

Final Examples

Announcements

Trees

Tree-Structured Data

Tree-Structured Data

```
def tree(label, branches=[]):
    return [label] + list(branches)

def label(t):
    return t[0]

def branches(t):
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def is_leaf(t):
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class Tree:
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  <li>Midterm <b>1</b></li>
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Tree processing often involves recursive calls on subtrees

Tree Processing

Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than all labels of their ancestor nodes.

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def big(t):
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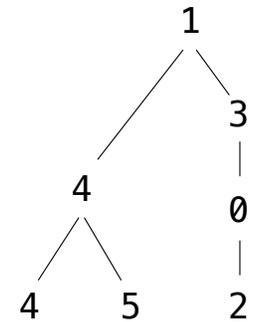
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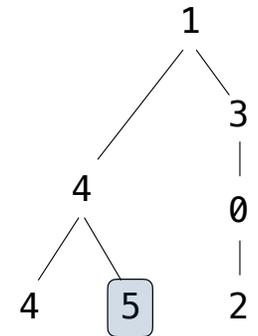
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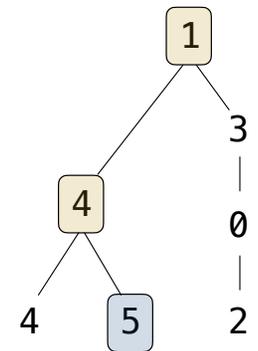
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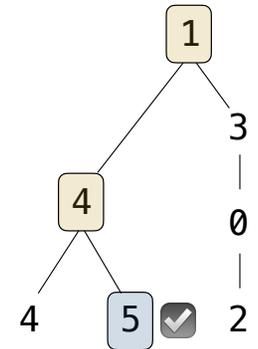
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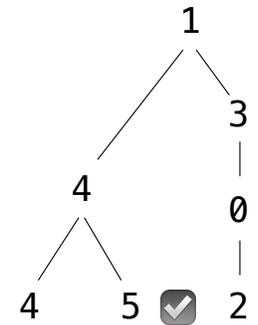
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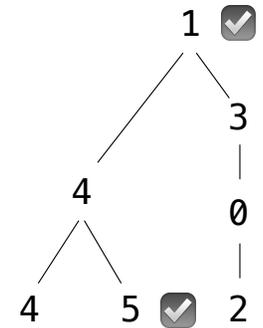
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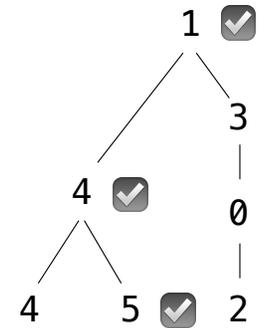
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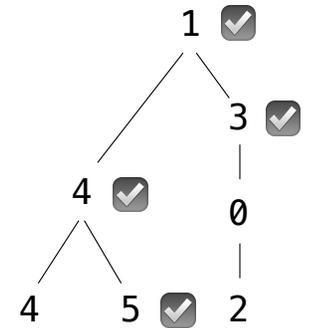
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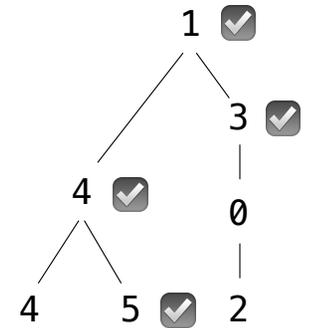
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The root label is always larger than all of its ancestors



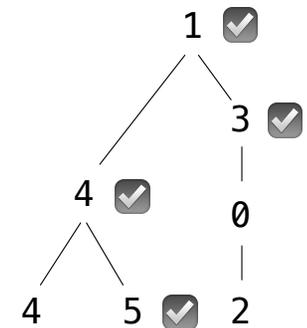
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if t.is_leaf():  
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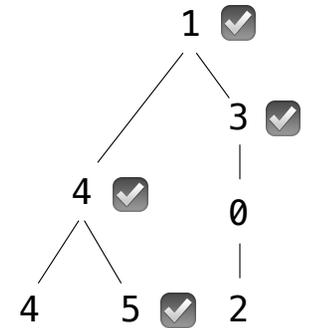
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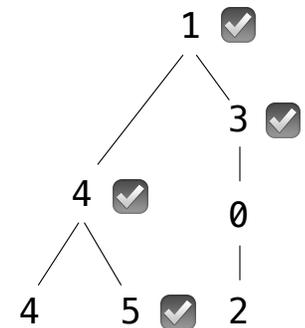
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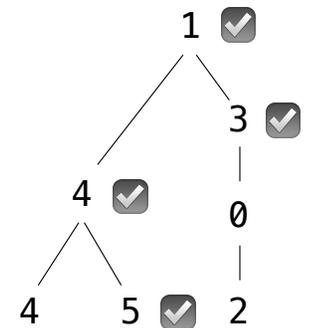
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    if node.label > max(ancestors):
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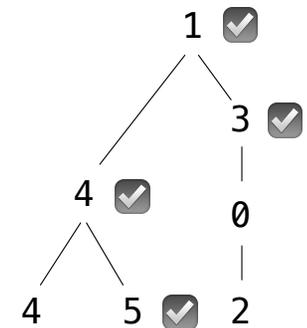
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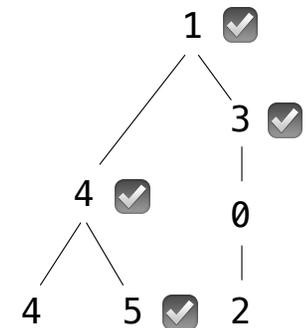
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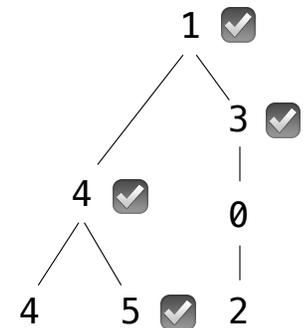
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if node.label > max(ancestors):
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Somehow track the
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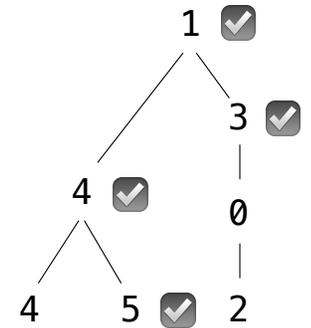
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    if _____:
```

```
        return 1 + _____
```

```
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```

```
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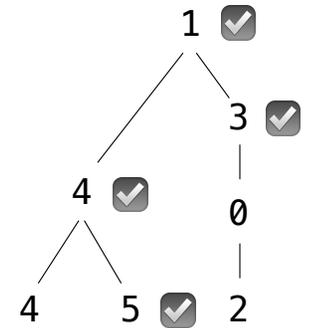
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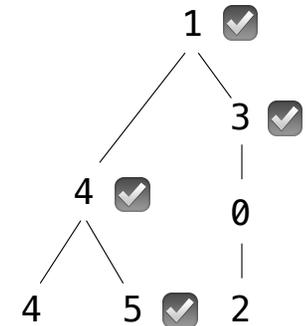
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node.label > max_ancestors



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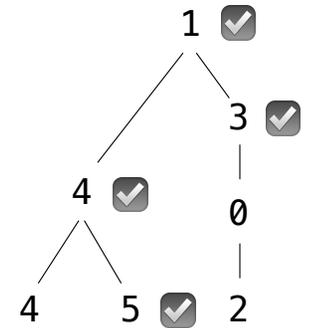
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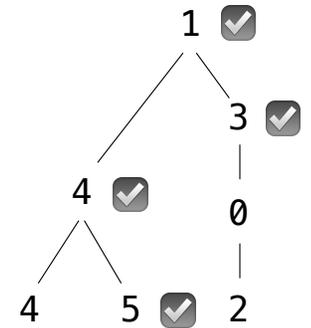
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```
    >>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
```

```
    >>> big(a)
```

```
    4
```

```
    """
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if a.label > x:
```

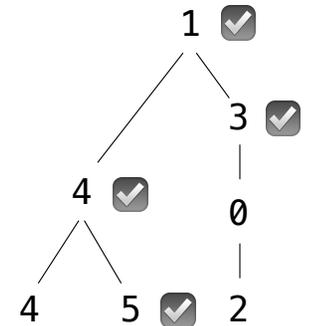
node.label > max_ancestors

```
            return 1 + _____
```

```
        else:
```

```
            return _____
```

```
    return f(t, _____)
```



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):
```

```
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
    >>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
```

```
    >>> big(a)
```

```
    4
```

```
    """
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if a.label > x:
```

node.label > max_ancestors

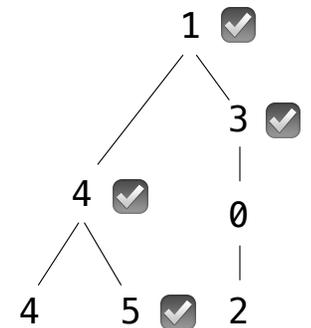
```
            return 1 + _____
```

```
        else:
```

```
            return _____
```

```
    return f(t, _____)
```

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):
```

```
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
    >>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
```

```
    >>> big(a)
```

```
    4
```

```
    """
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if a.label > x:
            node.label > max_ancestors
```

```
            return 1 + _____
```

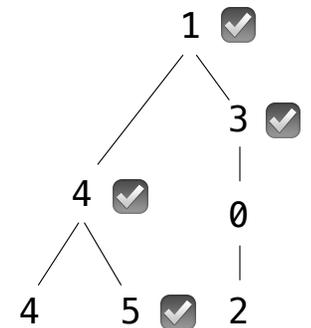
```
        else:
```

Somehow increment the total count

```
            return _____
```

```
    return f(t, _____)
```

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):
```

```
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
    >>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
```

```
    >>> big(a)
```

```
    4
```

```
    """
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if a.label > x:
            node.label > max_ancestors
```

```
            return 1 + sum([f(b, a.label) for b in a.branches])
```

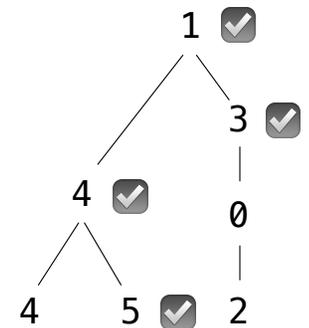
```
        else:
```

Somehow increment the total count

```
            return
```

```
    return f(t, )
```

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):
```

```
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
    >>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
```

```
    >>> big(a)
```

```
    4
```

```
    """
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if a.label > x:
            node.label > max_ancestors
```

```
            return 1 + sum([f(b, a.label) for b in a.branches])
```

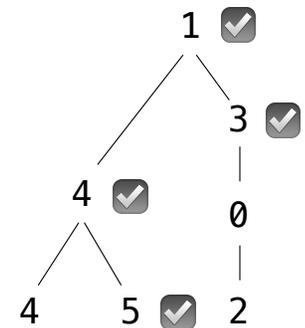
```
        else:
```

Somehow increment the total count

```
            return sum([f(b, x) for b in a.branches])
```

```
    return f(t, )
```

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):
```

```
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
    >>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
```

```
    >>> big(a)
```

```
    4
```

```
    """
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if a.label > x:
            node.label > max_ancestors
```

```
            return 1 + sum([f(b, a.label) for b in a.branches])
```

```
        else:
```

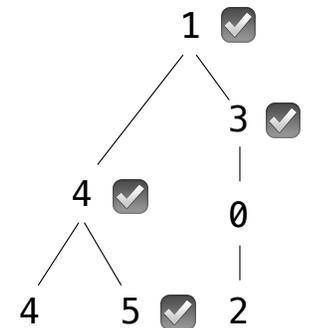
Somehow increment the total count

```
            return sum([f(b, x) for b in a.branches])
```

```
    return f(t,
```

Root label is always larger than its ancestors

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):
```

```
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
    >>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
```

```
    >>> big(a)
```

```
    4
```

```
    """
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if a.label > x:
```

A node in t

max_ancestor

node.label > max_ancestors

```
            return 1 + sum([f(b, a.label) for b in a.branches])
```

```
        else:
```

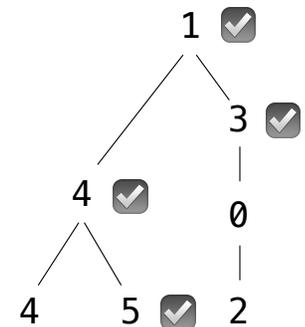
Somehow increment the total count

```
            return sum([f(b, x) for b in a.branches])
```

```
    return f(t, t.label - 1)
```

Root label is always larger than its ancestors

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):  
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
```

```
>>> big(a)
```

```
4
```

```
"""
```

```
def f(a, x):
```

Somehow track the largest ancestor

```
    if a.label > x: node.label > max_ancestors
```

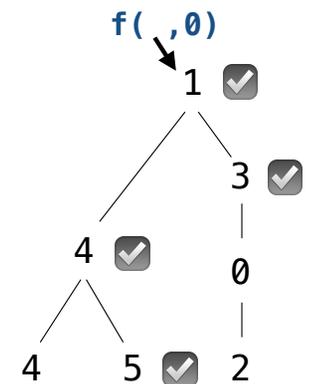
```
        return 1 + sum([f(b, a.label) for b in a.branches])
```

```
    else:           Somehow increment the total count
```

```
        return sum([f(b, x) for b in a.branches])
```

```
    return f(t, t.label - 1)  Root label is always larger than its ancestors
```

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):  
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])  
>>> big(a)
```

```
4
```

```
"""
```

```
def f(a, x):
```

A node in t

max_ancestor

```
if a.label > x
```

node.label > max_ancestors

```
:
```

```
    return 1 + sum([f(b, a.label) for b in a.branches])
```

```
else:
```

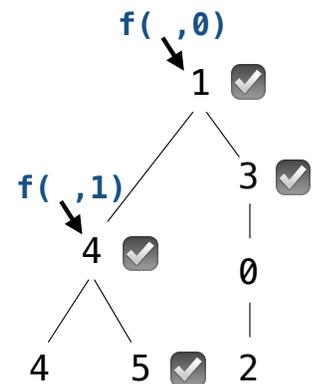
Somehow increment the total count

```
    return sum([f(b, x) for b in a.branches])
```

```
return f(t, t.label - 1)
```

Root label is always larger than its ancestors

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def bigs(t):  
    """Return the number of nodes in t that are larger than all their ancestors.  
    """
```

```
>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])  
>>> bigs(a)
```

```
4  
"""  
def f(a, x):  
    """  
    """
```

Somehow track the largest ancestor

```
A node in t →  
if a.label > x:  # max_ancestor  
    node.label > max_ancestors
```

```
    return 1 + sum([f(b, a.label) for b in a.branches])
```

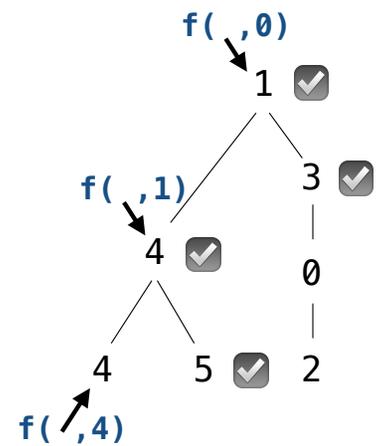
```
else:  
    """  
    """
```

Somehow increment the total count

```
    return sum([f(b, x) for b in a.branches])
```

```
return f(t, t.label - 1)  # Root label is always larger than its ancestors
```

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement **big**s, which takes a Tree instance *t* containing integer labels. It returns the number of nodes in *t* whose labels are larger than any labels of their ancestor nodes.

```
def bigs(t):
```

```
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
    >>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
```

```
    >>> bigs(a)
```

```
    4
```

```
    """
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if a.label > x:
            node.label > max_ancestors
```

```
            return 1 + sum([f(b, a.label) for b in a.branches])
```

```
        else:
```

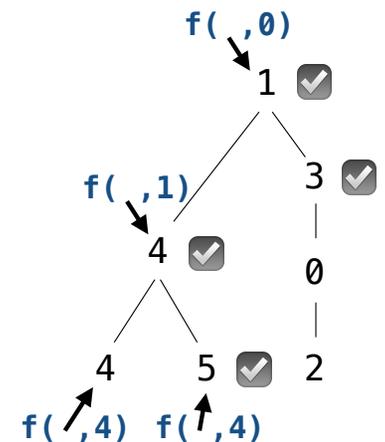
Somehow increment the total count

```
            return sum([f(b, x) for b in a.branches])
```

```
    return f(t, t.label - 1)
```

Root label is always larger than its ancestors

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):  
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])  
>>> big(a)
```

```
4
```

```
"""
```

```
def f(a, x):
```

Somehow track the largest ancestor

```
    if a.label > x:
```

A node in t

max_ancestor

node.label > max_ancestors

```
        return 1 + sum([f(b, a.label) for b in a.branches])
```

```
    else:
```

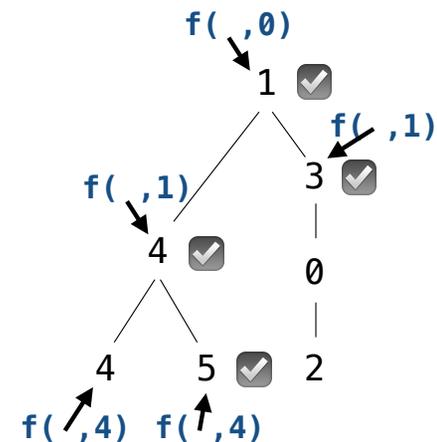
Somehow increment the total count

```
        return sum([f(b, x) for b in a.branches])
```

```
    return f(t, t.label - 1)
```

Root label is always larger than its ancestors

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])]
>>> big(a)
```

```
4
```

```
"""
```

```
def f(a, x):
```

A node in t

max_ancestor

```
if a.label > x
```

node.label > max_ancestors

```
:
```

```
    return 1 + sum([f(b, a.label) for b in a.branches])
```

```
else:
```

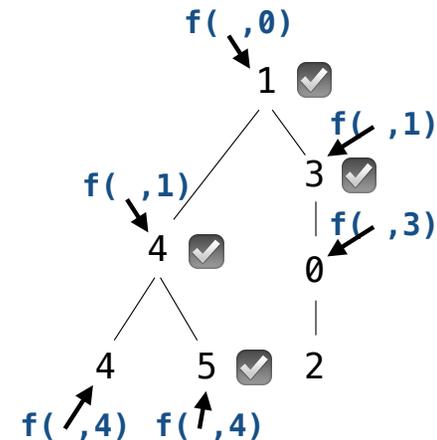
Somehow increment the total count

```
    return sum([f(b, x) for b in a.branches])
```

```
return f(t, t.label - 1)
```

Root label is always larger than its ancestors

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):  
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])  
>>> big(a)
```

```
4  
"""
```

```
def f(a, x):  
    if a.label > x:
```

A node in t

max_ancestor

node.label > max_ancestors

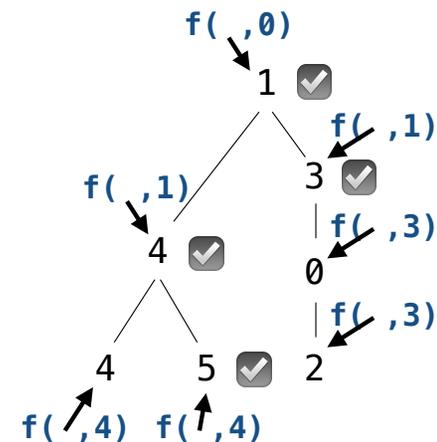
```
        return 1 + sum([f(b, a.label) for b in a.branches])
```

```
    else:
```

```
        return sum([f(b, x) for b in a.branches])
```

```
return f(t, t.label - 1)
```

Some initial value for the largest ancestor so far...



Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):  
    """Return the number of nodes in t that are larger than all their ancestors.
```

```
>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])  
>>> big(a)
```

```
4
```

```
"""
```

```
def f(a, x):
```

A node in t

max_ancestor

```
if a.label > x
```

node.label > max_ancestors

```
:
```

```
    return 1 + sum( f(b, a.label) for b in a.branches )
```

```
else:
```

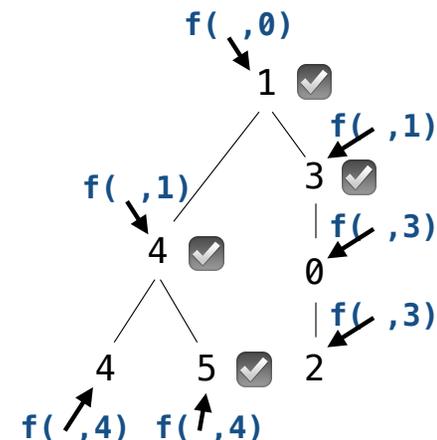
Somehow increment the total count

```
    return sum( f(b, x) for b in a.branches )
```

```
return f(t, t.label - 1)
```

Root label is always larger than its ancestors

Some initial value for the largest ancestor so far...



Recursive Accumulation

Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):  
    """Return the number of nodes in t that are larger than all their ancestors."""  
    n = 0  
  
    def f(a, x):  
  
        _____  
  
        if _____:  
            n += 1  
  
        _____:  
            f(_____)  
  
        _____  
  
    return n
```

Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):  
    """Return the number of nodes in t that are larger than all their ancestors."""  
    n = 0  
    def f(a, x):  
        Somehow track the largest ancestor  
        _____  
        if _____:  
            n += 1  
        _____:  
            f(_____)  
        _____  
    return n
```

Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):  
    """Return the number of nodes in t that are larger than all their ancestors."""  
    n = 0  
    def f(a, x):  
         Somehow track the largest ancestor  
        _____  
        if _____:  node.label > max_ancestors  
            n += 1  
        _____:  
            f(_____)  
        _____  
    return n
```

Solving Tree Problems

Implement `biggs`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def biggs(t):
```

```
    """Return the number of nodes in t that are larger than all their ancestors."""
```

```
    n = 0
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if _____:
```

node.label > max_ancestors

```
            n += 1
```

Somehow increment the total count

```
            f(_____)
```

```
    return n
```

Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):
```

```
    """Return the number of nodes in t that are larger than all their ancestors."""
```

```
    n = 0
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if _____:
```

node.label > max_ancestors

```
            n += 1
```

Somehow increment the total count

```
            f(_____)
```

Root label is always larger than its ancestors

```
    return n
```

Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):
```

```
    """Return the number of nodes in t that are larger than all their ancestors."""
```

```
    n = 0
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if a.label > x:
```

node.label > max_ancestors

```
            n += 1
```

Somehow increment the total count

```
            f(a.parent, x)
```

Root label is always larger than its ancestors

```
    return n
```

Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):
```

```
    """Return the number of nodes in t that are larger than all their ancestors."""
```

```
    n = 0
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if a.label > x:
```

node.label > max_ancestors

```
            n += 1
```

Somehow increment the total count

```
            f(a.left, x)
```

```
    f(t, t.label - 1)
```

Root label is always larger than its ancestors

```
    return n
```

Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):
```

```
    """Return the number of nodes in t that are larger than all their ancestors."""
```

```
    n = 0
```

```
    def f(a, x):
```

Somehow track the largest ancestor

```
        if a.label > x:
```

node.label > max_ancestors

```
            n += 1
```

```
        for b in a.branches:
```

Somehow increment the total count

```
            f(b, x)
```

```
    f(t, t.label - 1)
```

Root label is always larger than its ancestors

```
    return n
```

Solving Tree Problems

Implement `big`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):  
    """Return the number of nodes in t that are larger than all their ancestors."""  
    n = 0  
    def f(a, x):  
        if a.label > x:  
            n += 1  
            for b in a.branches:  
                f(b, max(a.label, x))  
    f(t, t.label - 1)  
    return n
```

Somehow track the largest ancestor

node.label > max_ancestors

Somehow increment the total count

Root label is always larger than its ancestors

Solving Tree Problems

Implement `biggs`, which takes a `Tree` instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def biggs(t):  
    """Return the number of nodes in t that are larger than all their ancestors."""  
    n = 0  
    def f(a, x):  
        nonlocal n  
  
        if a.label > x:  
            n += 1  
            for b in a.branches:  
                f(b, max(a.label, x))  
  
    f(t, t.label - 1)  
  
    return n
```

Somehow track the largest ancestor

node.label > max_ancestors

Somehow increment the total count

Root label is always larger than its ancestors

Designing Functions

How to Design Programs

<https://htdp.org/2018-01-06/Book/>

How to Design Programs

From Problem Analysis to Data Definitions

Identify the information that must be represented and how it is represented in the chosen programming language. Formulate data definitions and illustrate them with examples.

How to Design Programs

From Problem Analysis to Data Definitions

Identify the information that must be represented and how it is represented in the chosen programming language. Formulate data definitions and illustrate them with examples.

Signature, Purpose Statement, Header

State what kind of data the desired function consumes and produces. Formulate a concise answer to the question *what* the function computes. Define a stub that lives up to the signature.

How to Design Programs

From Problem Analysis to Data Definitions

Identify the information that must be represented and how it is represented in the chosen programming language. Formulate data definitions and illustrate them with examples.

Signature, Purpose Statement, Header

State what kind of data the desired function consumes and produces. Formulate a concise answer to the question *what* the function computes. Define a stub that lives up to the signature.

Functional Examples

Work through examples that illustrate the function's purpose.

How to Design Programs

From Problem Analysis to Data Definitions

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Testing

Articulate the examples as tests and ensure that the function passes all. Doing so discovers mistakes. Tests also supplement examples in that they help others read and understand the definition when the need arises—and it will arise for any serious program.

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Applying the Design Process

Designing a Function

Implement `smalls`, which takes a `Tree` instance `t` containing integer labels. It returns the non-leaf nodes in `t` whose labels are smaller than any labels of their descendant nodes.

```
def smalls(t):  
    """Return the non-leaf nodes in t that are smaller than all their descendants.  
  
    >>> a = Tree(1, [Tree(2, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(6)])])])  
    >>> sorted([t.label for t in smalls(a)])  
    [0, 2]  
    """  
    result = []  
    def process(t):  
  
        process(t)  
    return result
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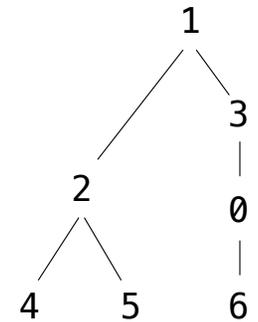
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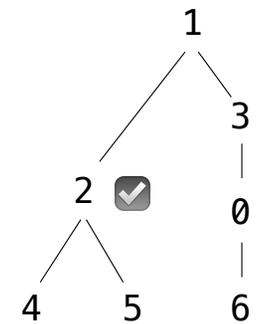
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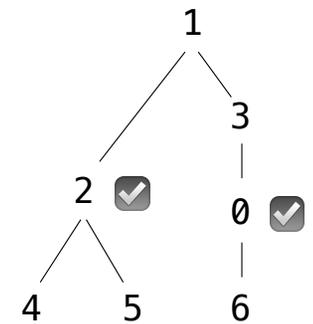
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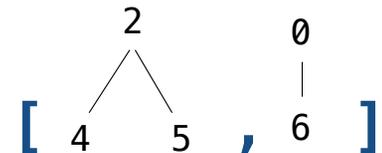
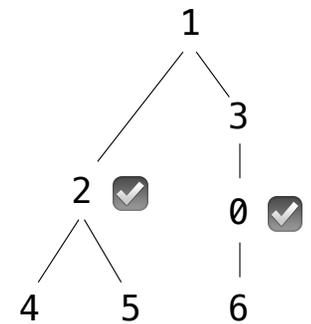
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Designing a Function

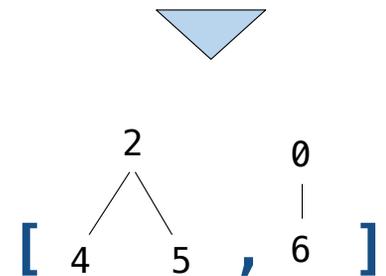
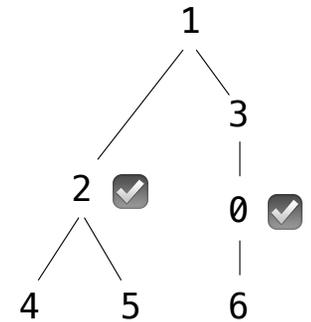
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    """ Signature: Tree -> number  
    result = []  
    def process(t):
```

```
        process(t)  
    return result
```



Designing a Function

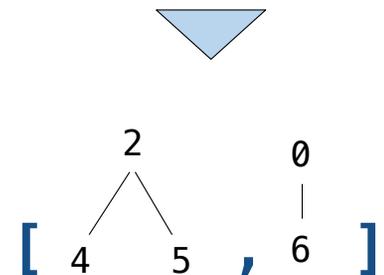
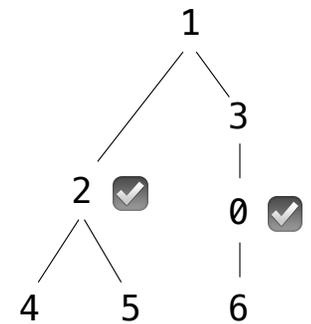
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    result = []  
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    process(t)  
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```



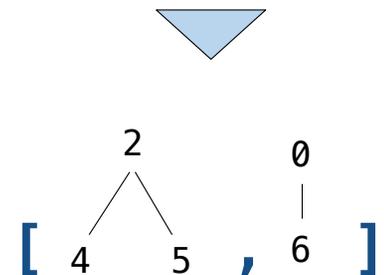
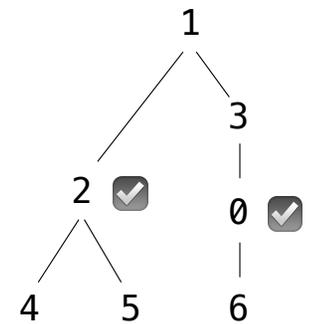
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    result = []
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    def process(t): "Find smallest label in t & maybe add t to result"
        if t.is_leaf():
            return t.label
        else:
            return min(...)
    process(t)
    return result
```



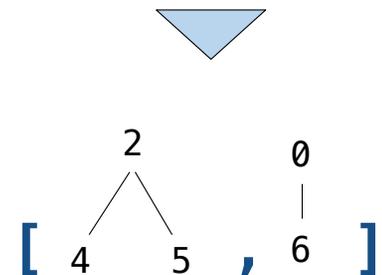
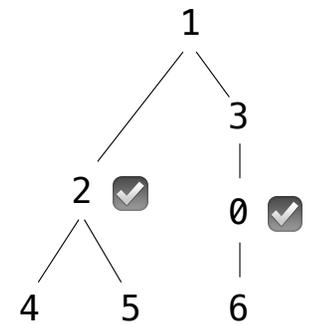
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    result = []
    Signature: Tree -> number
    def process(t): "Find smallest label in t & maybe add t to result"
        if t.is_leaf():
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        else:
            smallest = _____
            if _____:
                _____
            return min(smallest, t.label)
    process(t)
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```



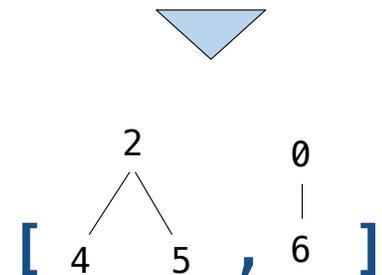
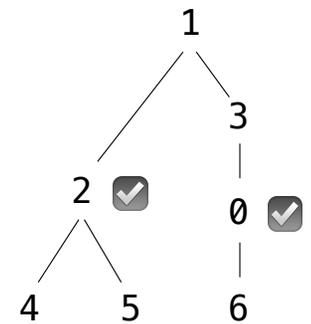
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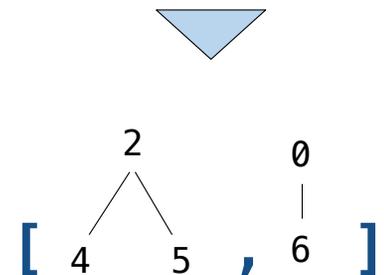
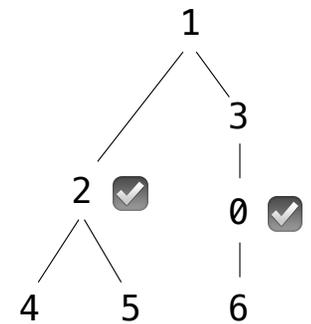
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        if t.is_leaf():
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            smallest = _____
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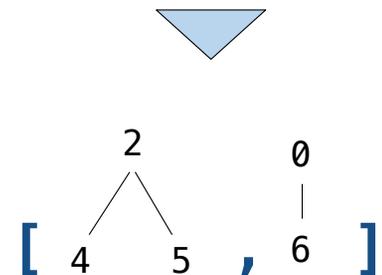
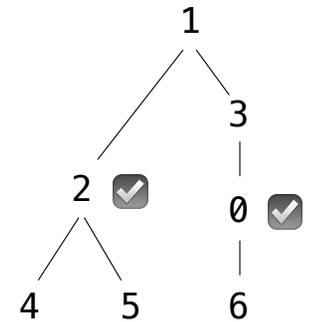
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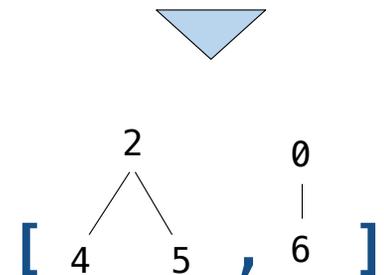
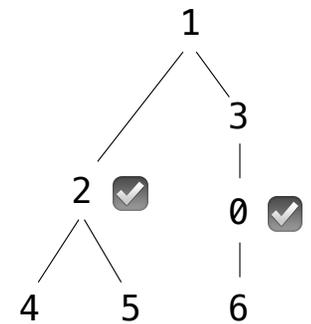
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        else:
            smallest = _____
            if _____:
                result.append(_____)
            return min(smallest, t.label)
    process(t)
    return result
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smallest label in a branch of t →



Designing a Function

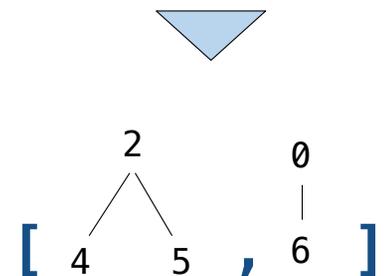
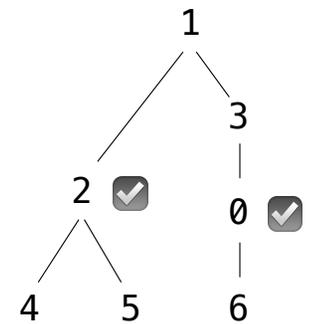
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            if _____:
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smallest label in a branch of t →



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    """
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        if t.is_leaf():
            return t.label
        else:
            smallest = min([process(b) for b in t.branches])
            smallest label in a branch of t
            if t.label < smallest:
                result.append( t )
            return min(smallest, t.label)
    process(t)
    return result
```

