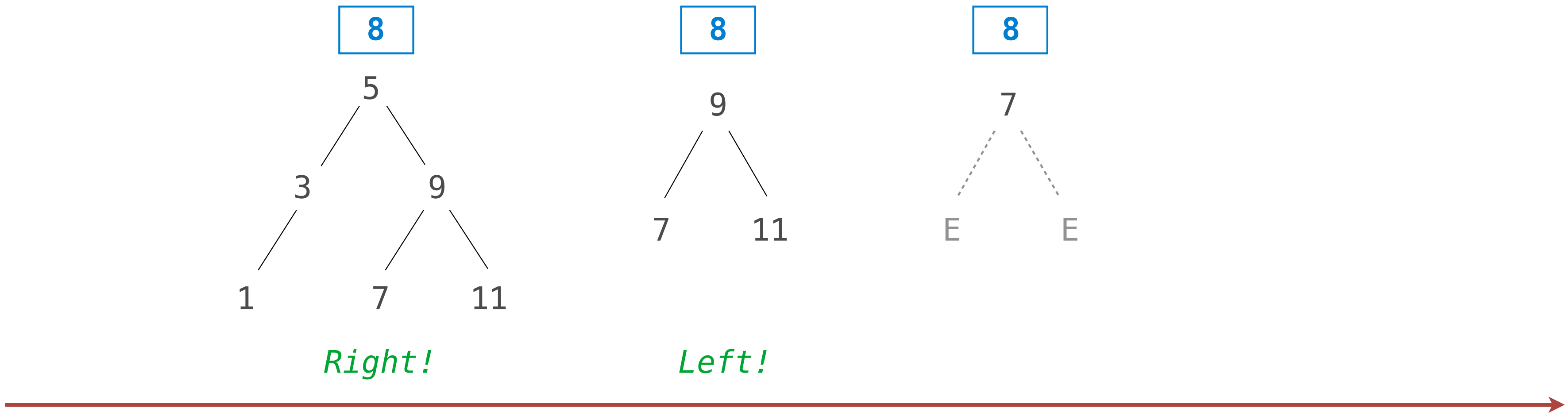

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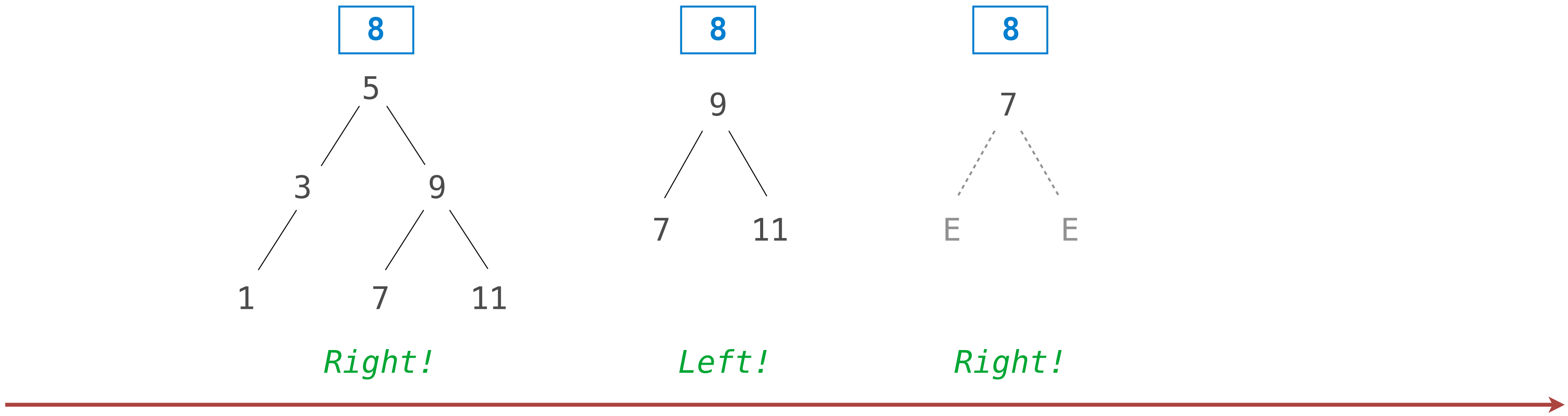

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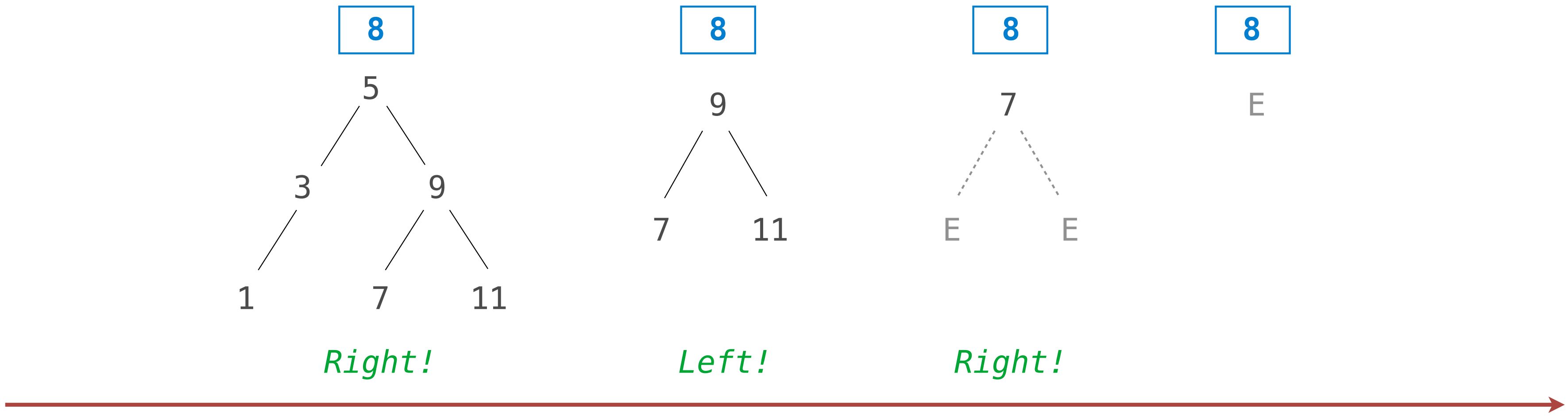

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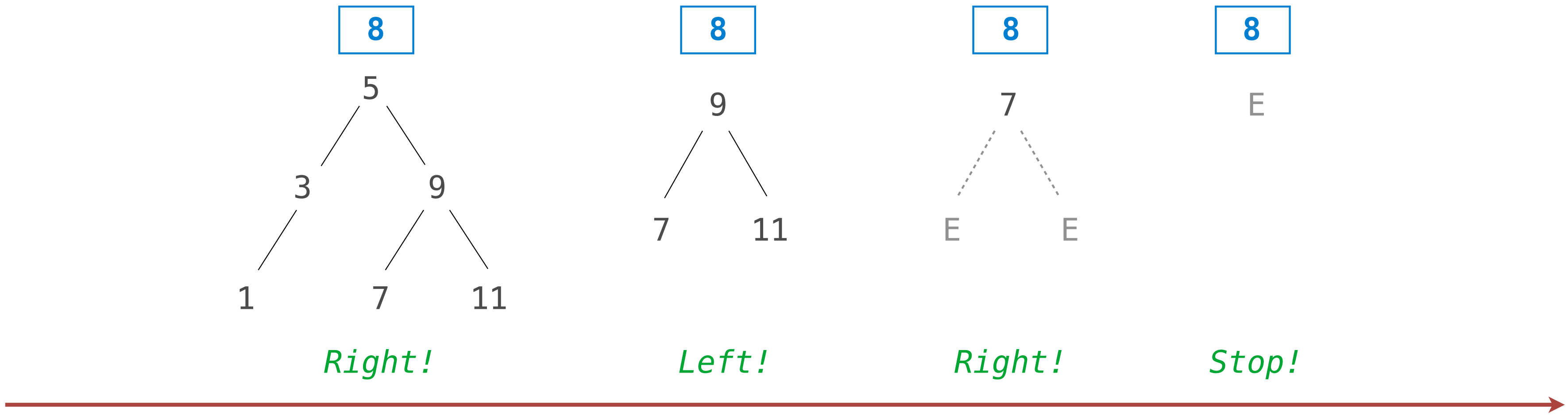

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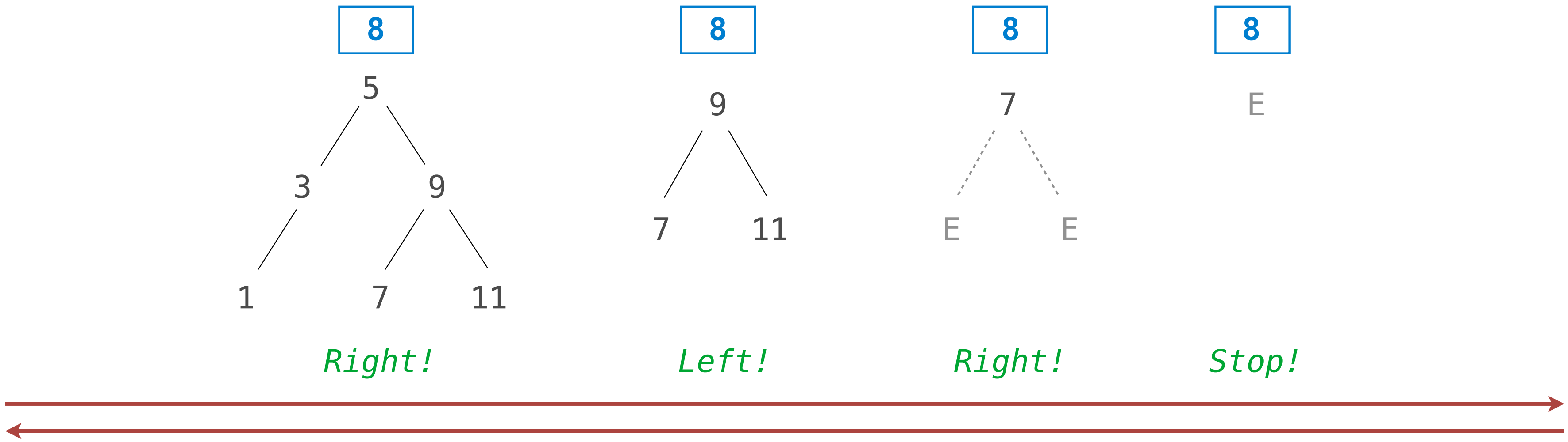

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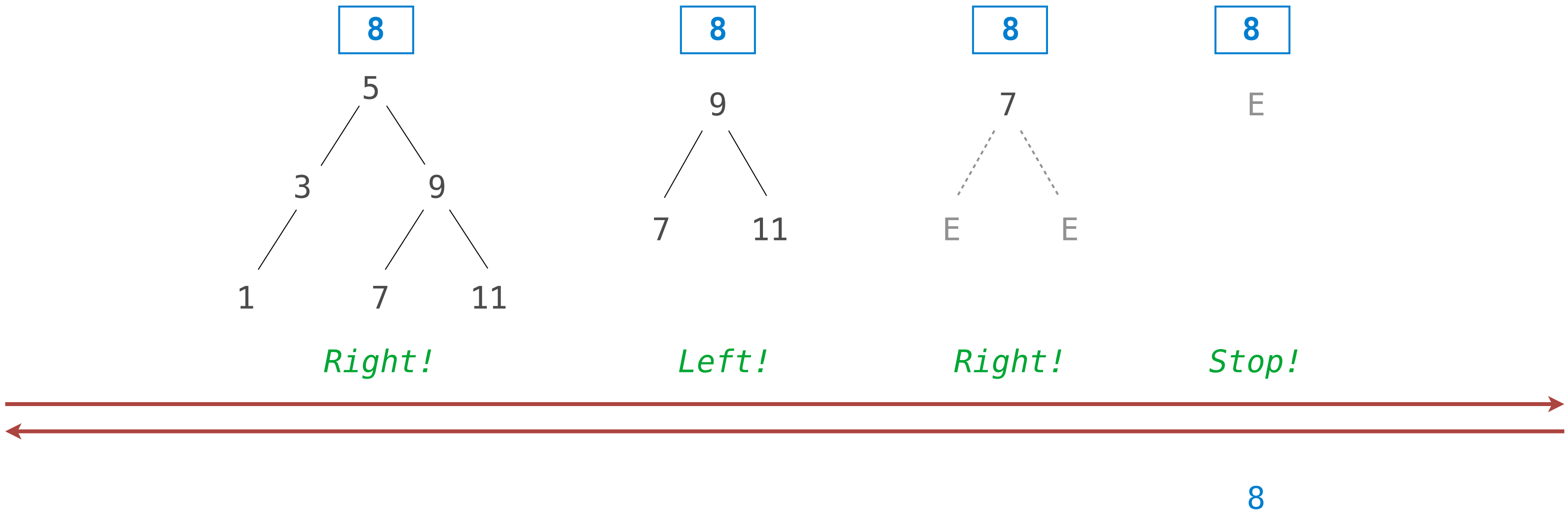

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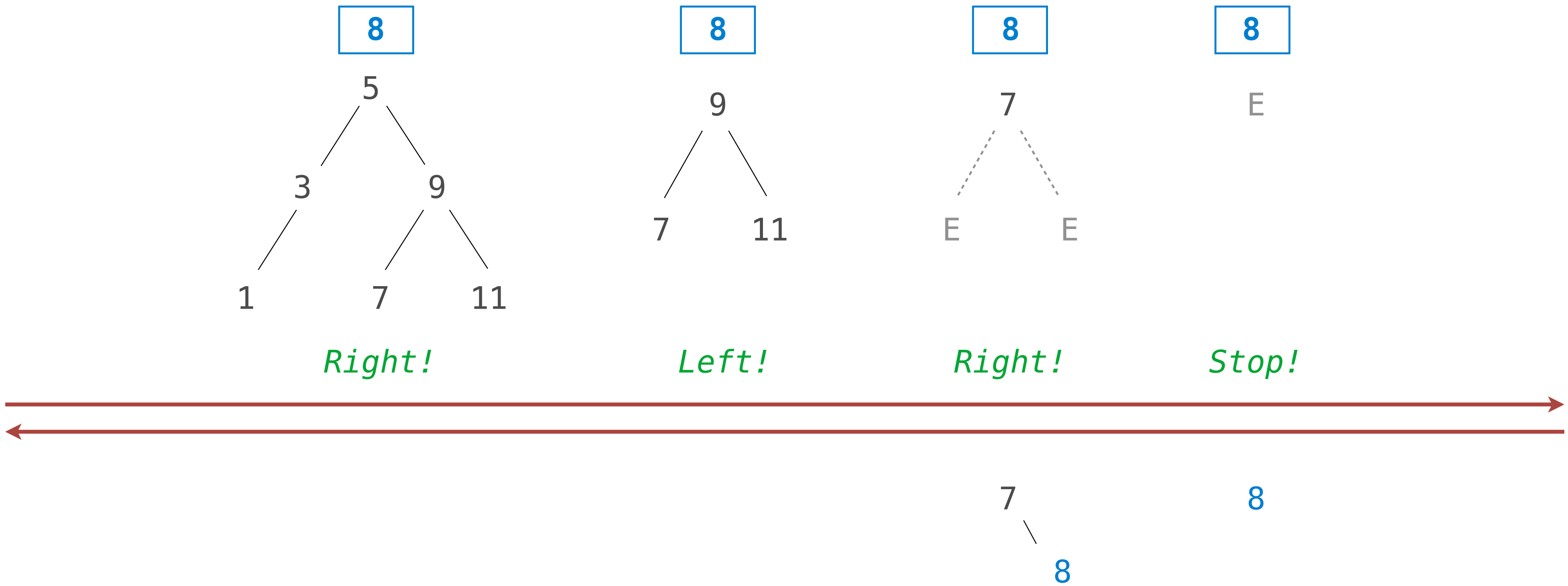

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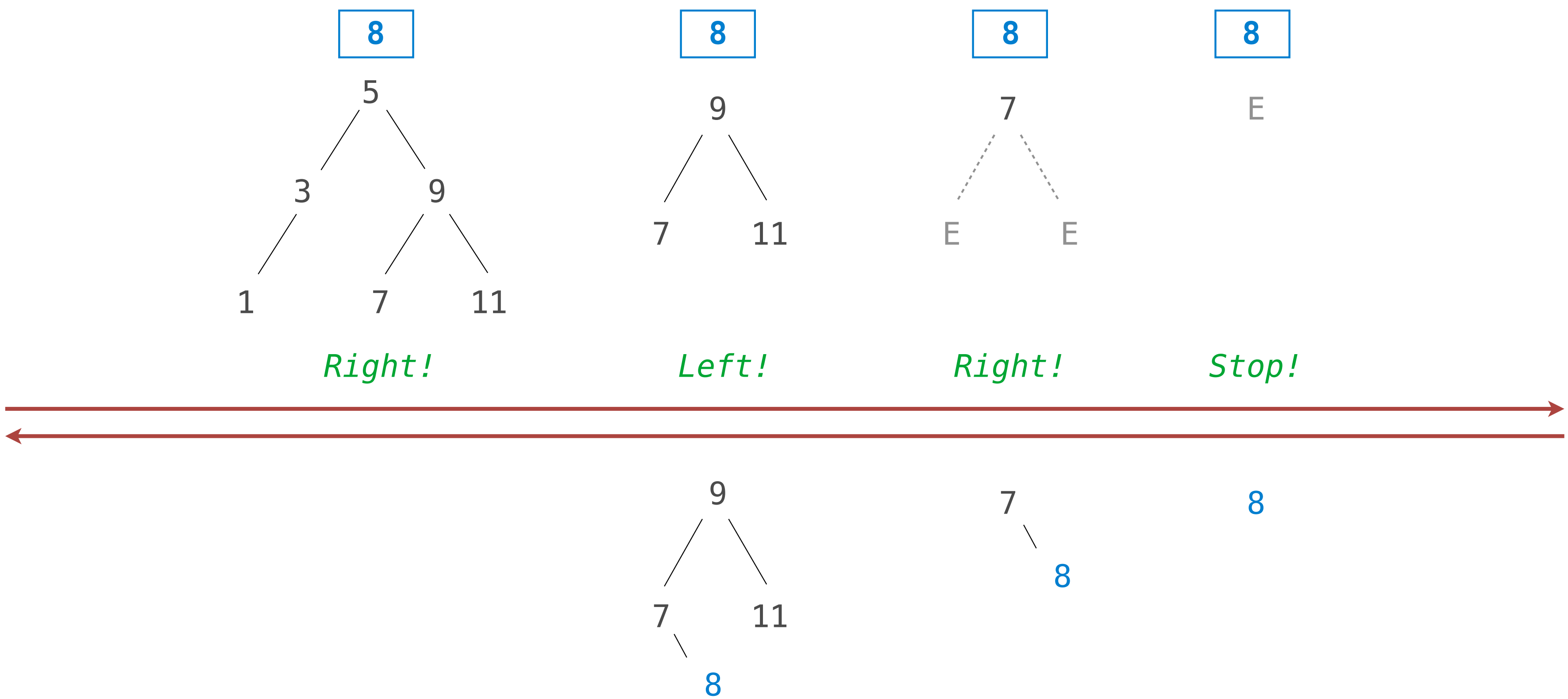

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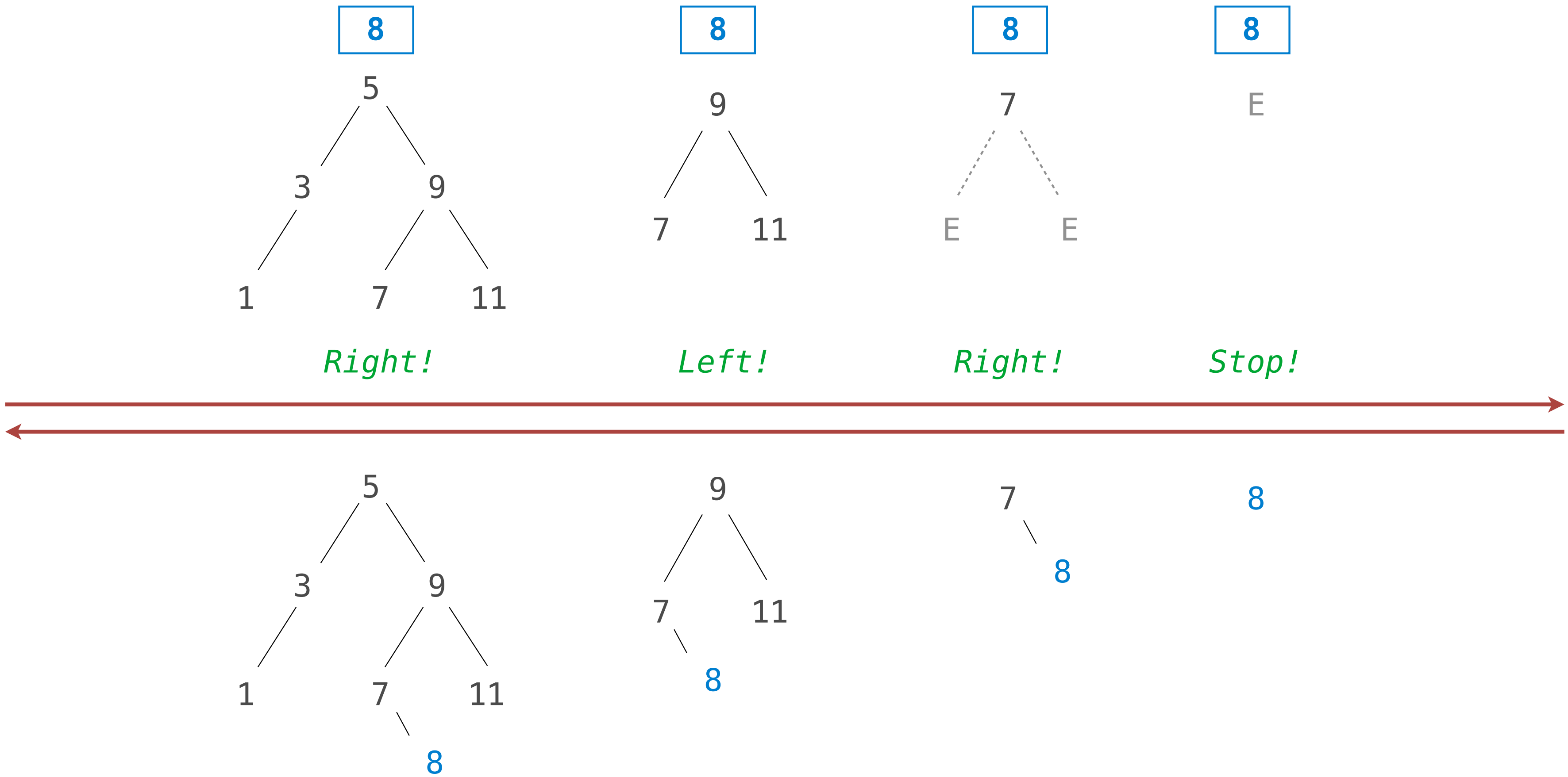

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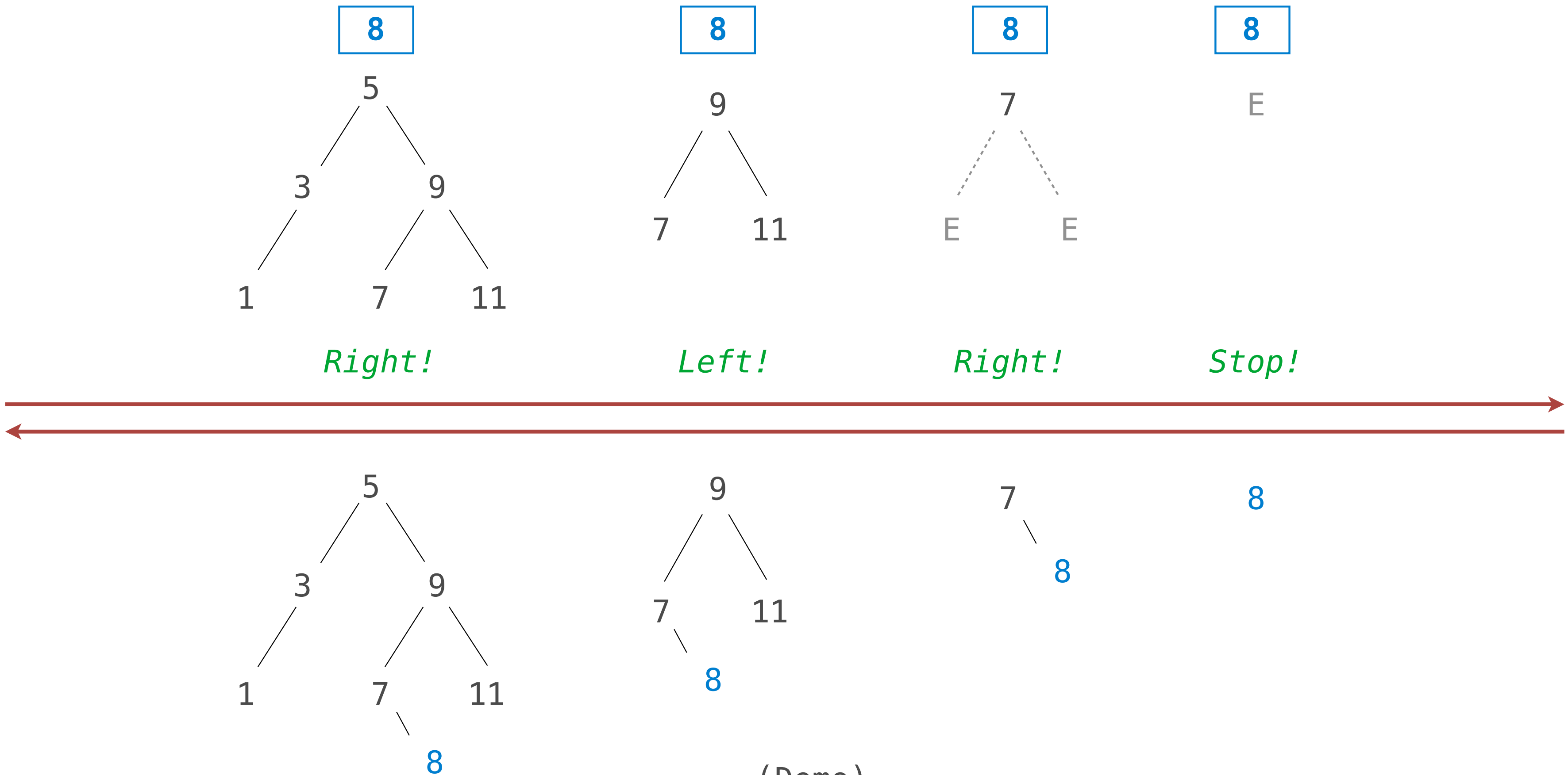

(Demo)