Data Abstraction

Announcements

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Data Abstraction

All Programmers

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All Programmers

Great Programmers

Rational Numbers

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$$\frac{\text{numerator}}{\text{denominator}}$$

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Exact representation of fractions

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Assume we can compose and decompose rational numbers:

- `rational(n, d)` returns a rational number `x`

Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- `rational(n, d)` returns a rational number `x`
- `numer(x)` returns the numerator of `x`

Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- rational(n, d) returns a rational number x
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Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Exact representation of fractions

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As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

Constructor → `rational(n, d)` returns a rational number x

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Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Exact representation of fractions

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As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

Constructor

- `rational(n, d)` returns a rational number x

Selectors

- `numer(x)` returns the numerator of x
- `denom(x)` returns the denominator of x

Rational Number Arithmetic

Example

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5}$$

Example

General Form

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$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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$$\frac{nx}{dx} * \frac{ny}{dy}$$

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Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

Example

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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Example

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

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General Form

Rational Number Arithmetic Implementation

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- `rational(n, d)` returns a rational number x
- `numer(x)` returns the numerator of x
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Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
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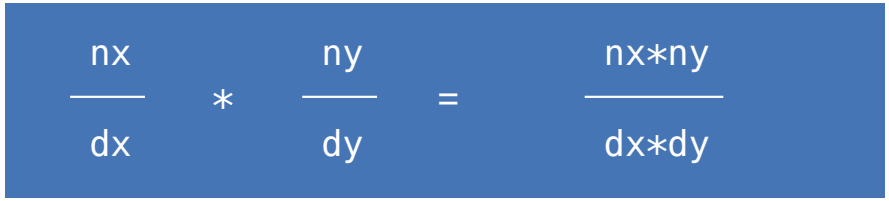
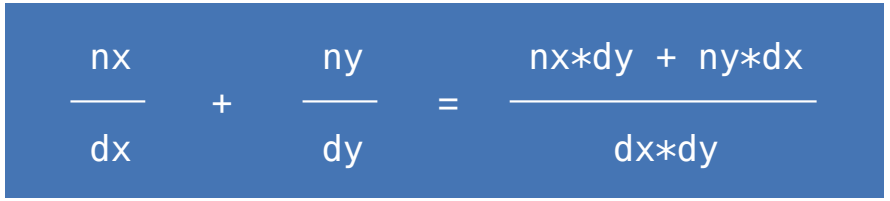
```





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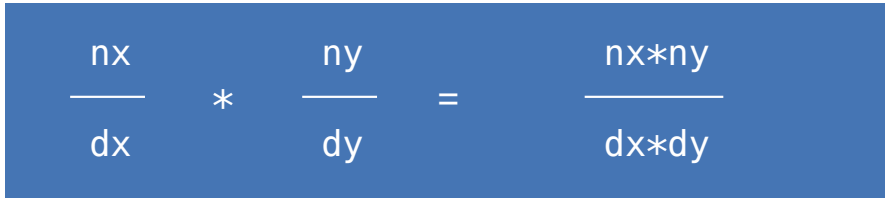

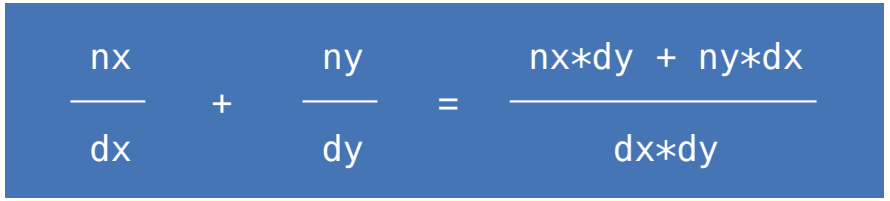

- rational(n, d) returns a rational number x
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These functions implement an abstract representation for rational numbers

Rational Number Arithmetic Implementation

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def mul_rational(x, y):
    return rational(numer(x) * numer(y),
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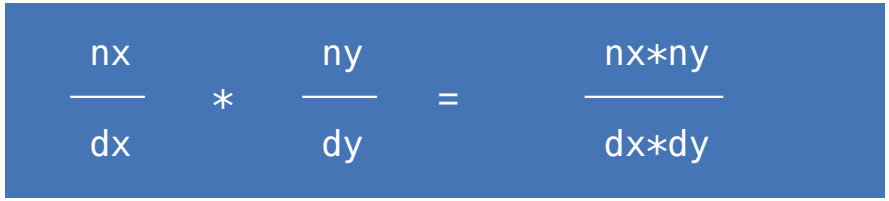
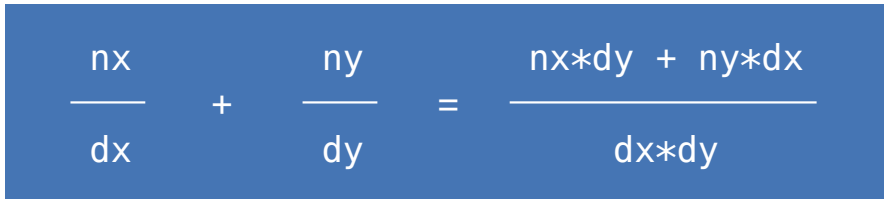
Constructor

Selectors




```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
```



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Rational Number Arithmetic Implementation

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def mul_rational(x, y):
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def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))
```

Constructor

Selectors

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

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def mul_rational(x, y):
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$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

- rational(n, d) returns a rational number x
- numer(x) returns the numerator of x
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These functions implement an abstract representation for rational numbers

Representing Rational Numbers

Representing Pairs Using Lists

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```
>>> pair = [1, 2]
```

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>>> pair
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A list literal:
Comma-separated expressions in brackets

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```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2
```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

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>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
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>>> y
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A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
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>>> y
2

>>> pair[0]
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```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

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>>> pair = [1, 2]
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>>> pair[1]
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A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Representing Pairs Using Lists

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[1, 2]
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>>> x, y = pair
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>>> y
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"Unpacking" a list

```
>>> pair[0]
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>>> pair[1]
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```

Element selection using the selection operator

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```
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
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    """Return the numerator of rational number X."""
    return x[0]
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Construct a list

```
def numer(x):
    """Return the numerator of rational number X."""
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def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
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Select item from a list

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Select item from a list

(Demo)

Reducing to Lowest Terms

Example:

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$$\frac{3}{2} * \frac{5}{3}$$

Reducing to Lowest Terms

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$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2} \qquad \frac{2}{5} + \frac{1}{10}$$

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from math import gcd
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Reducing to Lowest Terms

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def rational(n, d):
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Reducing to Lowest Terms

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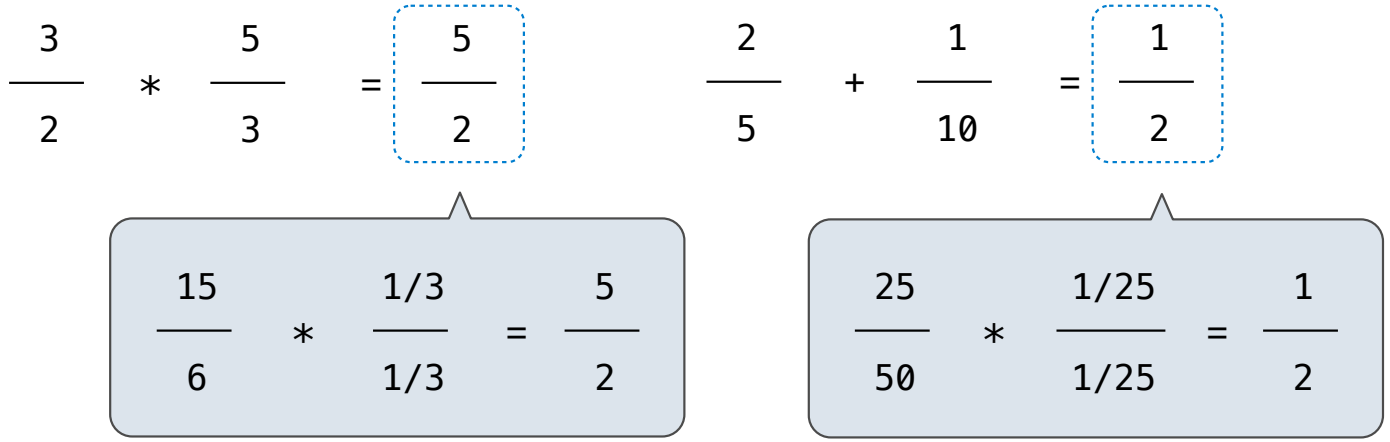


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from math import gcd

def rational(n, d):
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Reducing to Lowest Terms

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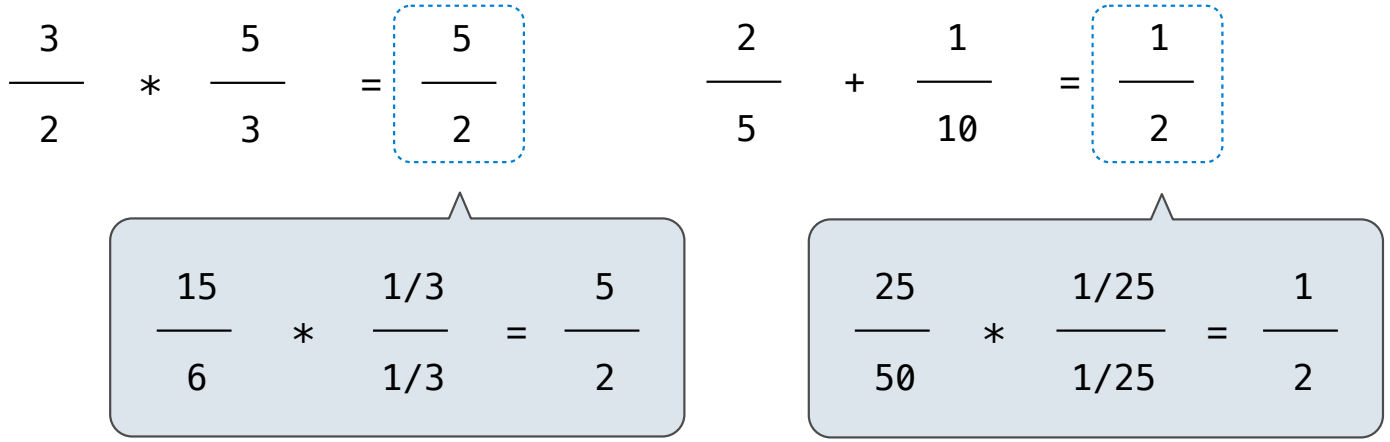


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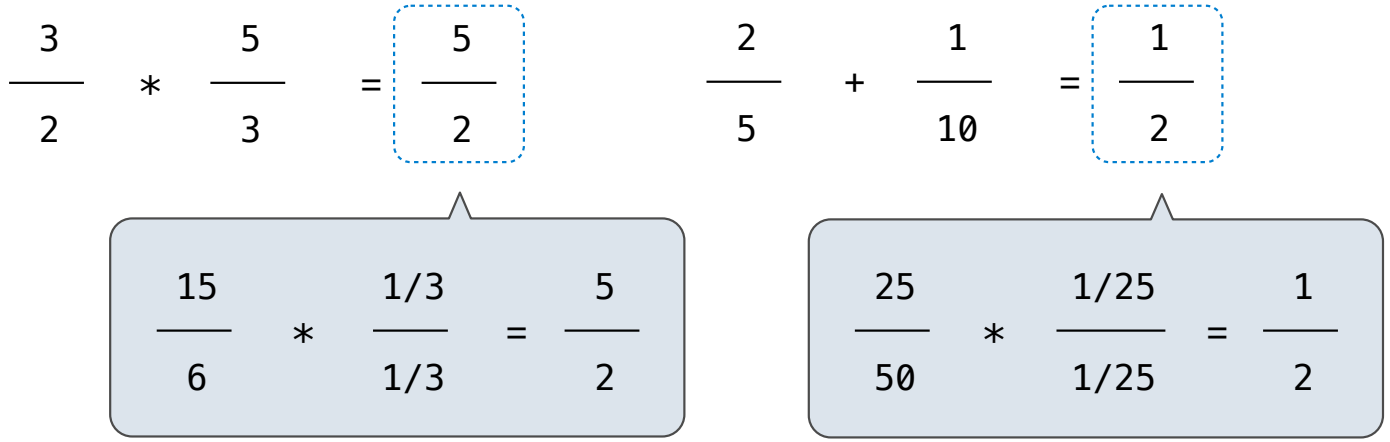


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Reducing to Lowest Terms

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Greatest common divisor

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Abstraction Barriers

Abstraction Barriers

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Parts of the program that...	Treat rationals as...	Using...

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation		

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	`add_rational`, `mul_rational` `rationals_are_equal`, `print_rational`

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Create rationals or implement rational operations		

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Create rationals or implement rational operations	numerators and denominators	`rational`, `numer`, `denom`
Implement selectors and constructor for rationals	two-element lists	list literals and element selection

Implementation of lists

Abstraction Barriers

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Violating Abstraction Barriers

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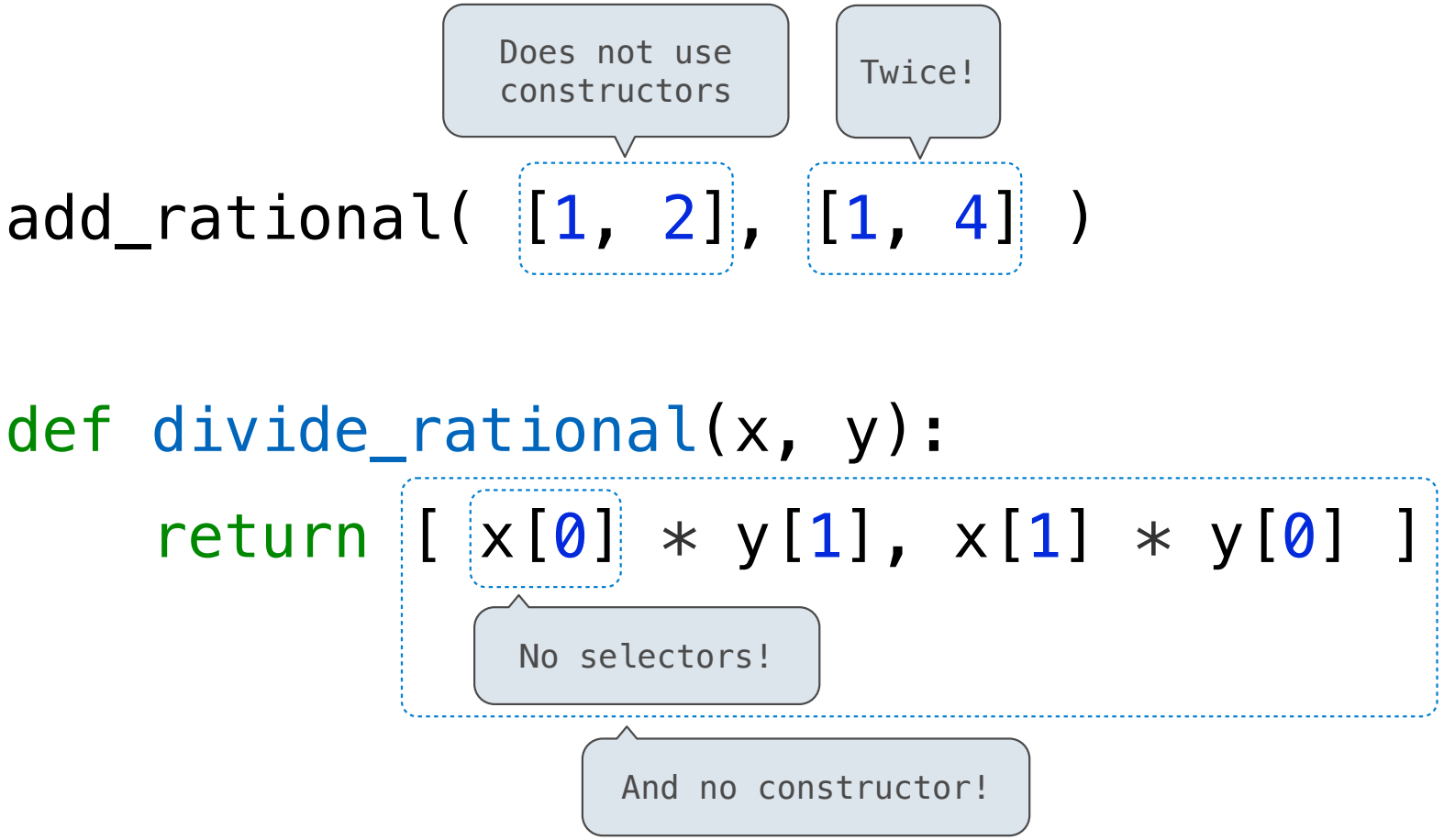

Violating Abstraction Barriers

Data Representations

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(Demo)

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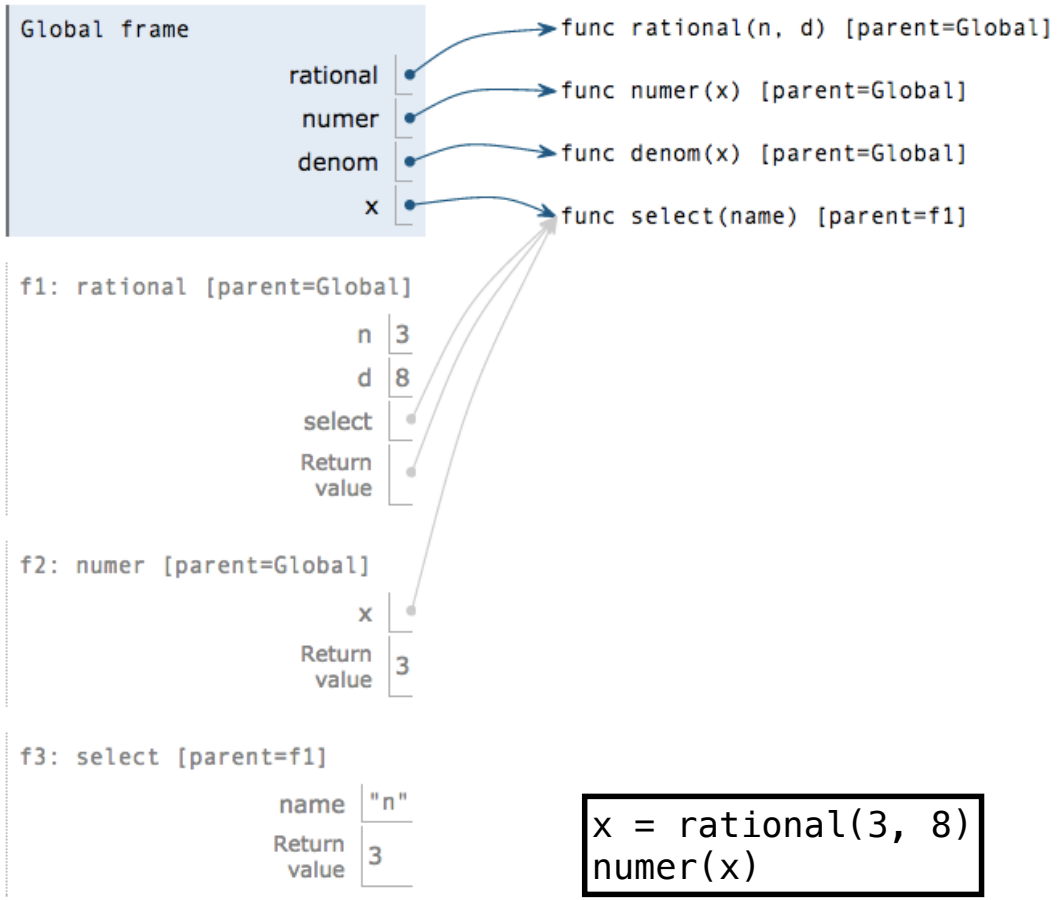


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pythontutor.com/composingprograms.html#code=def%20rational%28n,
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0A%20%20%20%20%0Adef%20numer%28x%29%3A%0A%20%20%20%20return%20x%28'n'%29%0A%0Adef%20denom%28x%29%3A%0A%20%20%20%20return%20x%28'd'%29%0A%20%20%20%20%0Ax%20%3D%20rational%283,%208%29%0Anumer%28x%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0