

Rational implementation using functions:

```
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
        elif name == 'd':
            return d
    return select
```

This function represents a rational number

```
def numer(x):
    return x('n')
```

Constructor is a higher-order function

```
def denom(x):
    return x('d')
```

Selector calls x

Lists:

```
>>> digits = [1, 8, 2, 8]
>>> len(digits)
4
>>> digits[3]
8
```

```
>>> [2, 7] + digits * 2
[2, 7, 1, 8, 2, 8, 1, 8, 2, 8]
>>> pairs = [[10, 20], [30, 40]]
>>> pairs[1]
[30, 40]
>>> pairs[1][0]
30
```

Executing a for statement:

```
for <name> in <expression>:
    <suite>
```

- Evaluate the header `<expression>`, which must yield an iterable value (a list, tuple, iterator, etc.)
- For each element in that sequence, in order:
 - Bind `<name>` to that element in the current frame
 - Execute the `<suite>`

Unpacking in a for statement:

```
>>> pairs = [[1, 2], [2, 2], [3, 2], [4, 4]]
>>> same_count = 0
```

A name for each element in a fixed-length sequence

```
>>> for x, y in pairs:
...     if x == y:
...         same_count = same_count + 1
>>> same_count
2
```

A sequence of fixed-length sequences

```
..., -3, -2, -1, 0, 1, 2, 3, 4, ...
```

range(-2, 2)

Length: ending value - starting value
Element selection: starting value + index

```
>>> list(range(-2, 2))
[-2, -1, 0, 1]
```

List constructor

```
>>> list(range(4))
[0, 1, 2, 3]
```

Range with a 0 starting value

Membership:

```
>>> digits = [1, 8, 2, 8]
>>> 2 in digits
True
>>> 1828 not in digits
True
```

Slicing:

```
>>> digits[0:2]
[1, 8]
>>> digits[1:]
[8, 2, 8]
```

Slicing creates a new object

Identity:

```
<exp0> is <exp1>
```

evaluates to True if both `<exp0>` and `<exp1>` evaluate to the same object

Equality:

```
<exp0> == <exp1>
```

evaluates to True if both `<exp0>` and `<exp1>` evaluate to equal values

Identical objects are always equal values

```
iter(iterable):
    Return an iterator over the elements of an iterable value
next(iterator):
    Return the next element
```

```
>>> s = [3, 4, 5]
>>> t = iter(s)
>>> next(t)
3
>>> next(t)
4
```

```
>>> d = {'one': 1, 'two': 2, 'three': 3}
>>> k = iter(d)
>>> next(k)
'one'
>>> next(k)
'two'
```

A generator function is a function that yields values instead of returning them.

```
>>> def plus_minus(x):
...     yield x
...     yield -x
>>> t = plus_minus(3)
>>> next(t)
3
>>> next(t)
-3
```

```
>>> def a_then_b(a, b):
...     yield from a
...     yield from b
>>> list(a_then_b([3, 4], [5, 6]))
[3, 4, 5, 6]
```

List comprehensions:

```
[<map exp> for <name> in <iter exp> if <filter exp>]
```

Short version: `[<map exp> for <name> in <iter exp>]`

A combined expression that evaluates to a list using this evaluation procedure:

- Add a new frame with the current frame as its parent
- Create an empty result list that is the value of the expression
- For each element in the iterable value of `<iter exp>`:
 - Bind `<name>` to that element in the new frame from step 1
 - If `<filter exp>` evaluates to a true value, then add the value of `<map exp>` to the result list

The result of calling `repr` on a value is what Python prints in an interactive session

```
>>> 12e12
12000000000000.0
>>> print(repr(12e12))
12000000000000.0
```

The result of calling `str` on a value is what Python prints using the `print` function

```
>>> today = datetime.date(2019, 10, 13)
>>> print(today)
2019-10-13
```

`str` and `repr` are both polymorphic; they apply to any object `repr` invokes a zero-argument method `__repr__` on its argument

```
>>> today.__repr__()
'datetime.date(2019, 10, 13)'
>>> today.__str__()
'2019-10-13'
```

Type dispatching: Look up a cross-type implementation of an operation based on the types of its arguments

Type coercion: Look up a function for converting one type to another, then apply a type-specific implementation.

Functions that aggregate iterable arguments

- `sum(iterable[, start])` -> value *sum of all values*
- `max(iterable[, key=func])` -> value *largest value*
- `max(a, b, c, ...[, key=func])` -> value
- `min(iterable[, key=func])` -> value *smallest value*
- `min(a, b, c, ...[, key=func])` -> value
- `all(iterable)` -> bool *whether all are true*
- `any(iterable)` -> bool *whether any is true*

```
>>> sum([1, 2])
3
>>> sum([1, 2], 3)
6
>>> sum([])
0
>>> all([False, True])
False
>>> all([])
True
```

Many built-in Python sequence operations return iterators that compute results lazily

`map(func, iterable)`: Iterate over `func(x)` for `x` in `iterable`

`filter(func, iterable)`: Iterate over `x` in `iterable` if `func(x)`

`zip(first_iter, second_iter)`: Iterate over co-indexed `(x, y)` pairs

`reversed(sequence)`: Iterate over `x` in a sequence in reverse order

`list(iterable)`: Create a list containing all `x` in `iterable`

`tuple(iterable)`: Create a tuple containing all `x` in `iterable`

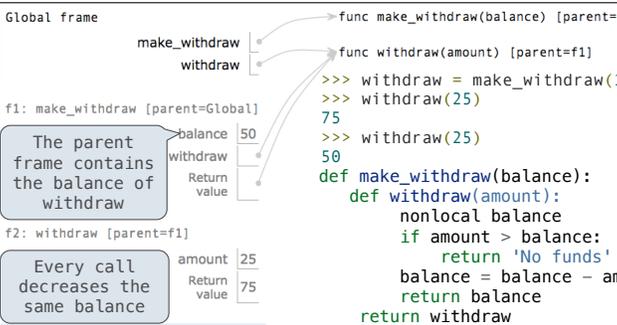
`sorted(iterable)`: Create a sorted list containing `x` in `iterable`

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
>>> cascade(123)
123
12
1
12
123
123
```

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```





Status: `x = 2`

- No nonlocal statement
- "x" is not bound locally

Effect: Create a new binding from name "x" to number 2 in the first frame of the current environment

- No nonlocal statement
- "x" is bound locally

Effect: Re-bind name "x" to object 2 in the first frame of the current environment

- nonlocal x
- "x" is bound in a non-local frame

Effect: Re-bind "x" to 2 in the first non-local frame of the current environment in which "x" is bound

- nonlocal x
- "x" is not bound in a non-local frame

Effect: SyntaxError: no binding for nonlocal 'x' found

- nonlocal x
- "x" is bound in a non-local frame
- "x" also bound locally

Effect: SyntaxError: name 'x' is parameter and nonlocal

List & dictionary mutation:

```
>>> a = [10]
>>> b = a
>>> a == b
True
>>> a.append(20)
>>> a == b
True
>>> a
[10, 20]
>>> b
[10, 20]
```

```
>>> a = [10]
>>> b = [10]
>>> a == b
True
>>> b.append(20)
>>> a
[10]
>>> b
[10, 20]
>>> a == b
False
```

```
>>> nums = {'I': 1.0, 'V': 5, 'X': 10}
>>> nums['X']
10
>>> nums['I'] = 1
>>> nums['L'] = 50
>>> nums
{'X': 10, 'L': 50, 'V': 5, 'I': 1}
>>> sum(nums.values())
66
>>> dict([(3, 9), (4, 16), (5, 25)])
{3: 9, 4: 16, 5: 25}
>>> nums.get('A', 0)
0
>>> nums.get('V', 0)
5
>>> {x: x*x for x in range(3,6)}
{3: 9, 4: 16, 5: 25}
```

```
>>> any([False, True])
True
>>> any([])
False
>>> max(1, 2)
2
>>> max([1, 2])
2
>>> max([1, -2], key=abs)
-2
```

You can copy a list by calling the list constructor or slicing the list from the beginning to the end.

```
>>> suits = ['coin', 'string', 'myriad']
>>> suits.pop()
'myriad'
>>> suits.remove('string')
>>> suits.append('cup')
>>> suits.extend(['sword', 'club'])
>>> suits[2] = 'spade'
>>> suits
['coin', 'cup', 'spade', 'club']
>>> suits[0:2] = ['diamond']
>>> suits
['diamond', 'spade', 'club']
>>> suits.insert(0, 'heart')
>>> suits
['heart', 'diamond', 'spade', 'club']
```

Remove and return the last element

Remove a value

Add all values

Replace a slice with values

Add an element at an index

False values:

- Zero
- False
- None
- An empty string, list, dict, tuple

```
>>> bool(0)
False
>>> bool(1)
True
>>> bool('')
False
>>> bool('0')
True
>>> bool([])
False
>>> bool([[]])
True
>>> bool({})
False
>>> bool(())
False
>>> bool(lambda x: 0)
True
```



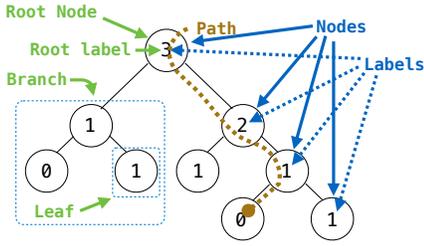
All other values are true values.

Recursive description:

- A tree has a root label and a list of branches
- Each branch is a tree
- A tree with zero branches is called a leaf

Relative description:

- Each location is a node
- Each node has a label
- One node can be the parent/child of another



```
def tree(label, branches=[]):
```

for branch in branches:
 assert is_tree(branch)
 return [label] + list(branches)

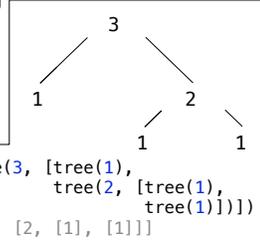
def label(tree):
 return tree[0]

def branches(tree):
 return tree[1:]

def is_tree(tree):
 if type(tree) != list or len(tree) < 1:
 return False
 for branch in branches(tree):
 if not is_tree(branch):
 return False
 return True

def is_leaf(tree):
 return not branches(tree)

def leaves(t):
 if n == 0 or n == 1:
 return tree(n)
 else:
 left = fib_tree(n-2),
 right = fib_tree(n-1)
 fib_n = label(left) + label(right)
 return tree(fib_n, [left, right])



```
class Tree:
```

def __init__(self, label, branches=[]):
 self.label = label
 for branch in branches:
 assert isinstance(branch, Tree)
 self.branches = list(branches)

def is_leaf(self):
 return not self.branches

def leaves(tree):
 "The leaf values in a tree."
 if tree.is_leaf():
 return [tree.label]
 else:
 return sum([leaves(b) for b in tree.branches], [])

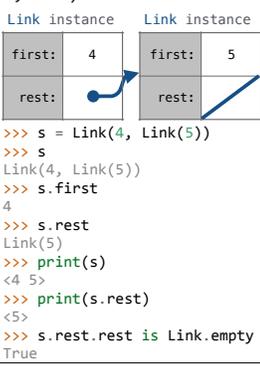
```
class Link:
```

empty = ()

def __init__(self, first, rest=empty):
 assert rest is Link.empty or isinstance(rest, Link)
 self.first = first
 self.rest = rest

def __repr__(self):
 if self.rest:
 rest = ', ' + repr(self.rest)
 else:
 rest = ''
 return 'Link(' + repr(self.first) + rest + ')'

def __str__(self):
 string = '<'
 while self.rest is not Link.empty:
 string += str(self.first) + '> '
 self = self.rest
 return string + str(self.first) + '>'



Anatomy of a recursive function:

- The def statement header is like any function
- Conditional statements check for base cases
- Base cases are evaluated without recursive calls
- Recursive cases are evaluated with recursive calls

Recursive decomposition: finding simpler instances of a problem. E.g., count_partitions(6, 4)

def count_partitions(n, m):
 if n == 0:
 return 1
 elif n < 0:
 return 0
 elif m == 0:
 return 0
 else:
 with_m = count_partitions(n-m, m)
 without_m = count_partitions(n, m-1)
 return with_m + without_m

Python object system:

Idea: All bank accounts have a balance and an account holder; the Account class should add those attributes to each of its instances

```
>>> a = Account('Jim')
>>> a.holder
'Jim'
>>> a.balance
0
```

When a class is called:
 1. A new instance of that class is created:
 2. The __init__ method of the class is called with the new object as its first argument (named self), along with any additional arguments provided in the call expression.

```
class Account:
    def __init__(self, account_holder):
        self.balance = 0
        self.holder = account_holder
    def deposit(self, amount):
        self.balance = self.balance + amount
        return self.balance
    def withdraw(self, amount):
        if amount > self.balance:
            return 'Insufficient funds'
        self.balance = self.balance - amount
        return self.balance
```

```
>>> type(Account.deposit)
<class 'function'>
>>> type(a.deposit)
<class 'method'>
```

```
>>> Account.deposit(a, 5)
10
>>> a.deposit(2)
12
```

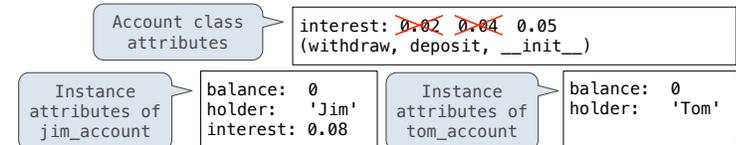
`<expression> . <name>`

The <expression> can be any valid Python expression. The <name> must be a simple name. Evaluates to the value of the attribute looked up by <name> in the object that is the value of the <expression>.

- To evaluate a dot expression:
1. Evaluate the <expression> to the left of the dot, which yields the object of the dot expression
 2. <name> is matched against the instance attributes of that object; if an attribute with that name exists, its value is returned
 3. If not, <name> is looked up in the class, which yields a class attribute value
 4. That value is returned unless it is a function, in which case a bound method is returned instead

Assignment statements with a dot expression on their left-hand side affect attributes for the object of that dot expression

- If the object is an instance, then assignment sets an instance attribute
- If the object is a class, then assignment sets a class attribute



```
>>> jim_account = Account('Jim')
>>> tom_account = Account('Tom')
>>> tom_account.interest
0.02
>>> jim_account.interest
0.02
>>> Account.interest = 0.04
>>> tom_account.interest
0.04
>>> jim_account.interest
0.04
```

```
class CheckingAccount(Account):
    """A bank account that charges for withdrawals."""
    withdraw_fee = 1
    interest = 0.01
    def withdraw(self, amount):
        return Account.withdraw(self, amount + self.withdraw_fee)
        or
        return super().withdraw(amount + self.withdraw_fee)
```

To look up a name in a class:
 1. If it names an attribute in the class, return the attribute value.
 2. Otherwise, look up the name in the base class, if there is one.

```
>>> ch = CheckingAccount('Tom') # Calls Account.__init__
>>> ch.interest # Found in CheckingAccount
0.01
>>> ch.deposit(20) # Found in Account
20
>>> ch.withdraw(5) # Found in CheckingAccount
14
```

