Streams

Announcements

Efficient Sequence Processing

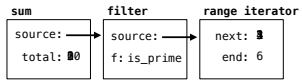
Sequence Operations

Map, filter, and reduce express sequence manipulation using compact expressions

Example: Sum all primes in an interval from **a** (inclusive) to **b** (exclusive)

```
def sum_primes(a, b):
    total = 0
    x = a
    while x < b:
        if is_prime(x):
            total = total + x
        x = x + 1
    return total
```

```
def sum_primes(a, b):
    return sum(filter(is_prime, range(a, b)))
sum_primes(1, 6)
```



Space: **Constant** **Also Constant**

(Demo)

Streams

Streams are Lazy Scheme Lists

A stream is a list, but the rest of the list is computed only when needed:

```
(car (cons 1 nil)) -> 1                    (car        (cons-stream 1 nil)) -> 1
(cdr (cons 1 nil)) -> ()                   (cdr-stream (cons-stream 1 nil)) -> ()
(cons 1 (cons 2 nil))                      (cons-stream 1 (cons-stream 2 nil))
```

Errors only occur when expressions are evaluated:

```
(cons        1 (cons        (/ 1 0) nil))              -> ERROR
(cons-stream 1 (cons-stream (/ 1 0) nil))              -> (1 . #[promise (not forced)])
(car        (cons-stream 1 (cons-stream (/ 1 0) nil))) -> 1
(cdr-stream (cons-stream 1 (cons-stream (/ 1 0) nil))) -> ERROR
```

(Demo)

Stream Ranges are Implicit

A stream can give on-demand access to each element in order

```
(define (range-stream a b)
  (if (>= a b)
      nil
      (cons-stream a (range-stream (+ a 1) b))))

(define lots (range-stream 1 1000000000000000000))

scm> (car lots)
1
scm> (car (cdr-stream lots))
2
scm> (car (cdr-stream (cdr-stream lots)))
3
```

Infinite Streams

Integer Stream

An integer stream is a stream of consecutive integers

The rest of the stream is not yet computed when the stream is created

```
(define (int-stream start)
  (cons-stream start (int-stream (+ start 1))))
```

(Demo)

Stream Processing

(Demo)

Recursively Defined Streams

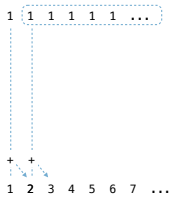
The rest of a constant stream is the constant stream

```
(define ones (cons-stream 1 ones))
```

Combine two streams by separating each into car and cdr

```
(define (add-streams s t)
  (cons-stream (+ (car s) (car t))
               (add-streams (cdr-stream s)
                            (cdr-stream t))))
```

```
(define ints (cons-stream 1 (add-streams ones ints)))
```



Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))
```

What's (prefix a 8)? (1 2 3 1 2 3 1 2)

What's (prefix (f a) 8)? (1 1 2 2 3 3 1 1)

What's (prefix (g a) 8)? (1 2 2 3 3 3 3 1)

Higher-Order Stream Functions

Higher-Order Functions on Streams

Implementations are identical, but change cons to cons-stream and change cdr to cdr-stream

```
(define (map-stream f s)
  (if (null? s)
      nil
      (cons-stream (f (car s))
                   (map-stream f
                               (cdr-stream s)))))

(define (filter-stream f s)
  (if (null? s)
      nil
      (if (f (car s))
          (cons-stream (car s)
                       (filter-stream f (cdr-stream s)))
          (filter-stream f (cdr-stream s)))))

(define (reduce-stream f start)
  (if (null? s)
      start
      (reduce-stream f
                     (cdr-stream s)
                     (f start (car s)))))
```

A Stream of Primes

For any prime k, any larger prime must not be divisible by k.

The stream of integers not divisible by any k <= n is:
The stream of integers not divisible by any k < n
Filtered to remove any element divisible by n
This recurrence is called the Sieve of Eratosthenes

2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, ~~9~~, ~~10~~, 11, ~~12~~, 13

(Demo)