

## 61A Lecture 27

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## Announcements

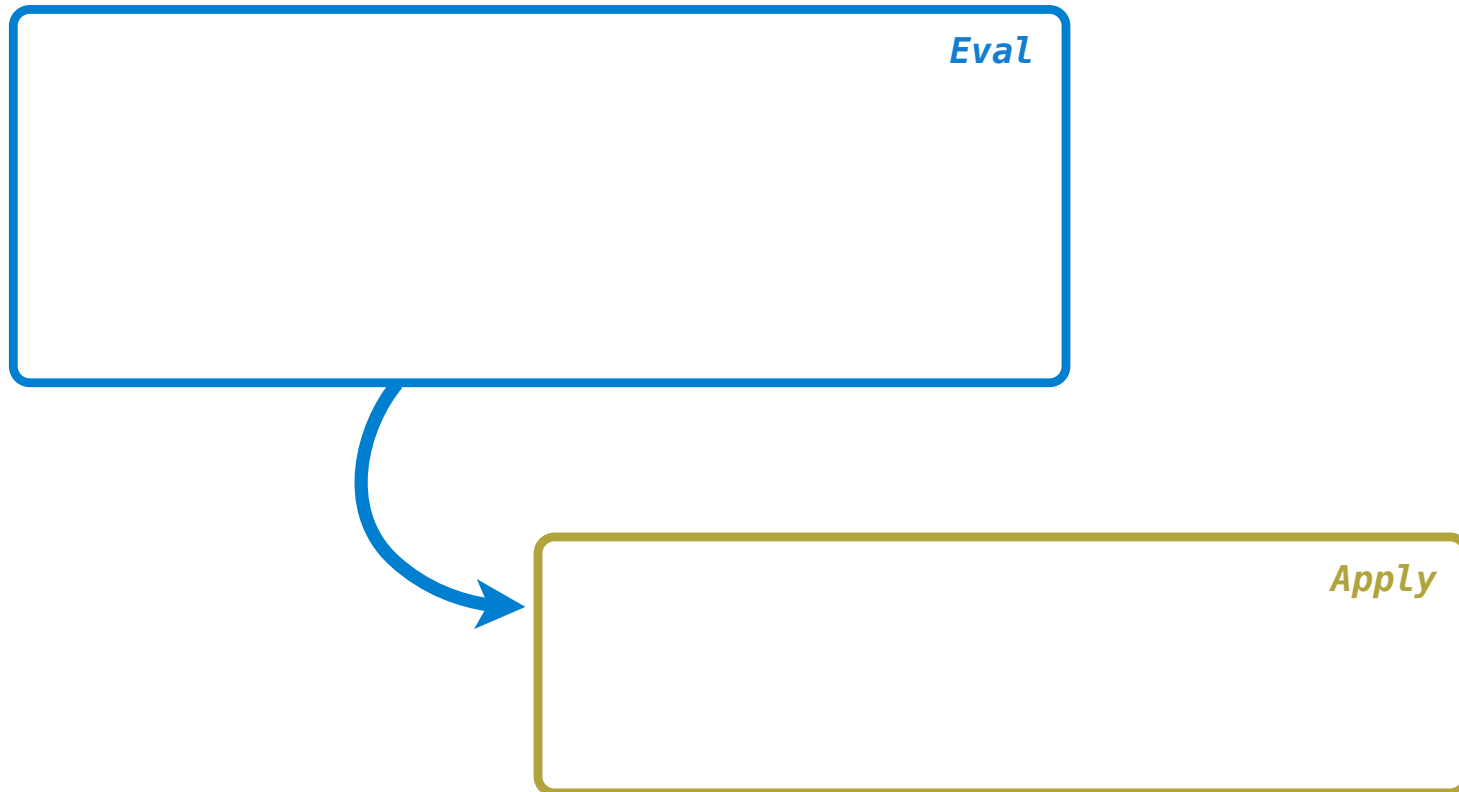
## Interpreting Scheme

## The Structure of an Interpreter

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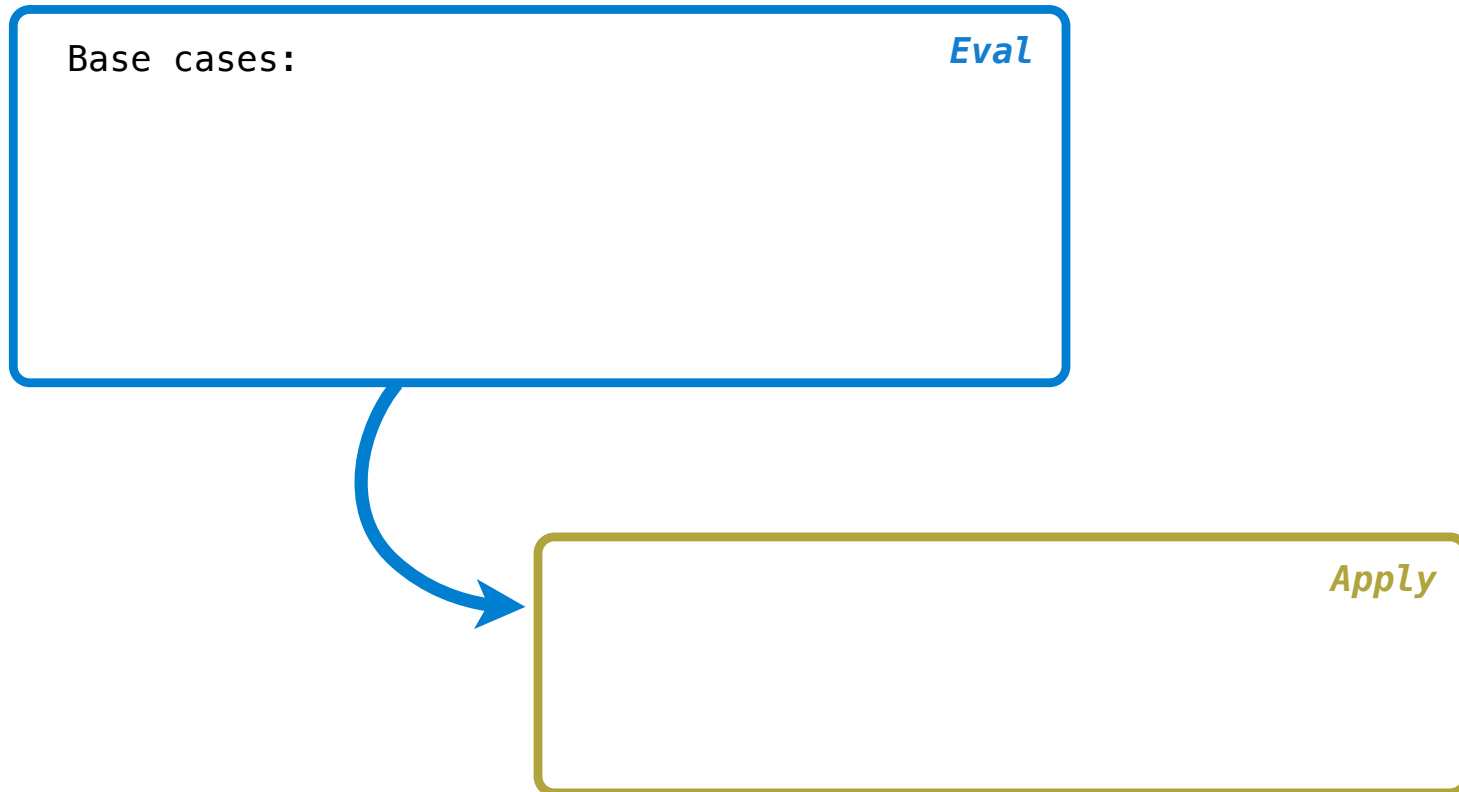
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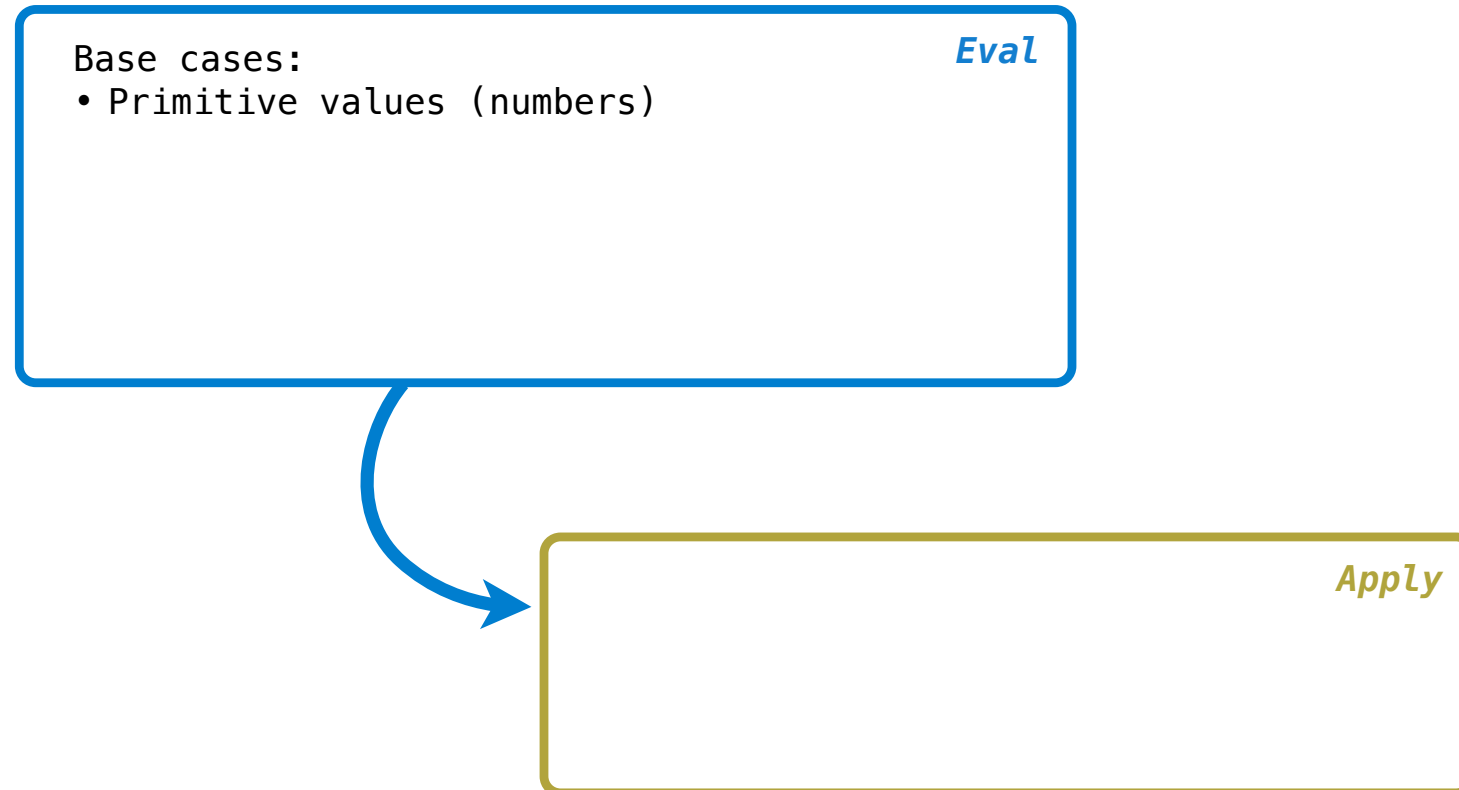
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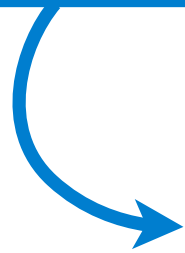
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Base cases:

- Primitive values (numbers)

Recursive calls:

*Eval*



*Apply*



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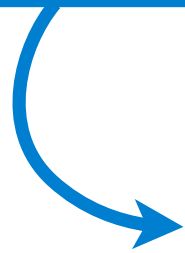
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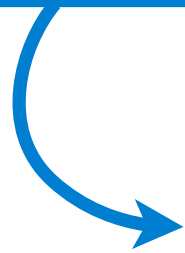
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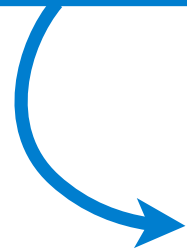
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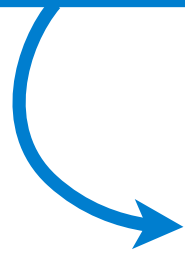
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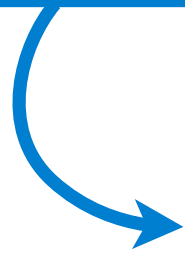
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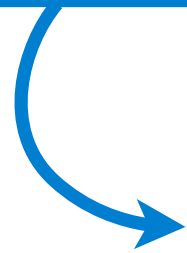
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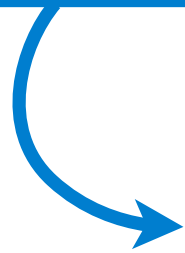
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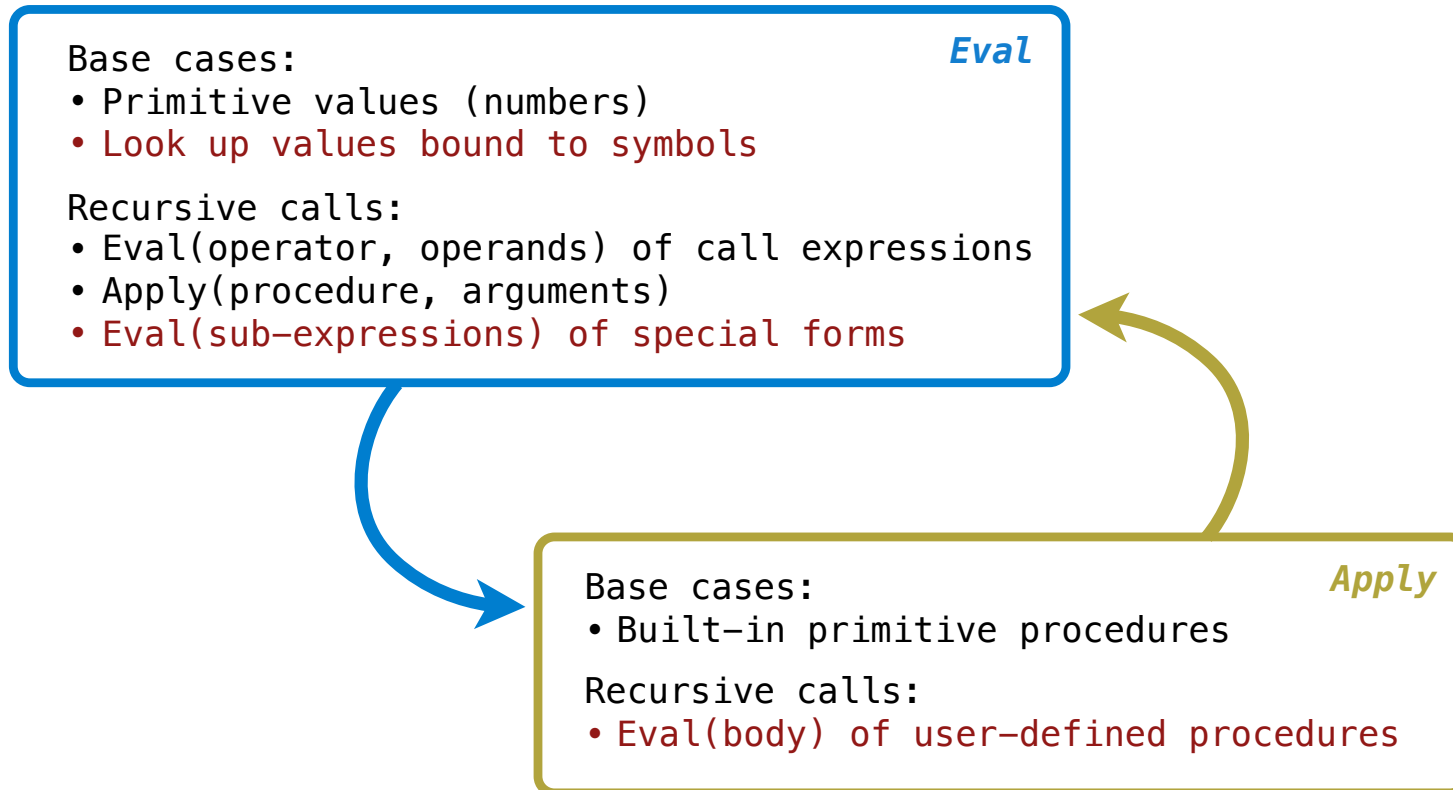
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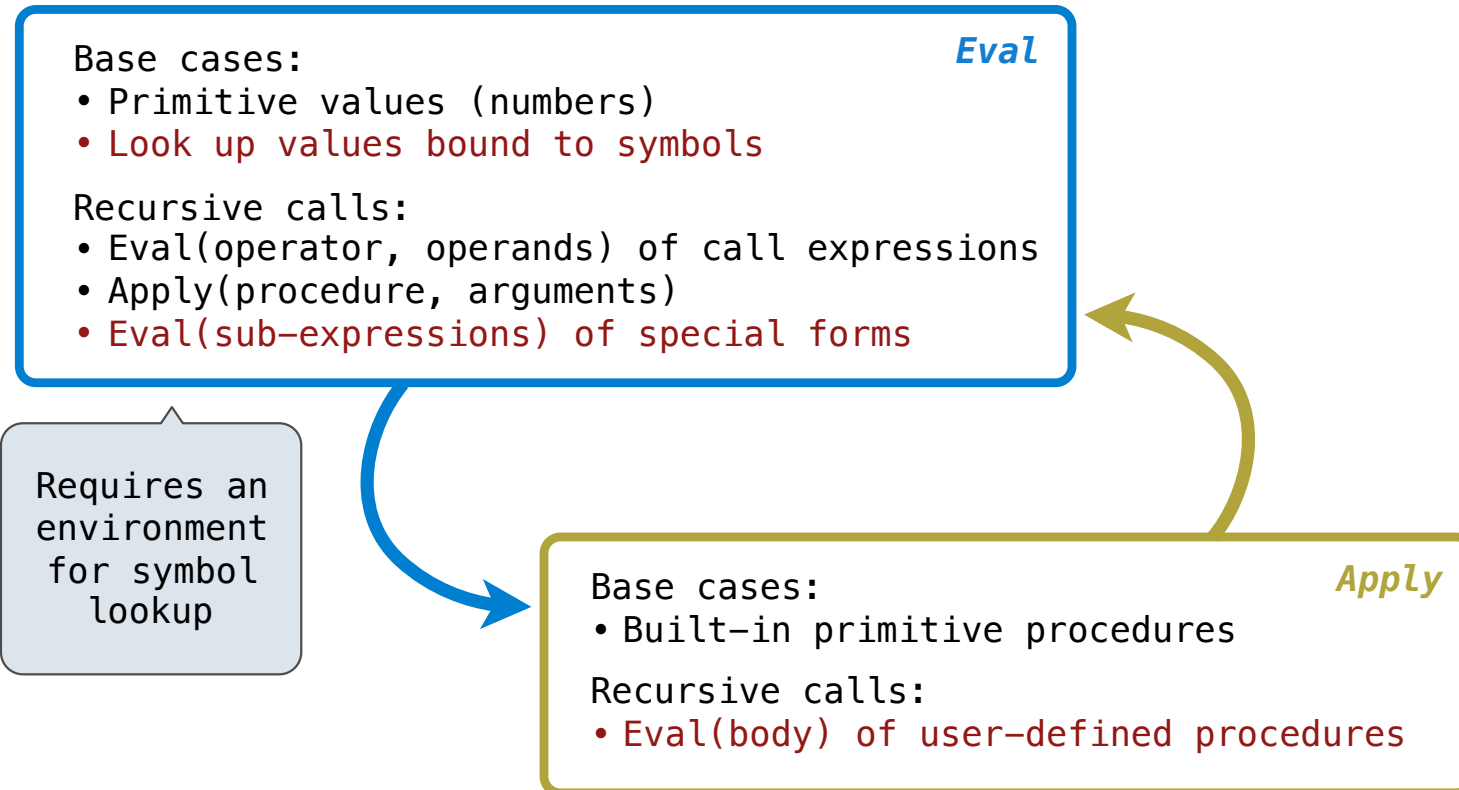
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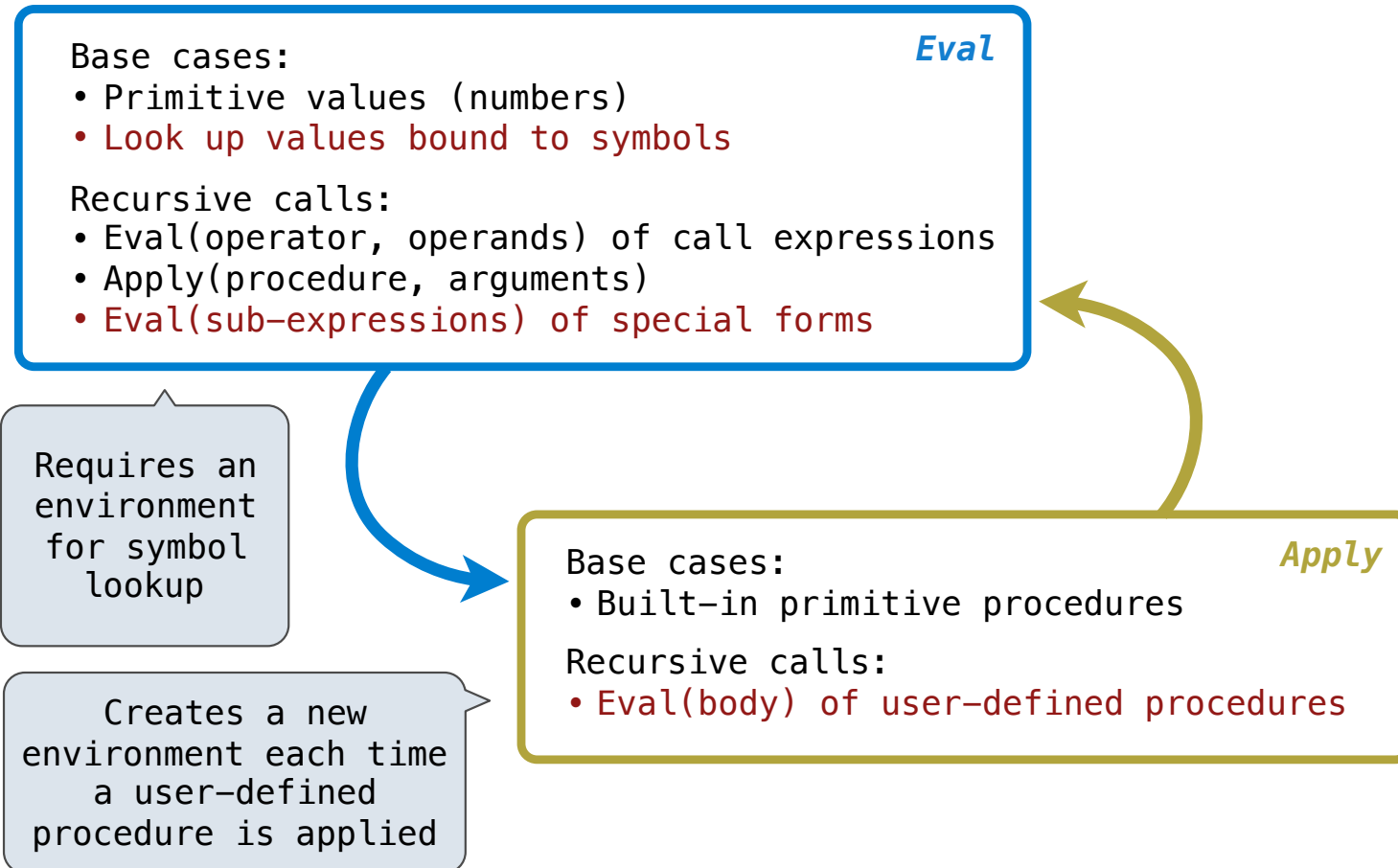


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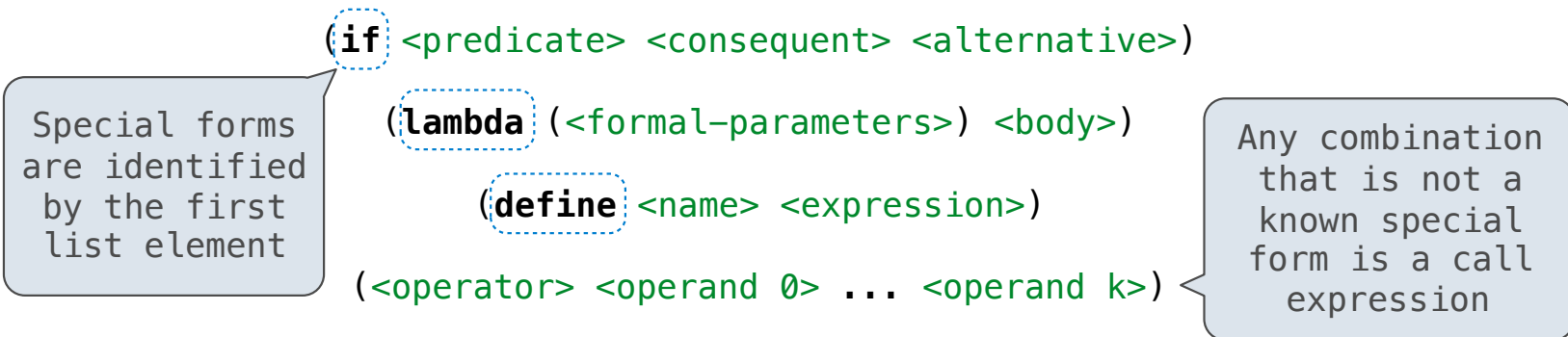
Any combination that is not a known special form is a call expression

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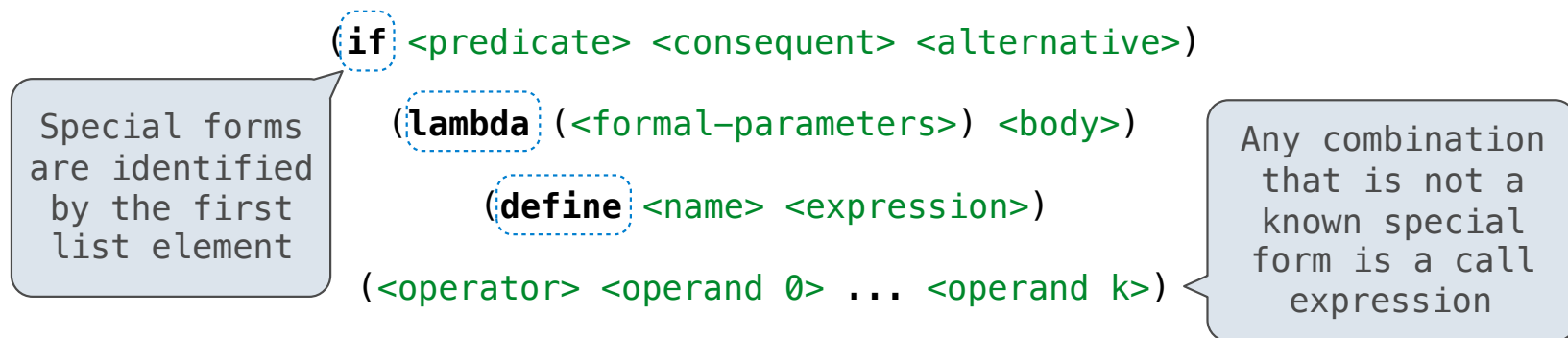


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(Demo)

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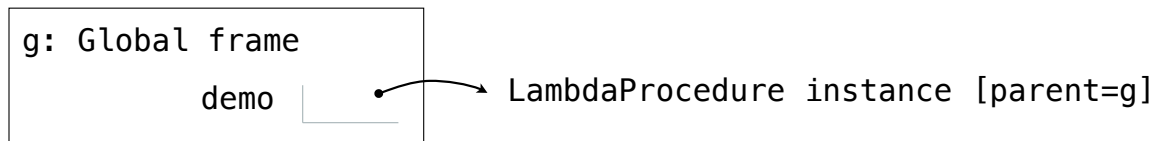
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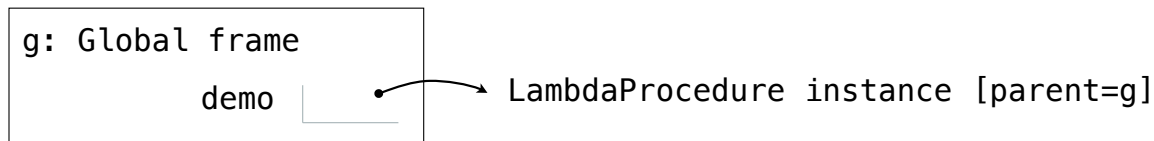
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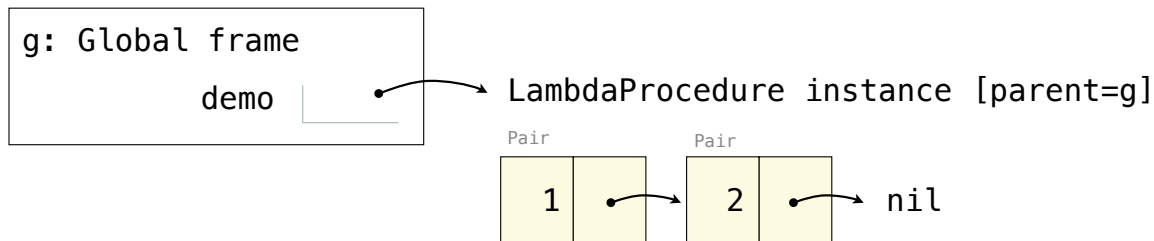
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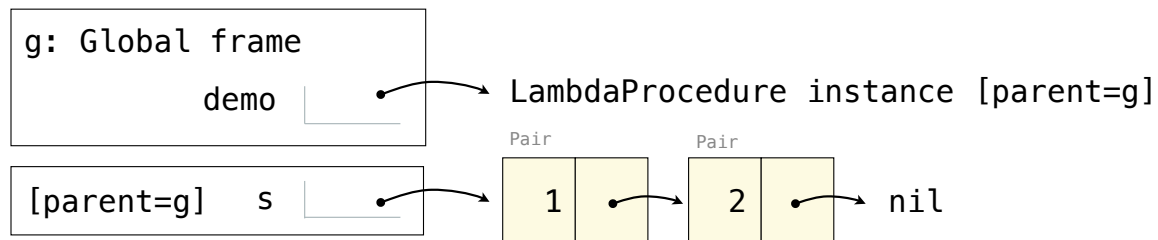


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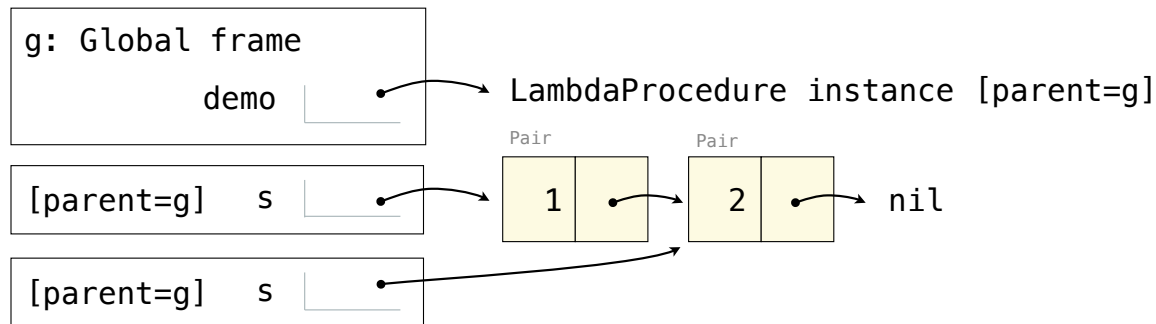


## Applying User-Defined Procedures

To apply a user-defined procedure, create a new frame in which formal parameters are bound to argument values, whose parent is the **env** attribute of the procedure

Evaluate the body of the procedure in the environment that starts with this new frame

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(define (demo s) (if (null? s) '(3) (cons (car s) (demo (cdr s)))))  
  
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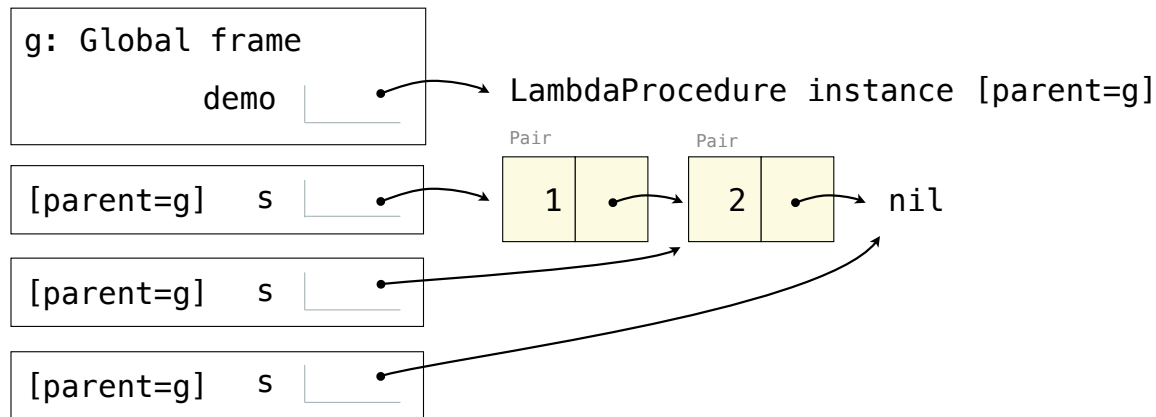


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## Eval/Apply in Lisp 1.5

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```
apply[fn;x;a] =
  [atom[fn] → [eq[fn;CAR] → caar[x];
               eq[fn;CDR] → cdar[x];
               eq[fn;CONS] → cons[car[x];cadr[x]];
               eq[fn;ATOM] → atom[car[x]];
               eq[fn;EQ] → eq[car[x];cadr[x]];
               T → apply[eval[fn;a];x;a]];
  eq[car[fn];LAMBDA] → eval[caddr[fn];pairlis[cadr[fn];x;a]];
  eq[car[fn];LABEL] → apply[caddr[fn];x;cons[cons[cadr[fn];
                                                    caddr[fn]];a]]]

eval[e;a] = [atom[e] → cdr[assoc[e;a]];
            atom[car[e]] →
              [eq[car[e],QUOTE] → cadr[e];
               eq[car[e];COND] → evcon[cdr[e];a];
               T → apply[car[e];evlis[cdr[e];a];a]];
            T → apply[car[e];evlis[cdr[e];a];a]]
```