61A Lecture 28

Announcements

Scheme Recursive Art Contest: Start Early!

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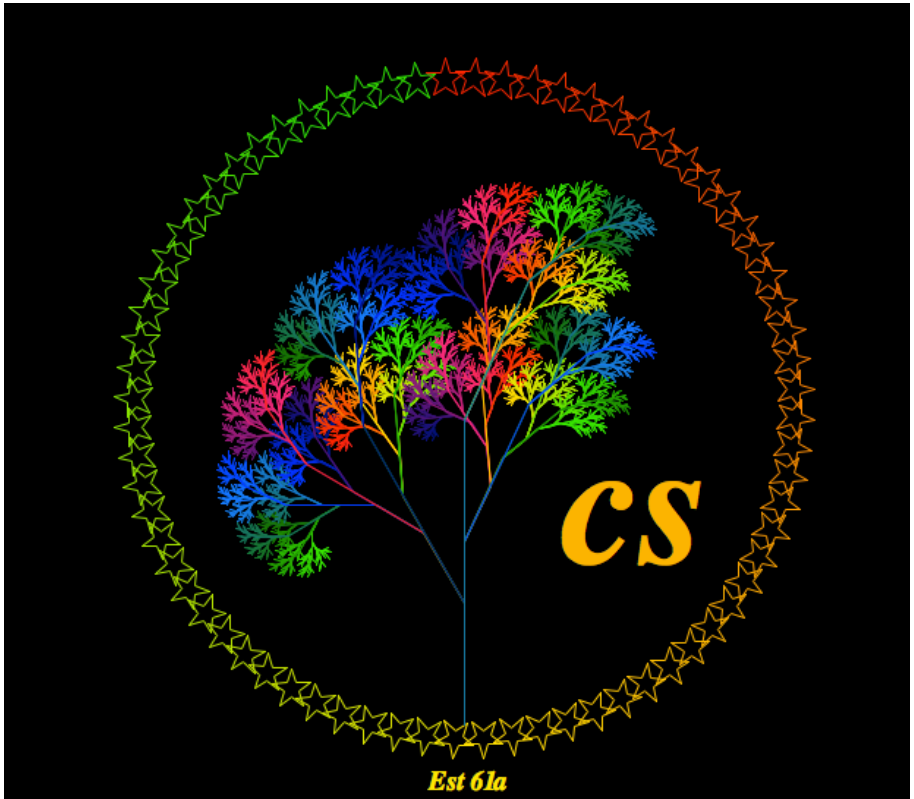
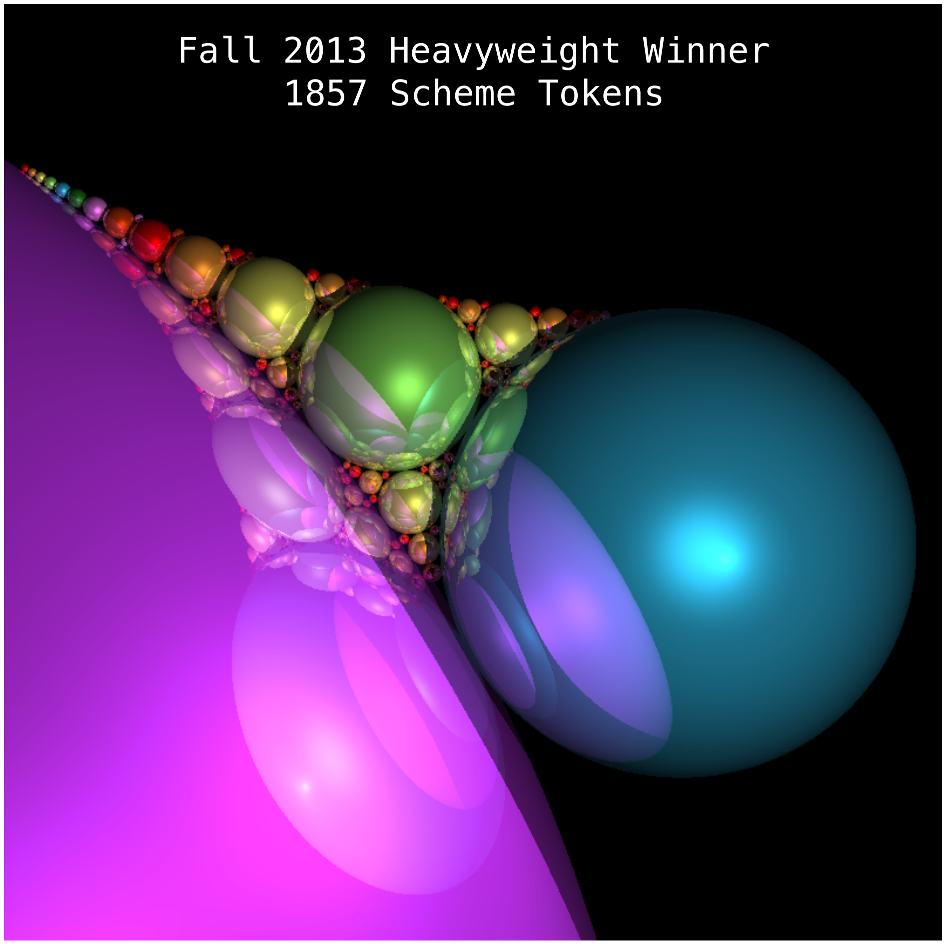


Fall 2012 Featherweight Winner
176 Scheme Tokens

Scheme Recursive Art Contest: Start Early!



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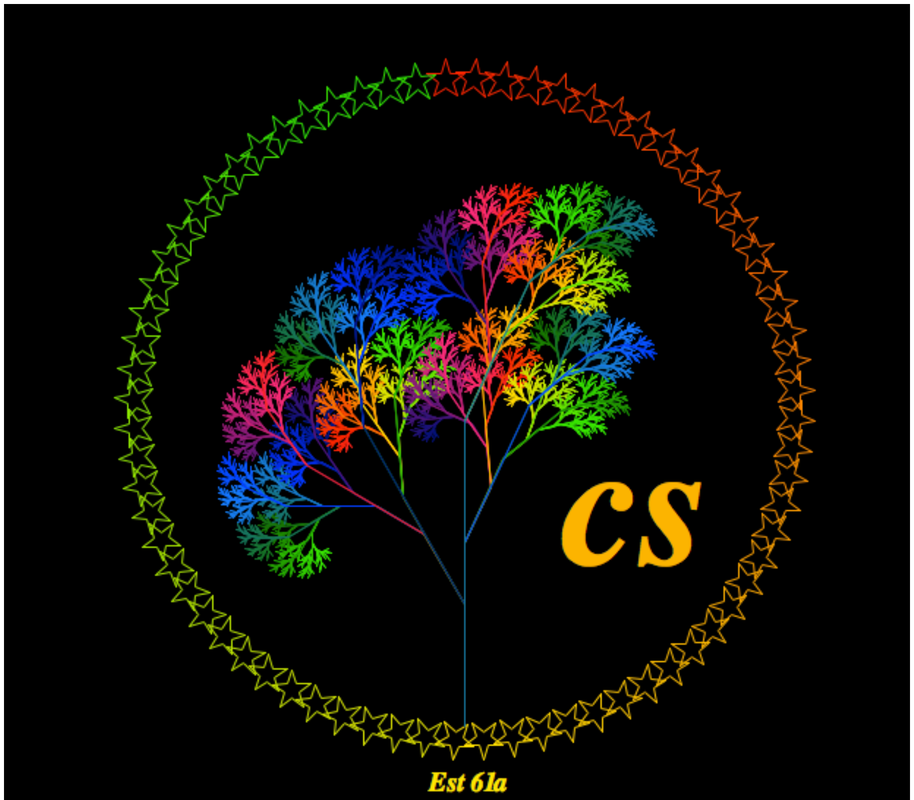

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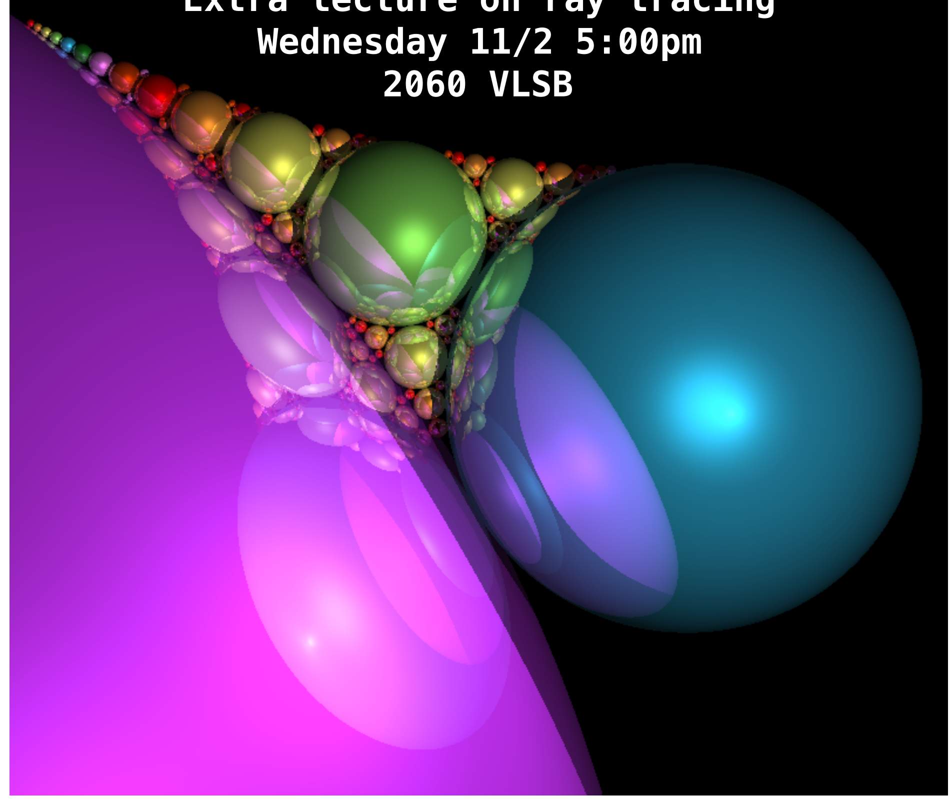

Fall 2013 Heavyweight Winner
1857 Scheme Tokens

Extra lecture on ray tracing
Wednesday 11/2 5:00pm
2060 VLSB

Dynamic Scope

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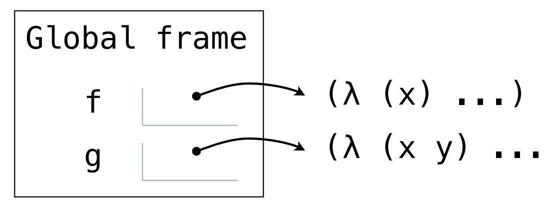


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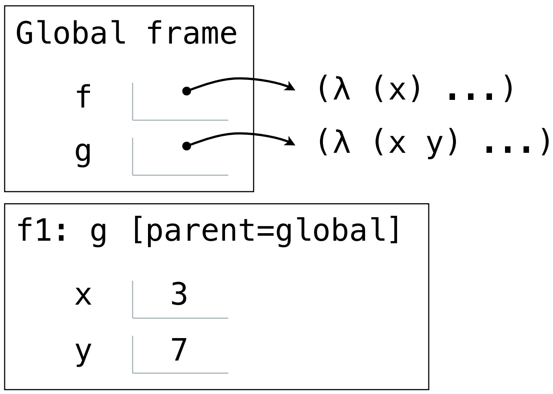

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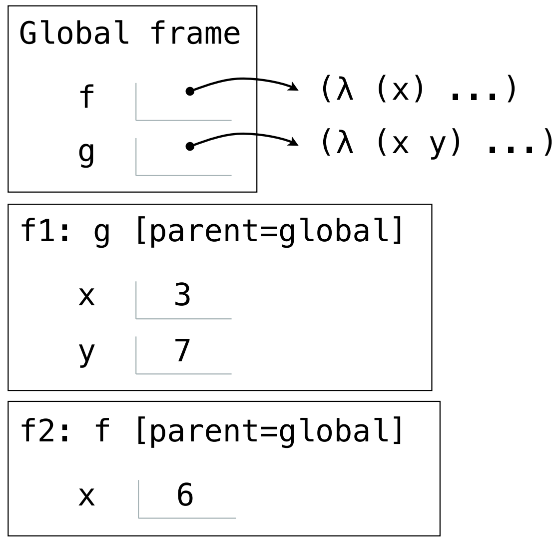

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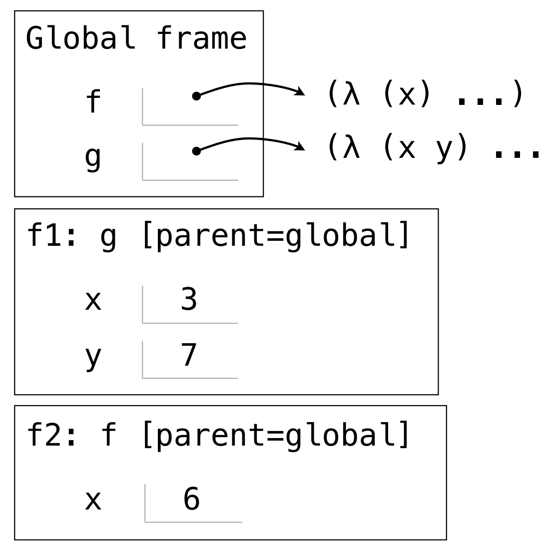

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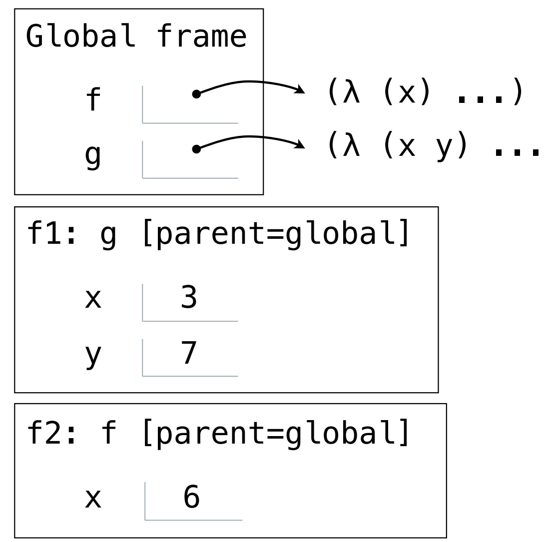

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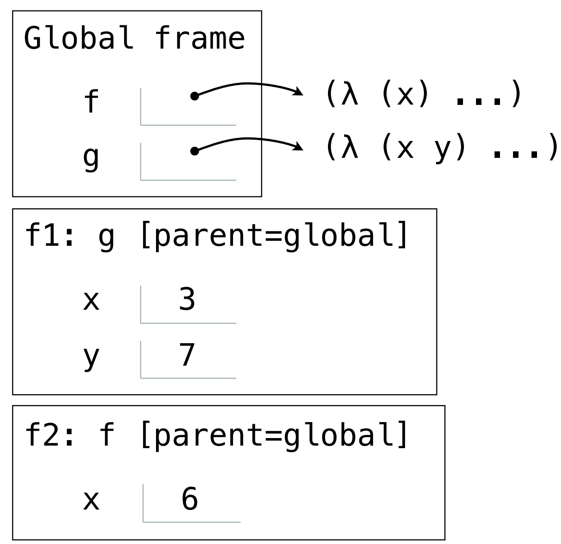

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Global frame

- f → (~~λ~~ *μ* (x) ...)
- g → (λ (x y) ...)

f1: g [parent=global]

- x 3
- y 7

f2: f [parent=global]

- x 6

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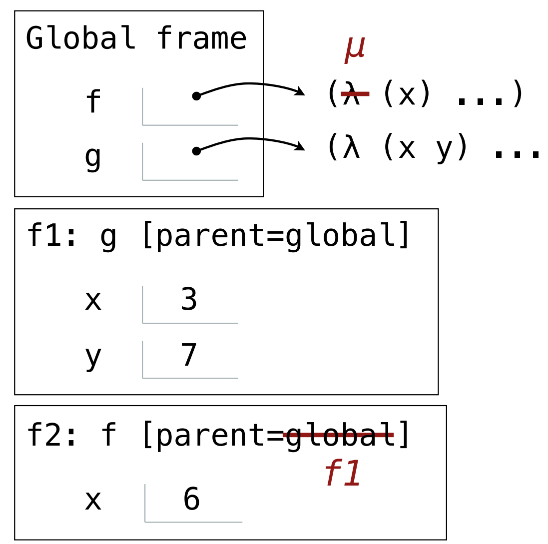

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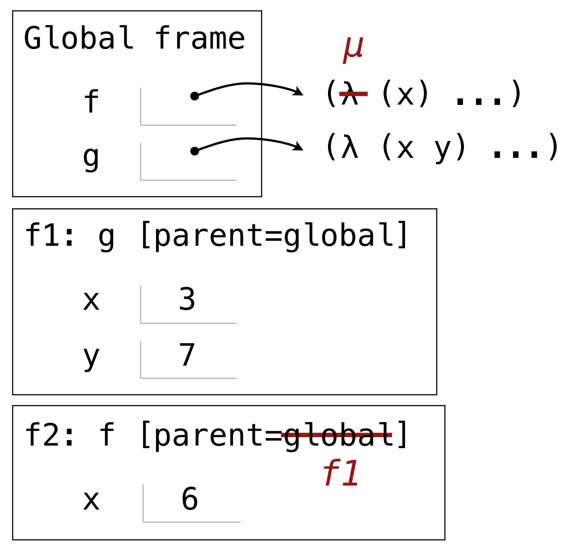

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13

Tail Recursion

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But... no for/while statements! Can we make basic iteration efficient? Yes!

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(Demo)

Interactive Diagram

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(define (length-tail s)
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Tail Recursion Examples

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Map and Reduce

Example: Reduce

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(define (reduce procedure s start)
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```
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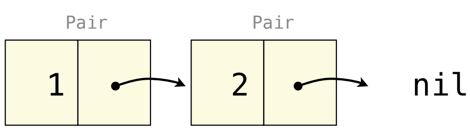

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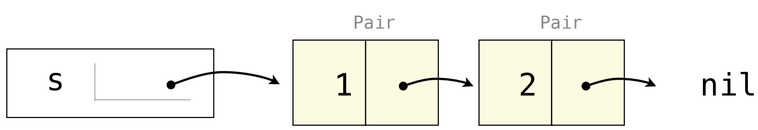

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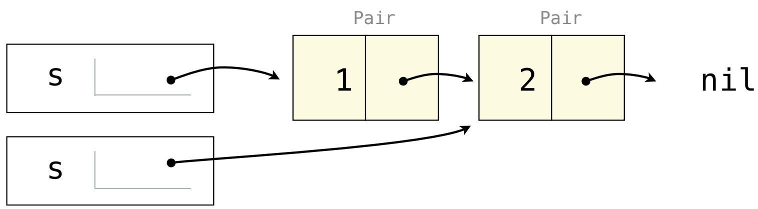

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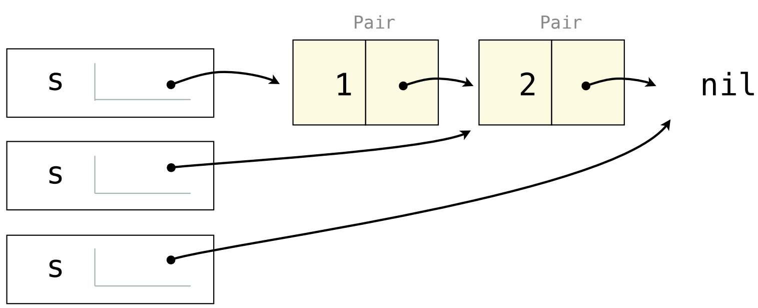

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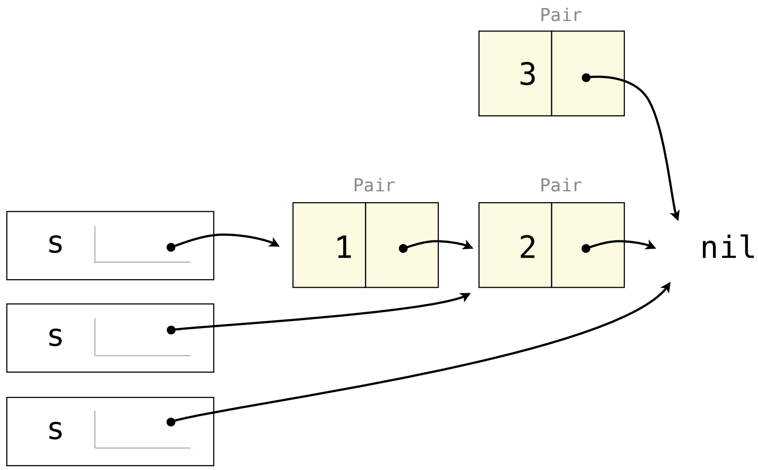

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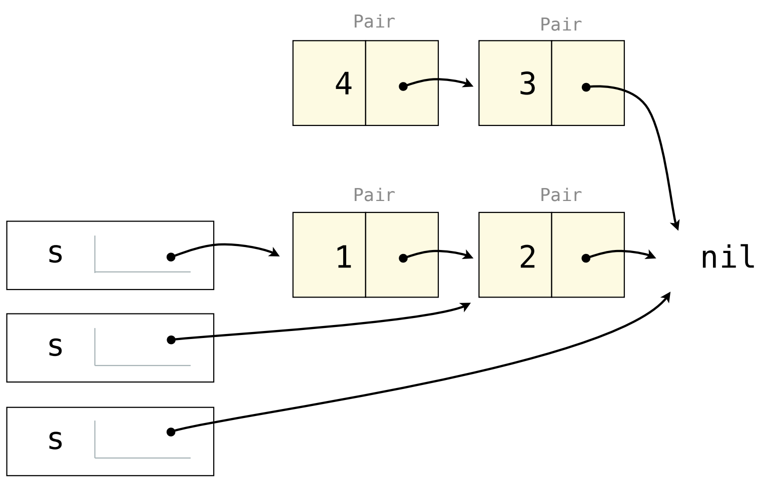

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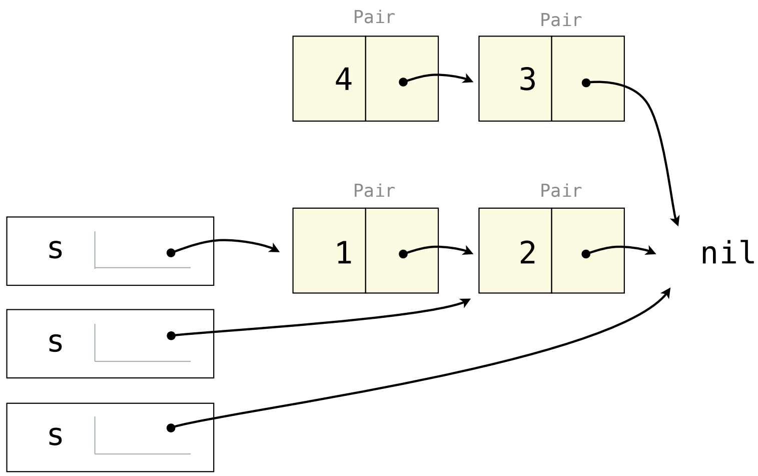

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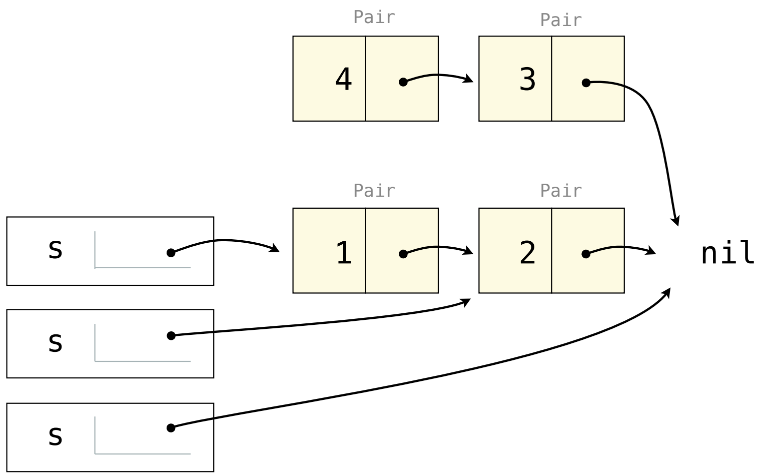

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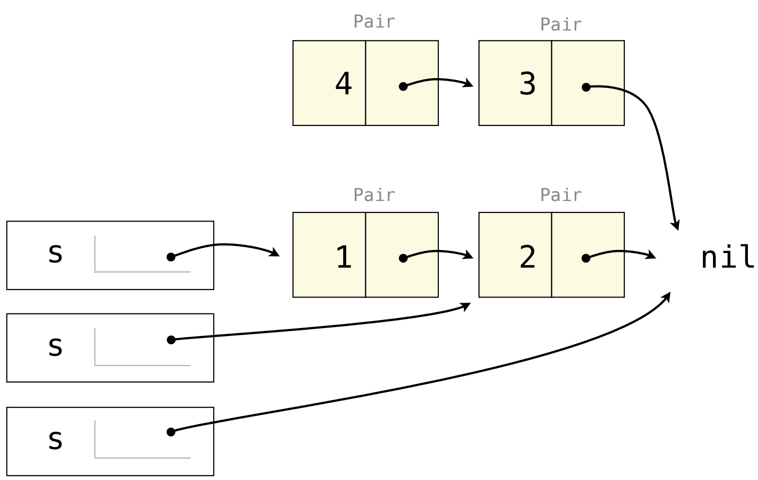

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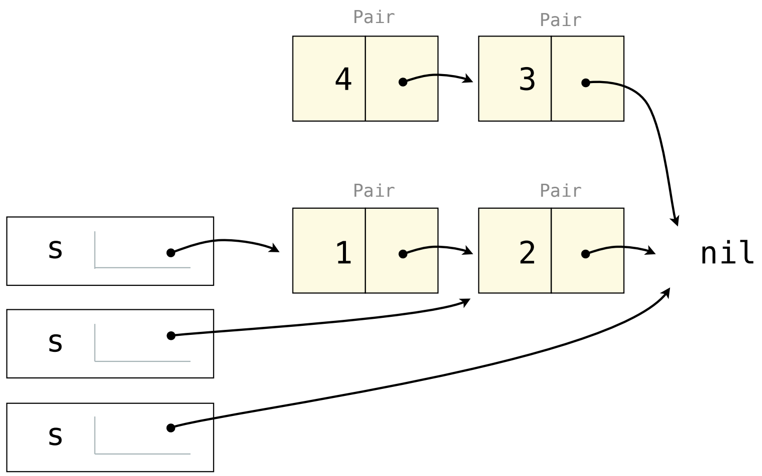

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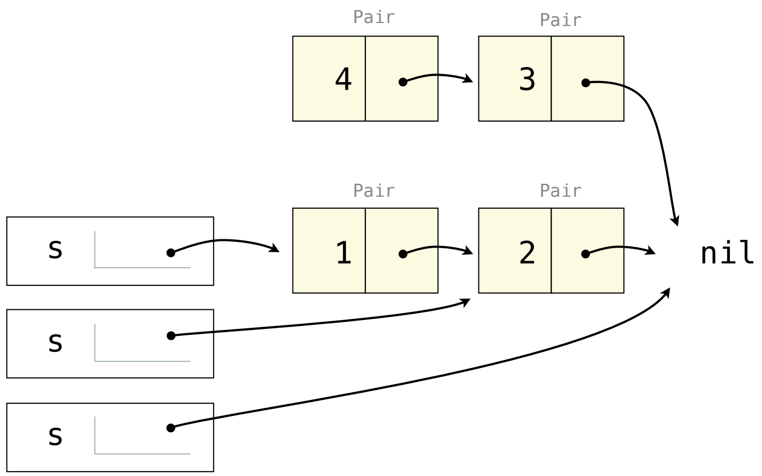

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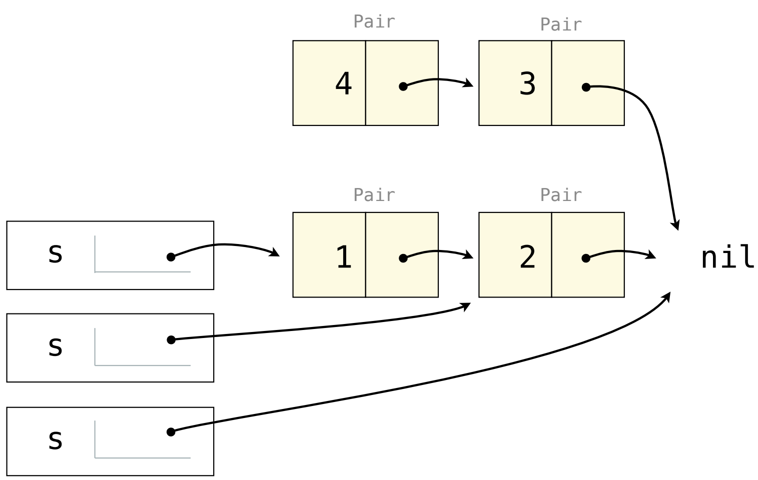

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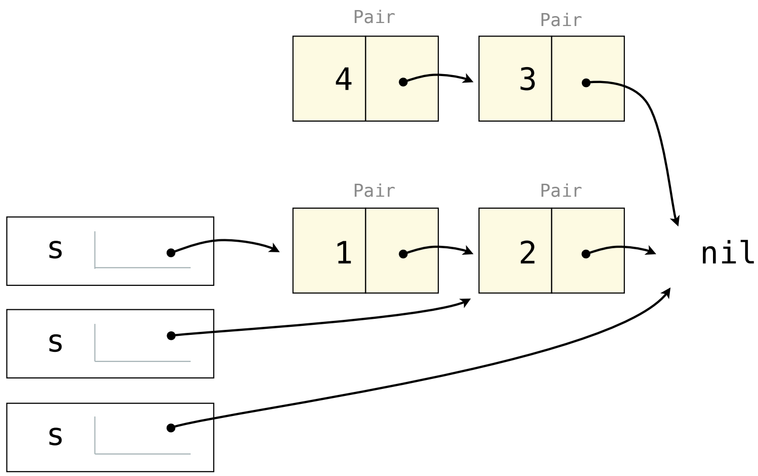

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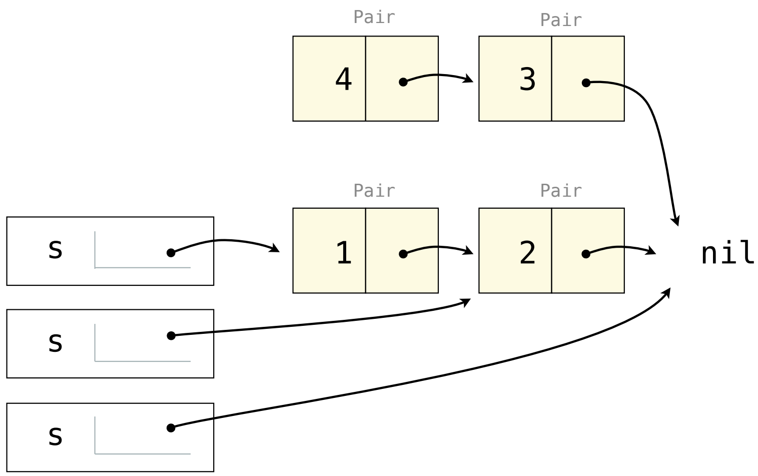


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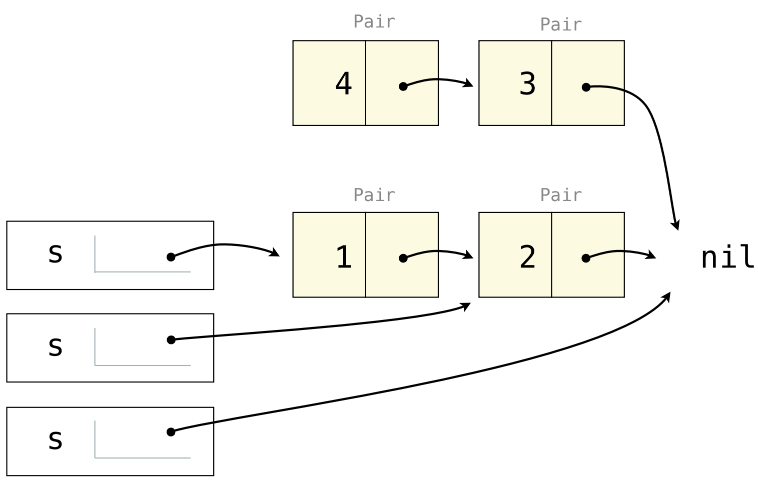

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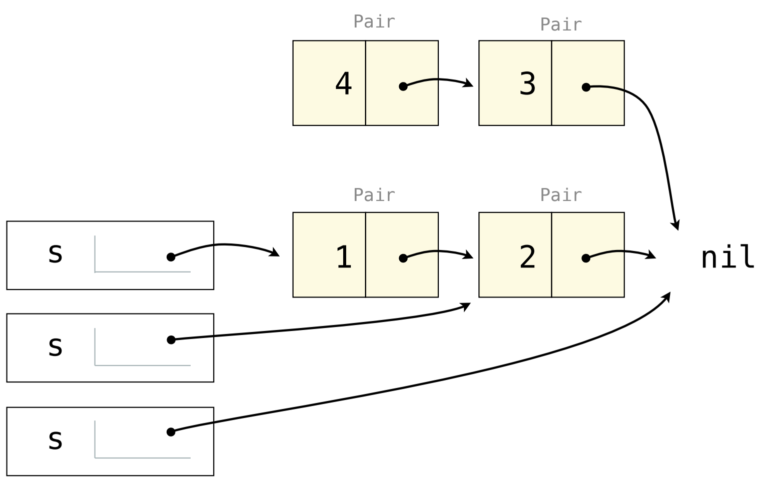


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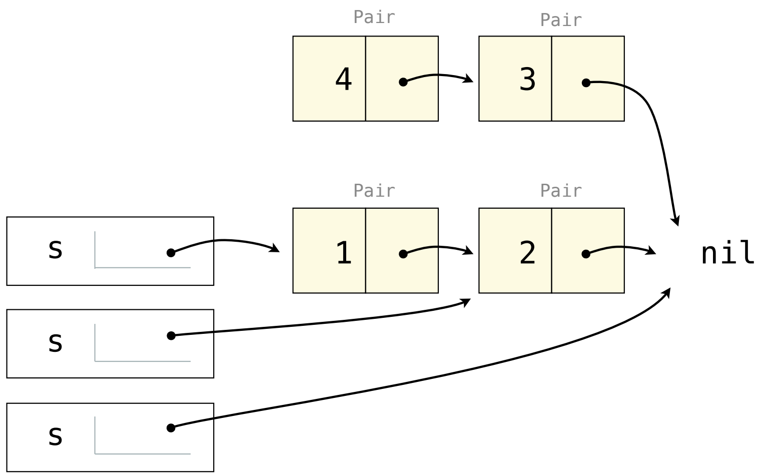


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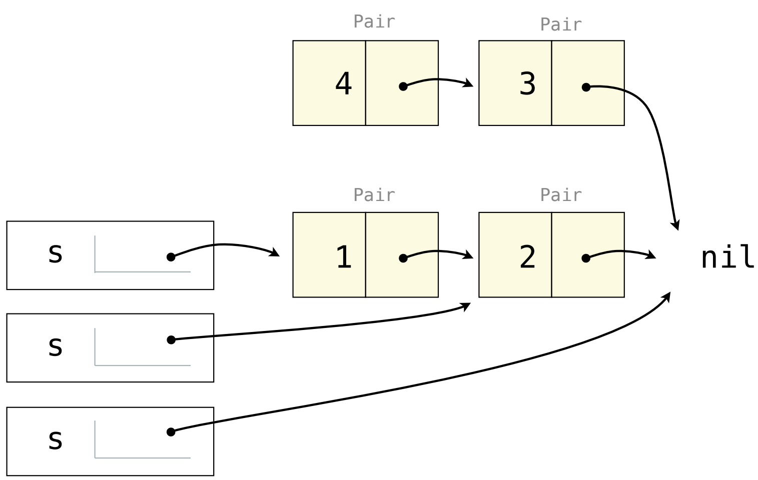


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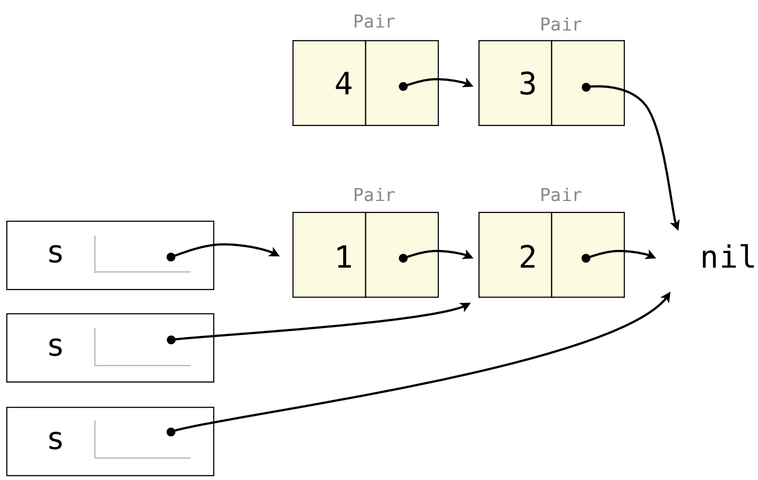


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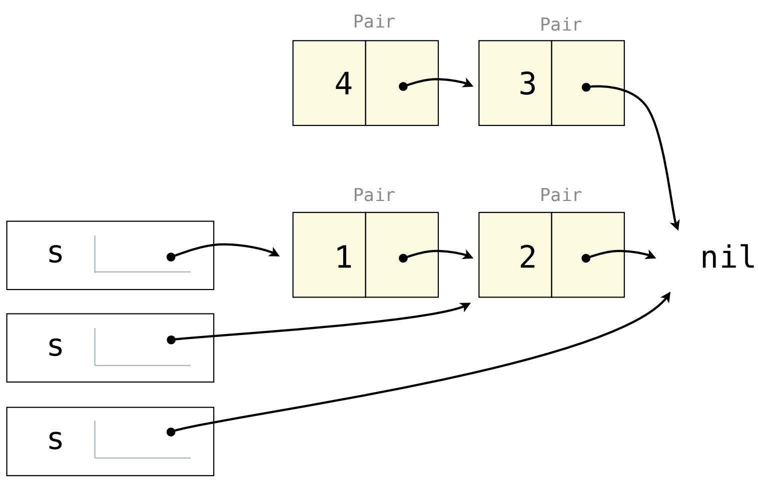


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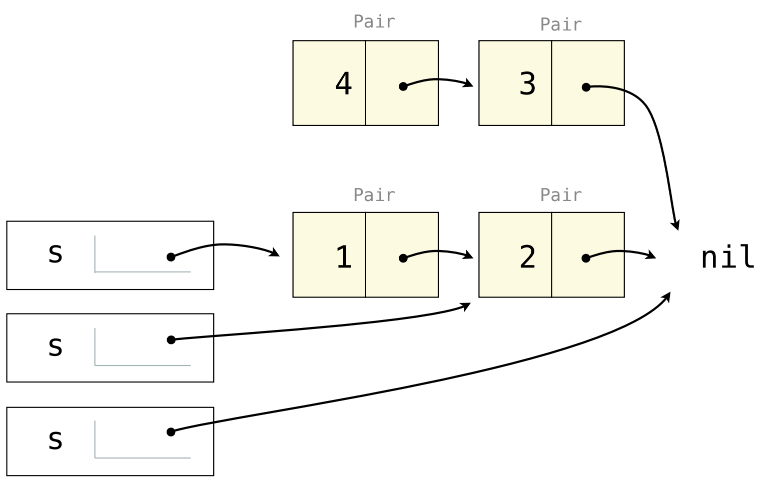


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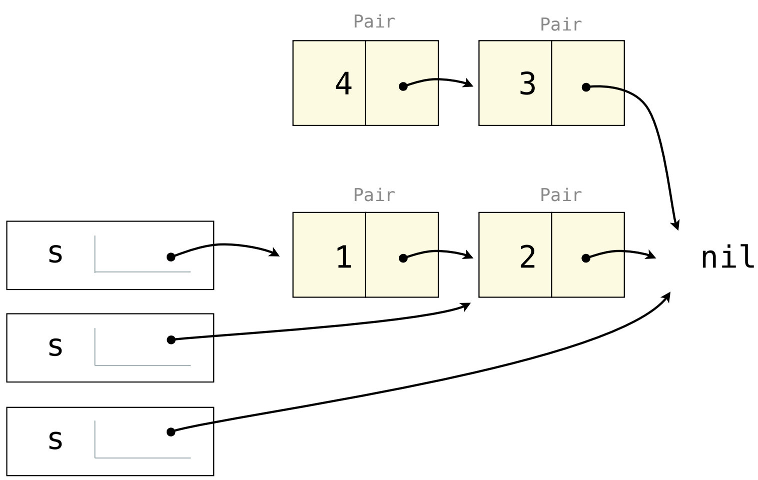


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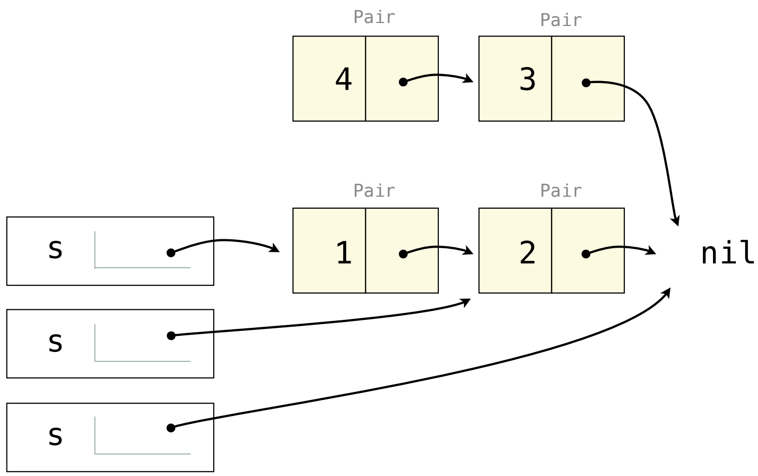


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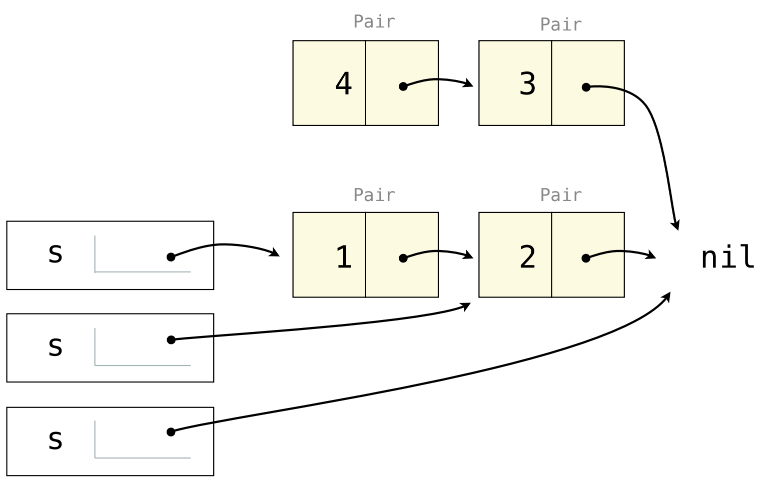


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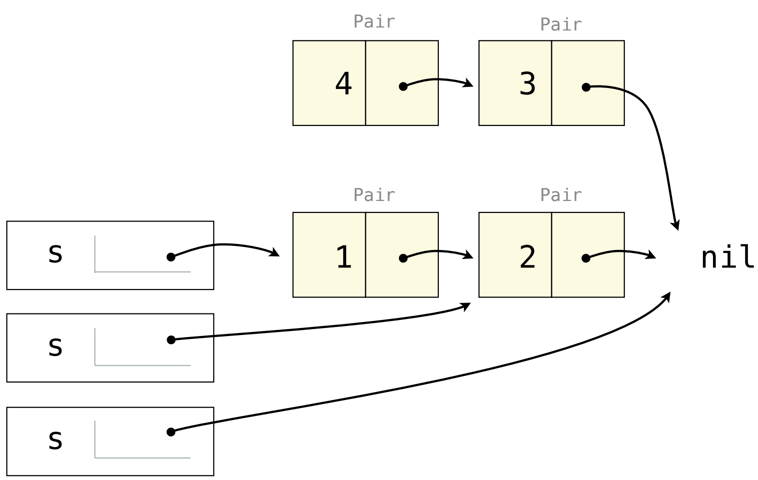


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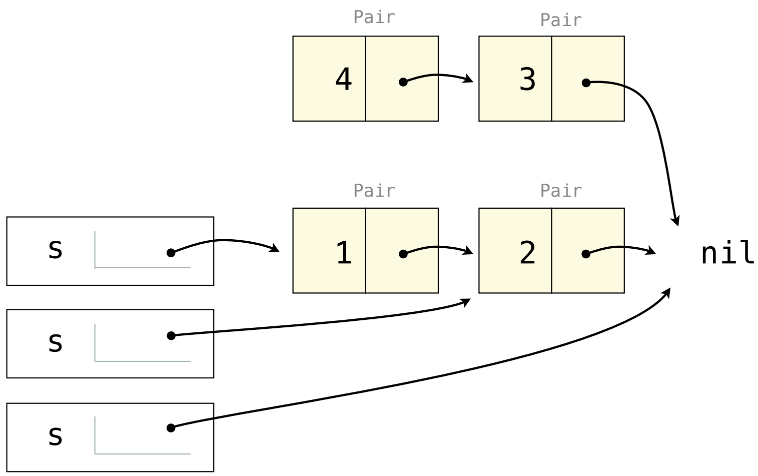


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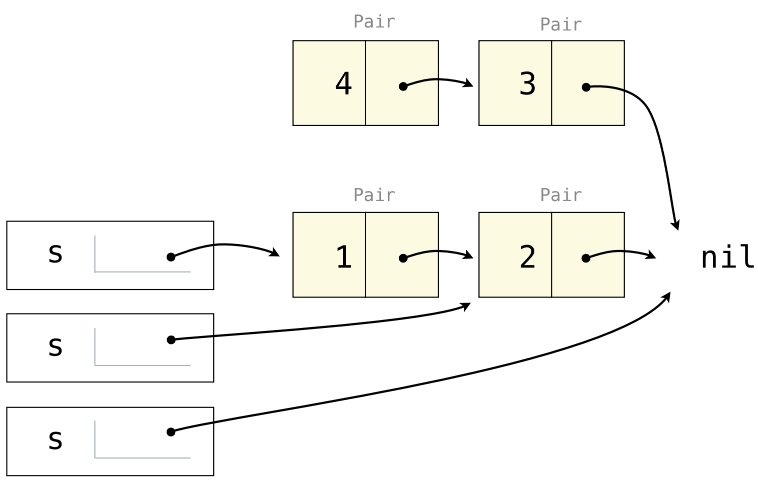


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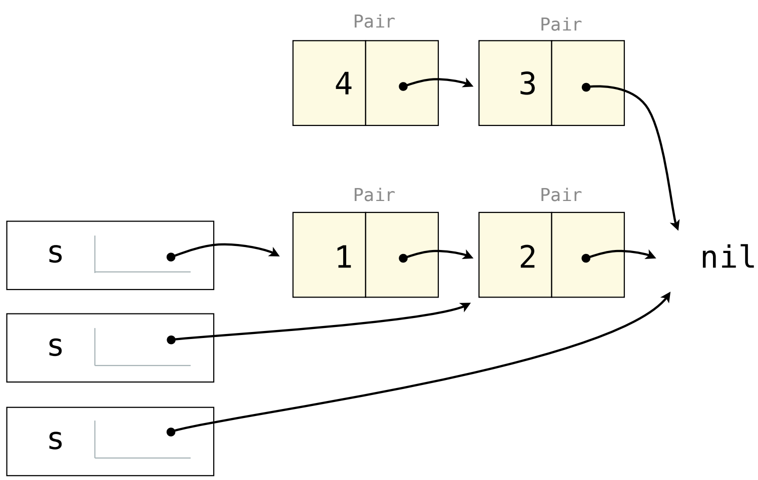


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General Computing Machines

An Analogy: Programs Define Machines

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```
Programs specify the logic of a computational device
```

An Analogy: Programs Define Machines

Programs specify the logic of a computational device

factorial

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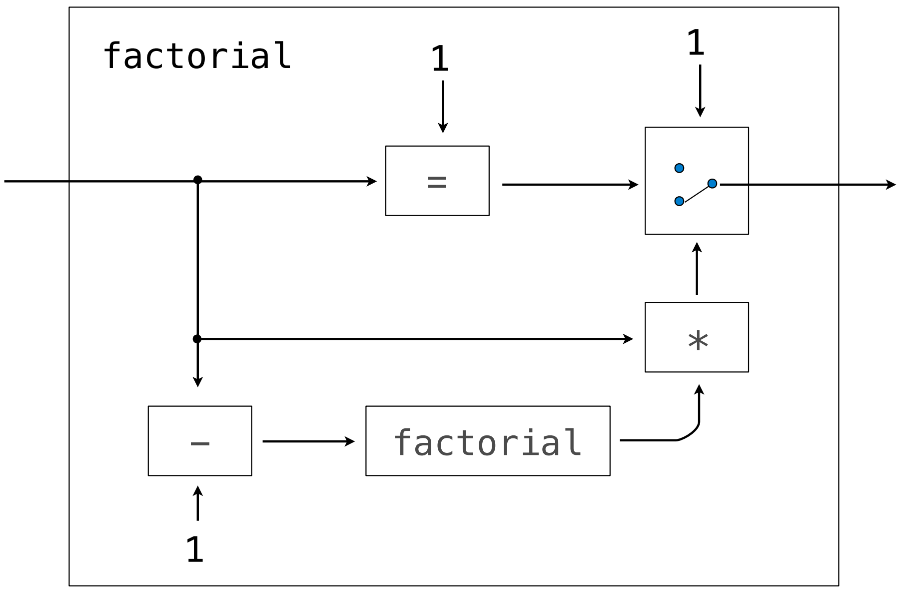

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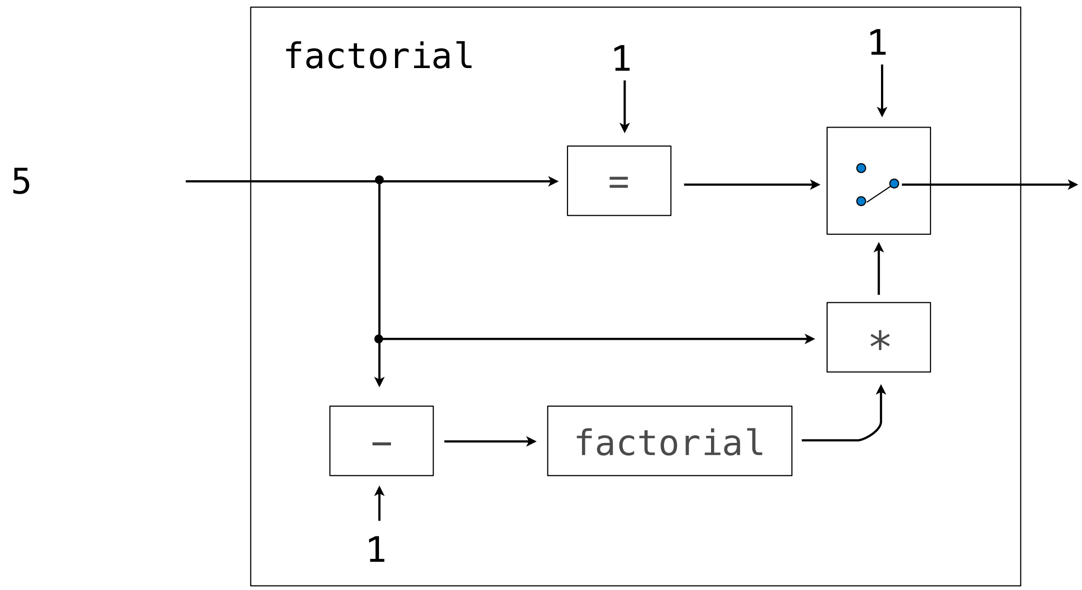

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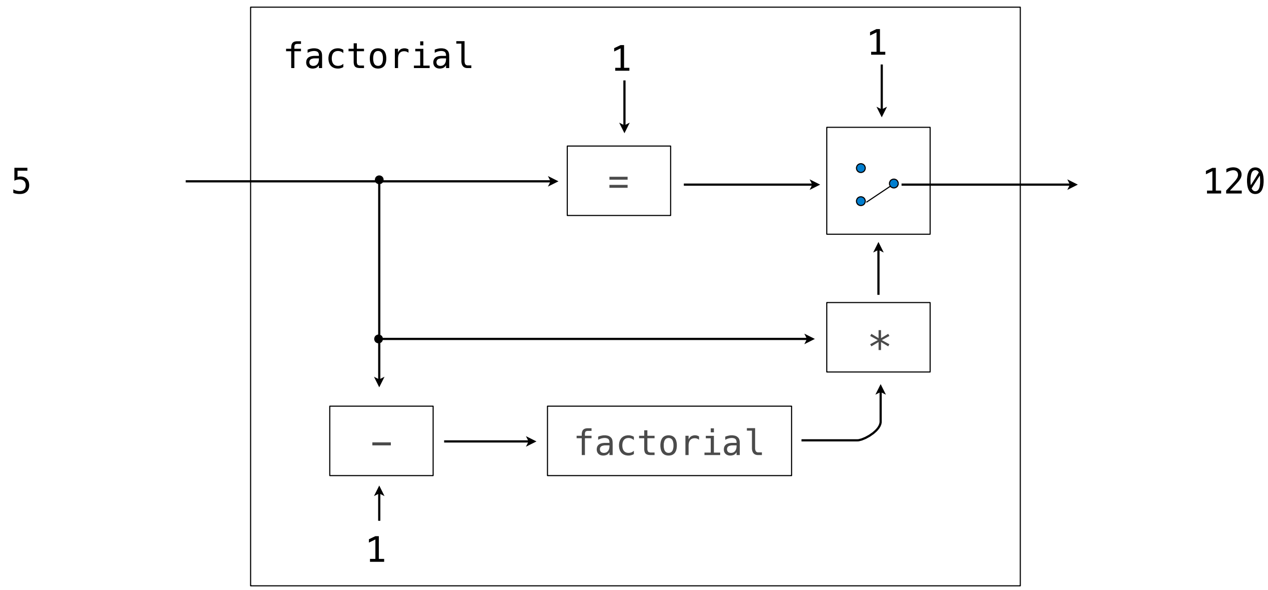

Interpreters are General Computing Machine

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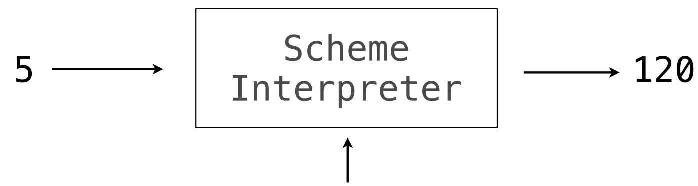


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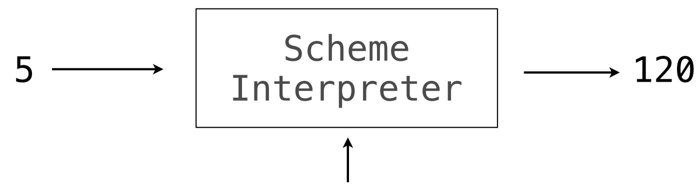


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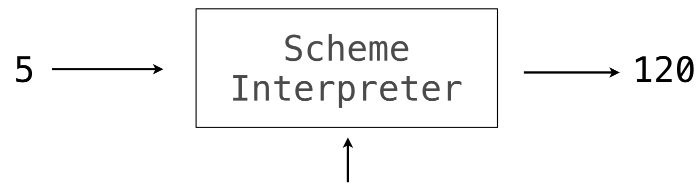


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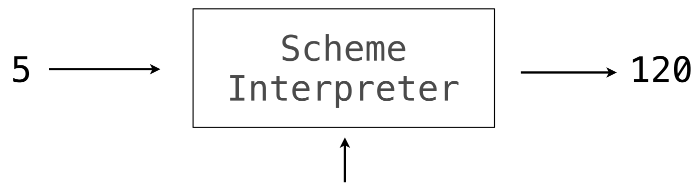
Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself

Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine




```
(define (factorial n)
  (if (zero? n) 1 (* n (factorial (- n 1)))))
```

Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself

Internally, it is just a set of evaluation rules