

## Environments

## Announcements

## Environments for Higher-Order Functions

### Environments Enable Higher-Order Functions

**Functions are first-class:** Functions are values in our programming language

**Higher-order function:** A function that takes a function as an argument value or  
A function that returns a function as a return value

*Environment diagrams describe how higher-order functions work!*

(Demo)

### Names can be Bound to Functional Arguments

```

1 def apply_twice(f, x):
2   return f(f(x))
3
4 def square(x):
5   return x * x
6
7 result = apply_twice(square, 2)
    
```

```

1 def apply_twice(f, x):
2   return f(f(x))
3
4 def square(x):
5   return x * x
6
7 result = apply_twice(square, 2)
    
```

## Environments for Nested Definitions

(Demo)

### Environment Diagrams for Nested Def Statements

```

1 def make_adder(n):
2   def adder(k):
3     return k + n
4   return adder
5
6 add_three = make_adder(3)
7 add_three(4)
    
```

- Every user-defined function has a parent frame (often global)
- The parent of a function is the frame in which it was defined
- Every local frame has a parent frame (often global)
- The parent of a frame is the parent of the function called

### How to Draw an Environment Diagram

When a function is defined:

Create a function value: `func <name>(<formal parameters>) [parent=<label>]`

Its parent is the current frame.

`f1: make_adder`      `func adder(k) [parent=f1]`

Bind <name> to the function value in the current frame

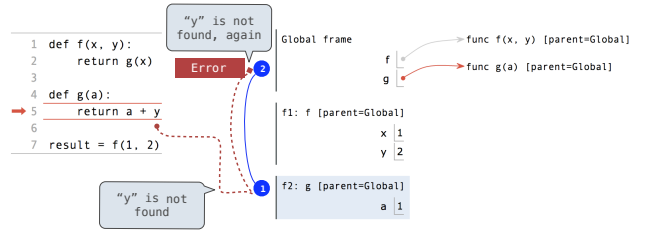
When a function is called:

1. Add a local frame, titled with the <name> of the function being called.
- ★ 2. Copy the parent of the function to the local frame: `[parent=<label>]`
3. Bind the <formal parameters> to the arguments in the local frame.
4. Execute the body of the function in the environment that starts with the local frame.

## Local Names

(Demo)

## Local Names are not Visible to Other (Non-Nested) Functions

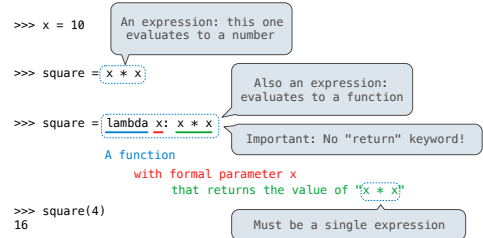


- An environment is a sequence of frames.
- The environment created by calling a top-level function (no def within def) consists of one local frame, followed by the global frame.

## Lambda Expressions

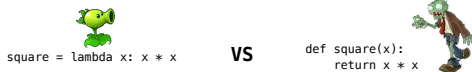
(Demo)

## Lambda Expressions

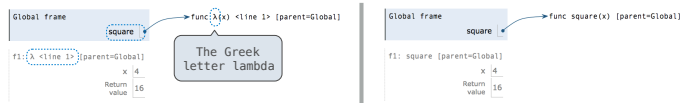


Lambda expressions are not common in Python, but important in general. Lambda expressions in Python cannot contain statements at all!

## Lambda Expressions Versus Def Statements



- Both create a function with the same domain, range, and behavior.
- Both bind that function to the name square.
- Only the def statement gives the function an intrinsic name, which shows up in environment diagrams but doesn't affect execution (unless the function is printed).



## Function Composition

(Demo)

## The Environment Diagram for Function Composition

