## Functional Abstraction

Announcements

## Lambda Function Environments

## Environment Diagrams with Lambda

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A lambda function's parent is the current frame in which the lambda expression is evaluated

```
a = 1
def f(g):
    a = 2
    return lambda y: a * g(y)
    f(lambda y: a + y)(a)
```


## Environment Diagrams with Lambda

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| 1 | $a=1$ |
| :--- | :--- |
| 2 | def $f(g):$ |
| 3 | $a=2$ |
| 4 | $\quad$ return lambda $y: a{ }^{*} g(y)$ |
| 5 | $f(l a m b d a y: a+y)(a)$ |



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| 4 | $\quad$ return lambda $y: a * g(y)$ |
| 5 | $f(l a m b d a y: a+y)(a)$ |


f3: $\lambda$ <line 5> [parent=Global]

| $y$ | 1 |
| ---: | :--- |
| Return <br> value | 2 |

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Return

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"""Print the final digits of N in reverse order until D is found.
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while $\mathrm{n}>0$ :
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if $\qquad$ :
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1. Evaluate the operator and then the operand subexpressions

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doesn't exist doesn't exist

"if" header expression

```
def if_(c, t, f):
    if c:
def if_(c, t, f): if c :
```

        return t
    else:
            return f return t else: return f
    
"if"
suite


## Evaluation Rule for Call Expressions:

1. Evaluate the operator and then the operand subexpressions
2. Apply the function that is the value of the operator to the arguments that are the values of the operands

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Control Expressions

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Abstraction

Functional Abstractions

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def square(x):
    return mul(x, x)
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def square(x):
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def sum_squares(x, y):
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What does sum_squares need to know about square?
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def square(x): def sum_squares(x, y):
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Square takes one argument.

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def square(x): def sum_squares(x, y):
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```

def sum_squares(x, y):
return square $(x)+$ square $(y)$

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def square(x):
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```

    def square(x):
        return mul( \(x, x-1\) ) \(+x\)
    
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```
def square(x):
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```

    def square(x):
    return mul(x, \(x-1)+x\)
    If the name "square" were bound to a built-in function,
sum_squares would still work identically.

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| $l, I, 0$ | $k, i, m$ |

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Repeated compound expressions:
if sqrt(square(a) + square(b)) > 1:
$x=x+\operatorname{sqrt}($ square $(a)+\operatorname{square(b))}$

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Repeated compound expressions:
if sqrt(square(a) + square(b)) > 1:
$x=x+\operatorname{sqrt}($ square $(a)+$ square $(b))$
$\square$
hypotenuse $=$ sqrt(square(a) + square(b))
if hypotenuse > 1:
$x=x+h y p o t e n u s e$

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if sqrt(square(a) + square(b)) > 1:
$x=x+\operatorname{sqrt}($ square $(a)+$ square $(b))$

hypotenuse = sqrt(square(a) + square(b))
if hypotenuse > 1:
$x=x+h y p o t e n u s e$

Meaningful parts of complex expressions:

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Repeated compound expressions:

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if sqrt(square(a) + square(b)) > 1:
    x = x + sqrt(square(a) + square(b))
```

    hypotenuse \(=\) sqrt(square(a) + square(b))
    if hypotenuse > 1:
        \(x=x+h y p o t e n u s e\)
    Meaningful parts of complex expressions:

```
x1 = (-b + sqrt(square(b) - 4*a*c)) / (2*a)
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Meaningful parts of complex expressions:

```
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discriminant $=$ square(b) - $4 * a * c$
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hypotenuse $=$ sqrt (square(a) + square (b))
if hypotenuse > 1:
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## More Naming Tips

- Names can be long if they help document your code:
average_age = average(age, students)
is preferable to
\# Compute average age of students aa $=\operatorname{avg}(a, s t)$

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x1 = (-b + sqrt(square(b) - 4*a*c)) / (2*a)
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discriminant $=$ square(b) - $4 * a * c$ x1 = (-b + sqrt(discriminant)) / (2 * a)

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- Names can be long if they help document your code:
average_age = average(age, students)
is preferable to
\# Compute average age of students aa $=\operatorname{avg}(a, s t)$
- Names can be short if they represent generic quantities: counts, arbitrary functions, arguments to mathematical operations, etc.
n, k, i - Usually integers
x, y, z - Usually real numbers
f, g, h - Usually functions


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Repeated compound expressions:

```
if sqrt(square(a) + square(b)) > 1:
    x = x + sqrt(square(a) + square(b))
```

```
hypotenuse = sqrt(square(a) + square(b))
```

if hypotenuse > 1:

$$
x=x+\text { hypotenuse }
$$

## More Naming Tips

- Names can be long if they help document your code:
average_age = average(age, students)
is preferable to
\# Compute average age of students aa $=\operatorname{avg}(a, s t)$
- Names can be short if they represent generic quantities: counts,

```
x1 = (-b + sqrt(square(b) - 4*a*c)) / (2*a)
``` arbitrary functions, arguments to mathematical operations, etc.
n, k, i - Usually integers
x, y, z - Usually real numbers
f, g, h - Usually functions

\section*{Errors \& Tracebacks}

\section*{Taxonomy of Errors}

\author{
Syntax Errors Detected by the Python \\ interpreter (or editor) \\ before the program executes \\ Runtime Errors \\ Logic \& Behavior Errors \\ Not detected by the Python interpreter; what tests are for
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(Demo)```

