

Final Examples

Announcements

Trees

Tree-Structured Data

Tree-Structured Data

```
def tree(label, branches=[]):
    return [label] + list(branches)

def label(t):
    return t[0]

def branches(t):
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def is_leaf(t):
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class Tree:
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A tree can contains other trees:

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<ul>
  <li>Midterm <b>1</b></li>
  <li>Midterm <b>2</b></li>
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Tree processing often involves recursive calls on subtrees

Tree Processing

Solving Tree Problems

Implement **big**s, which takes a Tree instance t containing integer labels. It returns the number of nodes in t whose labels are larger than all labels of their ancestor nodes.

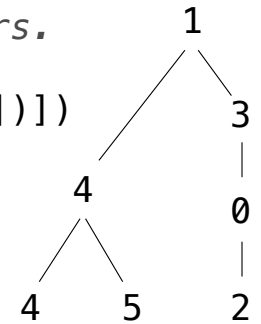
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>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
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```
>>> bigs(a)
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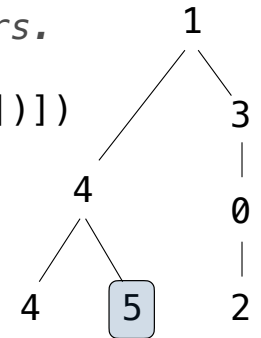
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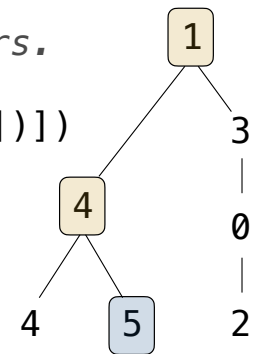
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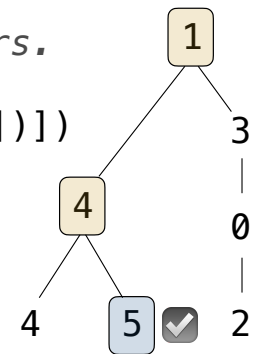
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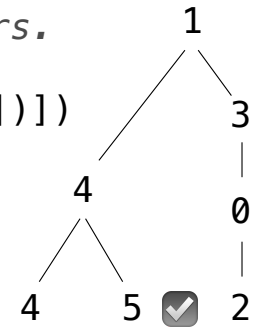
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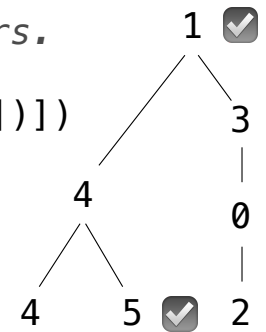
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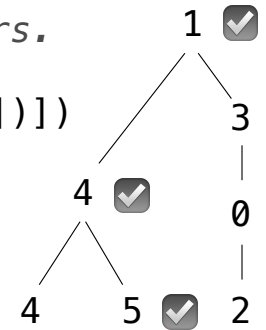
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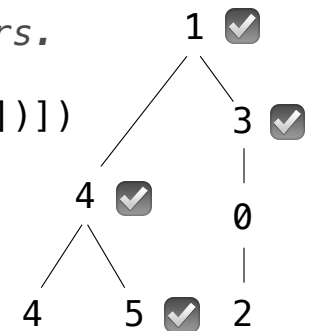
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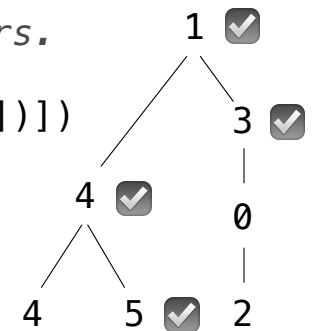
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The root label is always larger than all of its ancestors

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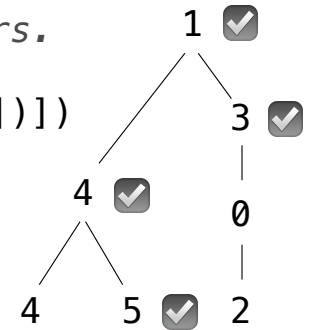
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```
    if t.is_leaf():
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```
    else:
```

```
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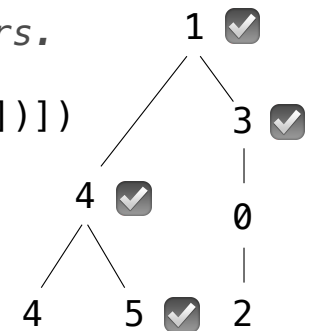
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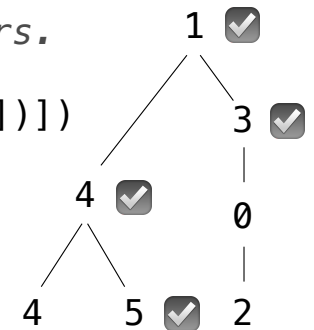
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Somehow increment
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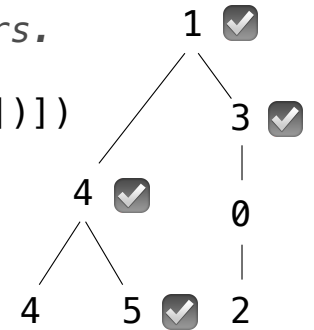
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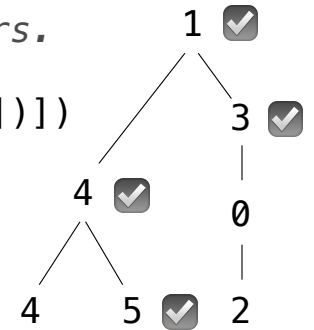
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Somehow track a
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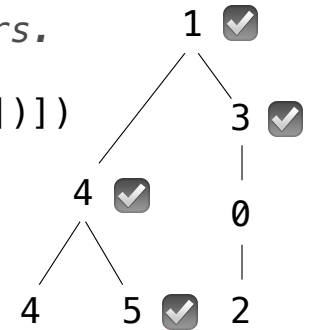
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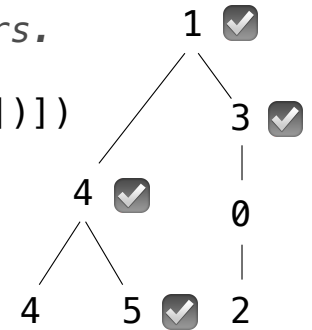
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Somehow track a
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```
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Somehow track the
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"""
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```
def f(a, x):
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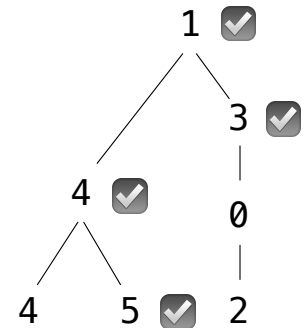
```
    if _____:
```

```
        return 1 + _____
```

```
    else:
```

```
        return _____
```

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```



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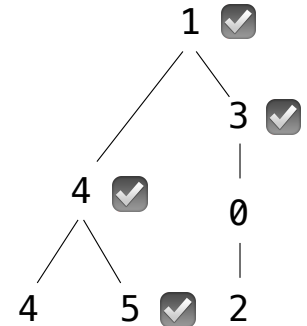
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Somehow track the
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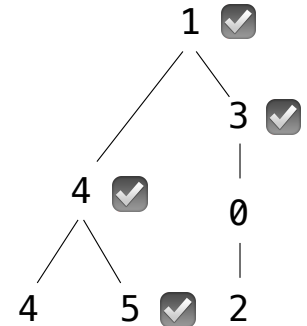
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Somehow track the
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node.label > max_ancestors



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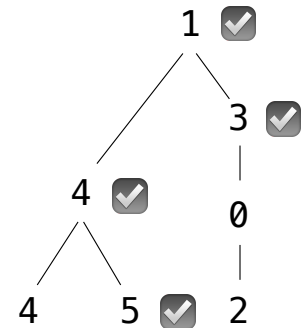
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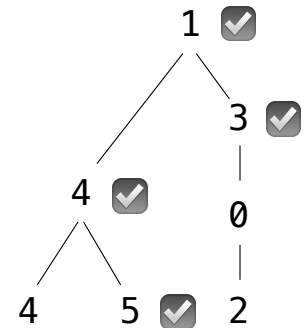
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def bigs(t):
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■■ ■■ ■■

```
def f(a, x):
```

A node   *max_ancestor*

if

Somehow track the largest ancestor

```
a.label > x
```

```
node.label > max_ancestors
```

```
return 1 +
```

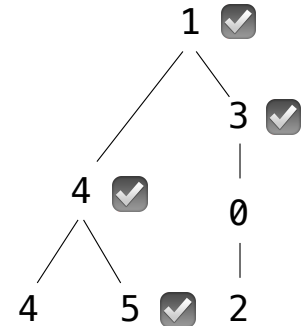
else:

return

return

 $f(t,$

)



Solving Tree Problems

Implement **big**s, which takes a Tree instance *t* containing integer labels. It returns the number of nodes in *t* whose labels are larger than all labels of their ancestor nodes.

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def bigs(t):
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```
>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
```

```
>>> bigs(a)
```

```
4
```

```
"""
```

```
def f(a, x):
```

```
    A node  $\nearrow$   $\nwarrow$  max_ancestor
```

```
    if a.label > x:
```

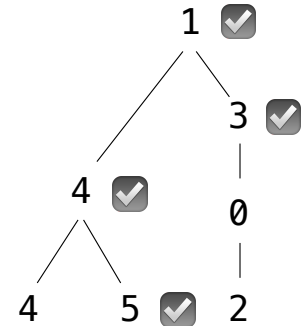
```
        return 1 +
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```
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```

```
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```

```
    return f(t,
```

Some initial value for the largest ancestor so far...



Solving Tree Problems

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```
    if
```

```
        a.label > x
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```
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```

```
    :
```

```
        return 1 +
```

```
            Somehow increment the total count
```

```
    else:
```

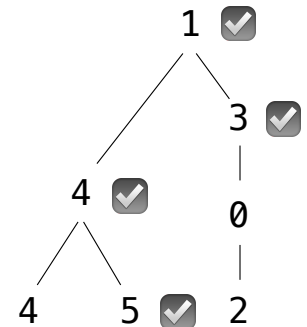
```
        return
```

```
    return
```

```
        f(t,
```

```
        )
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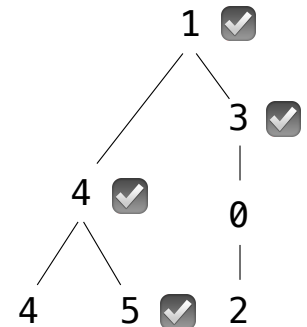
```
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```
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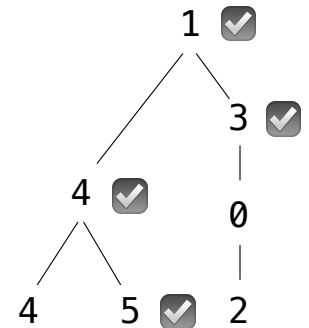
```
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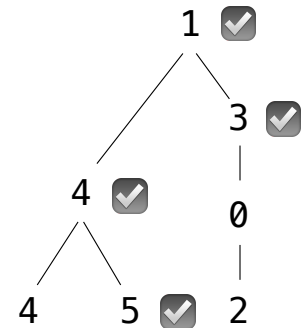
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```
        Root label is always larger than its ancestors
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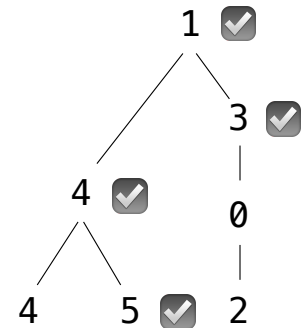
```
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    return f(t, t.label - 1)
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Somehow track the largest ancestor

node.label > max_ancestors

```
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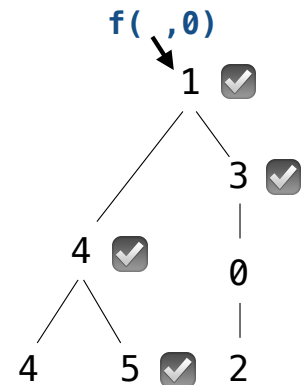
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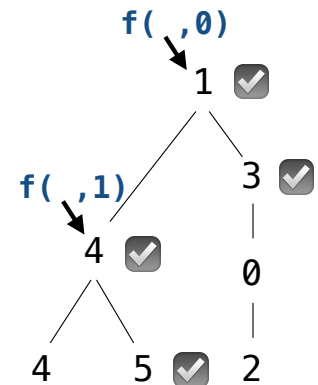
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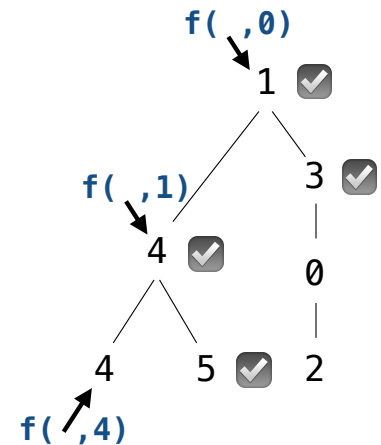
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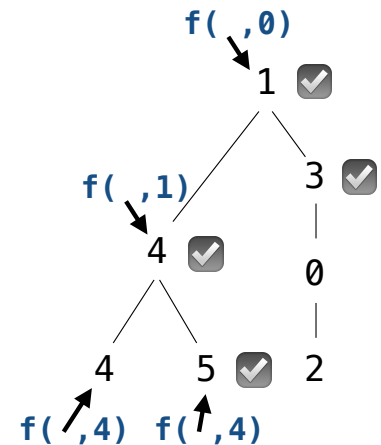
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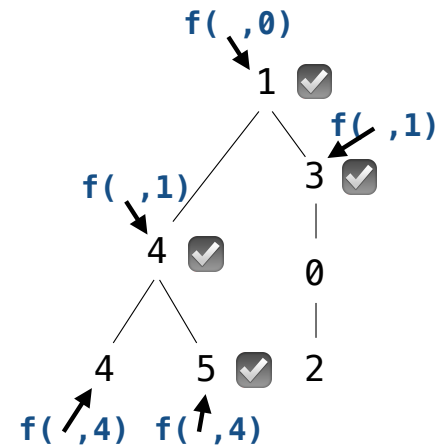
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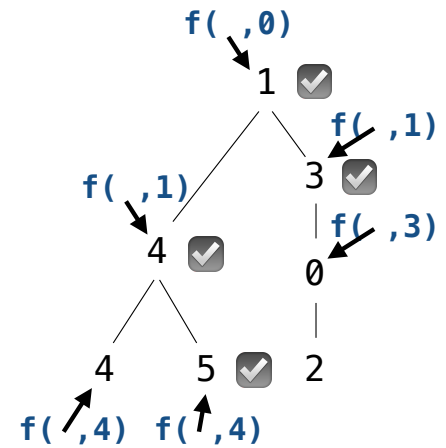
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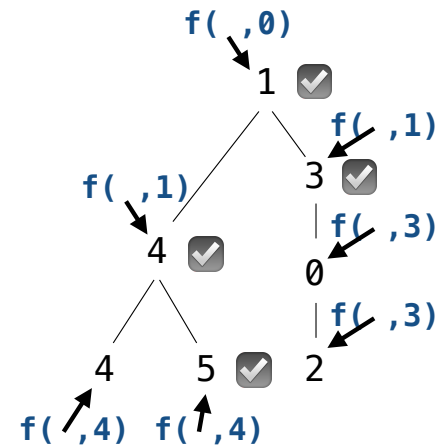
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Recursive Accumulation

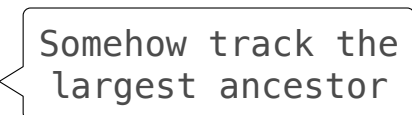
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def big(t):  
    """Return the number of nodes in t that are larger than all their ancestors."""  
    n = [0]  
  
    def f(a, x):  
        if _____:  
            _____  
            _____:  
                f(_____)  
        _____  
  
    return n[0]
```

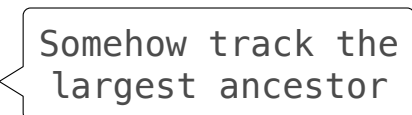
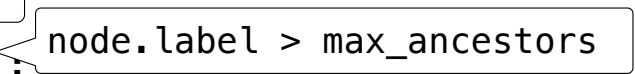
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            _____  
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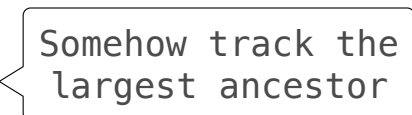
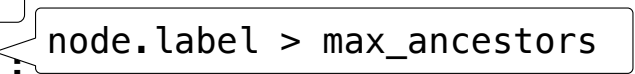
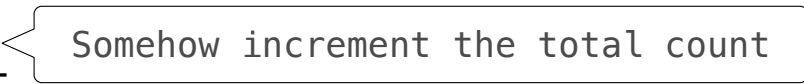
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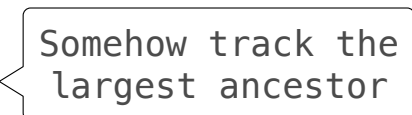
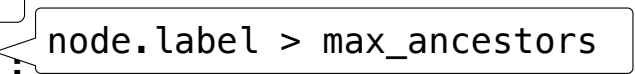
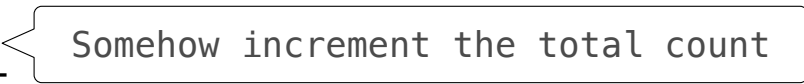

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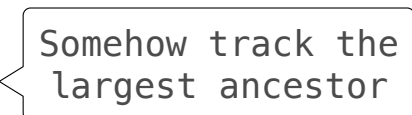
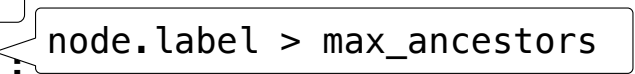
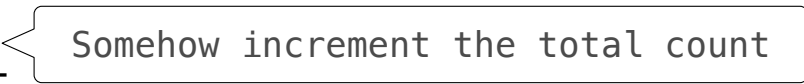

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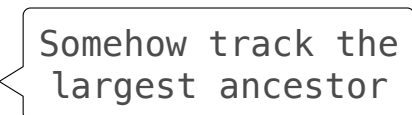
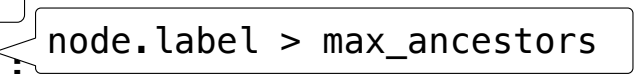
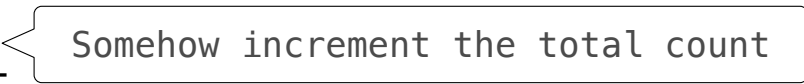

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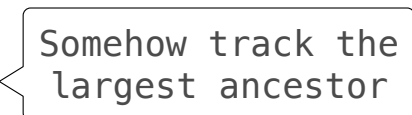
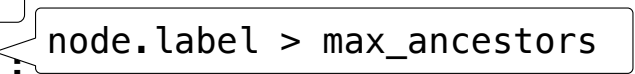
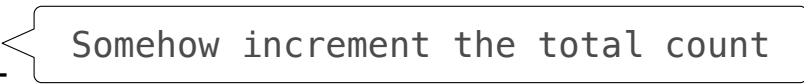

Solving Tree Problems

Implement **big**s, which takes a Tree instance *t* containing integer labels. It returns the number of nodes in *t* whose labels are larger than any labels of their ancestor nodes.

```
def bigs(t):  
    """Return the number of nodes in t that are larger than all their ancestors."""  
    n = [0]  
    def f(a, x):  
         Somehow track the largest ancestor  
        if a.label > x:  node.label > max_ancestors  
             Somehow increment the total count  
            n[0] += 1  
        f(a.left, a.label)  
        f(a.right, a.label)  
    f(t, -1)  Root label is always larger than its ancestors  
    return n[0]
```

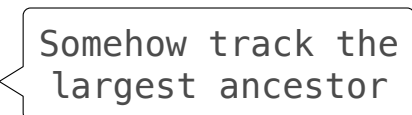
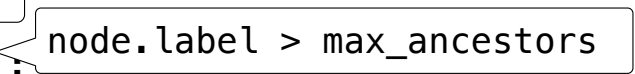
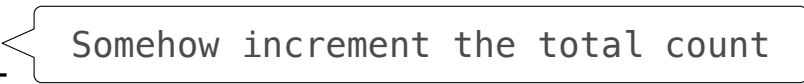

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    f(t, t.label - 1)  Root label is always larger than its ancestors  
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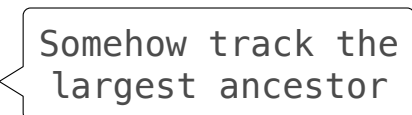
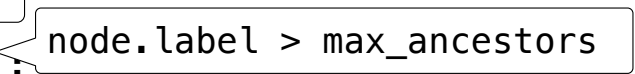
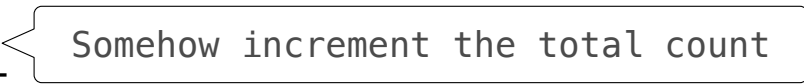

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             Somehow increment the total count  
            n[0] += 1  
        for b in a.branches:  
            f(_____)  Root label is always larger than its ancestors  
    f(t, t.label - 1)  
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             Somehow increment the total count  
            n[0] += 1  
        for b in a.branches:  
            f(b, max(a.label, x))  
    f(t, t.label - 1)  Root label is always larger than its ancestors  
  
    return n[0]
```

Designing Functions

How to Design Programs

<https://htdp.org/2018-01-06/Book/>

How to Design Programs

From Problem Analysis to Data Definitions

Identify the information that must be represented and how it is represented in the chosen programming language. Formulate data definitions and illustrate them with examples.

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Articulate the examples as tests and ensure that the function passes all. Doing so discovers mistakes. Tests also supplement examples in that they help others read and understand the definition when the need arises—and it will arise for any serious program.

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Applying the Design Process

Designing a Function

Implement `smalls`, which takes a `Tree` instance `t` containing integer labels. It returns the non-leaf nodes in `t` whose labels are smaller than any labels of their descendant nodes.

```
def smalls(t):  
    """Return the non-leaf nodes in t that are smaller than all their descendants.  
  
    >>> a = Tree(1, [Tree(2, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(6)])])])  
    >>> sorted([t.label for t in smalls(a)])  
    [0, 2]  
  
    """  
    result = []  
    def process(t):  
  
        process(t)  
    return result
```

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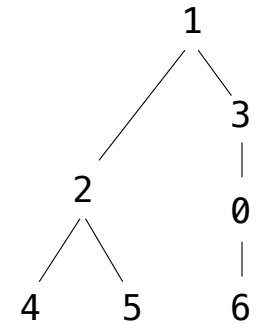
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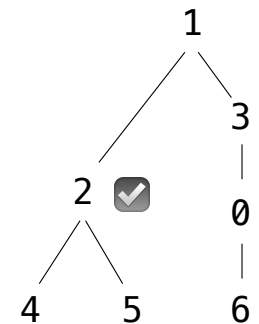
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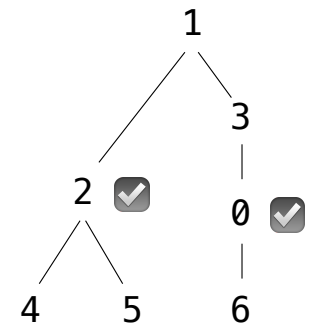
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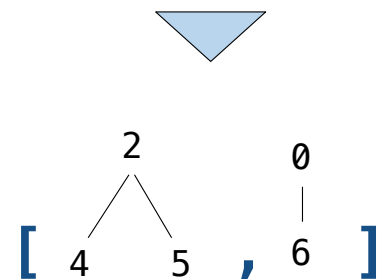
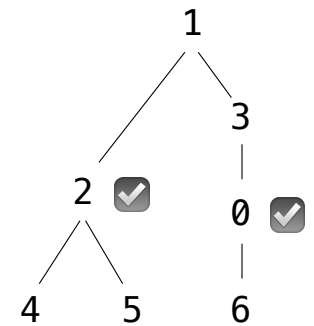
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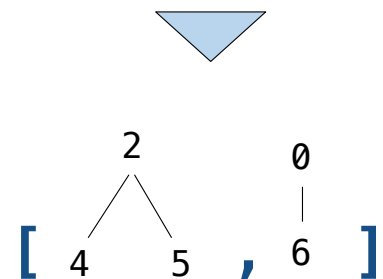
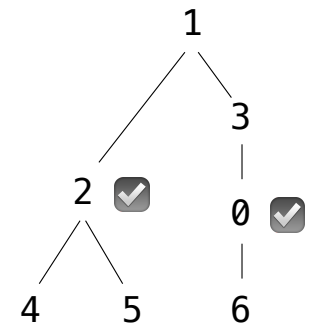
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    result = []
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        process(t)
    return result
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Designing a Function

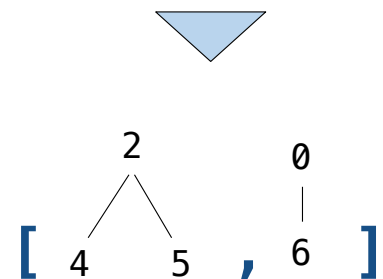
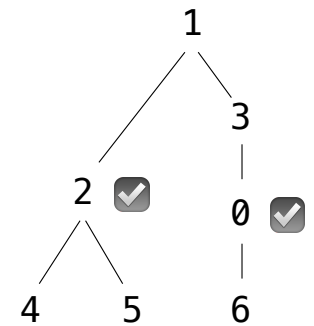
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    process(t)
    return result
```



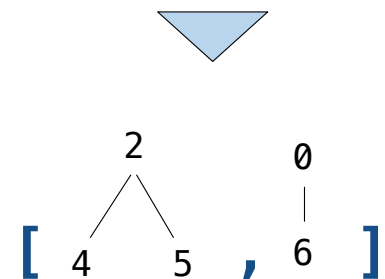
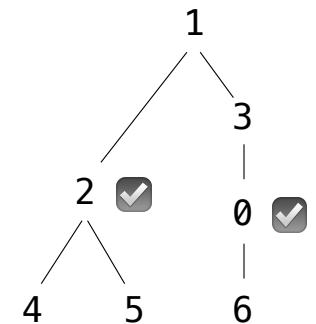
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    """
    result = []
    Signature: Tree -> number
    def process(t): "Find smallest label in t & maybe add t to result"
        if t.is_leaf():
            return t.label
        else:
            return min(...)
    process(t)
    return result
```



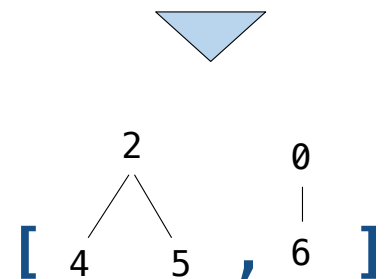
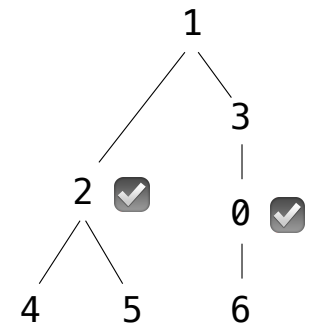
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    """
    result = []
    Signature: Tree -> number
    def process(t): "Find smallest label in t & maybe add t to result"
        if t.is_leaf():
            return _____
        else:
            smallest = _____
            if _____:
                _____
            return min(smallest, t.label)
    process(t)
    return result
```



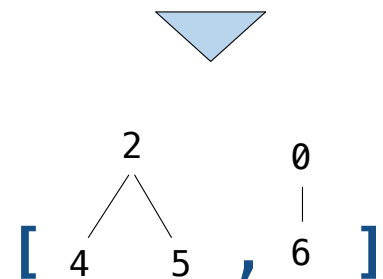
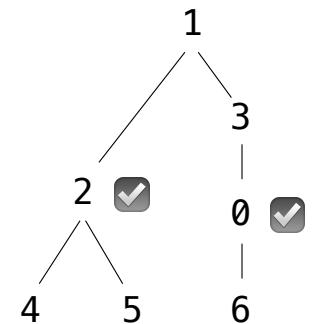
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            if _____:
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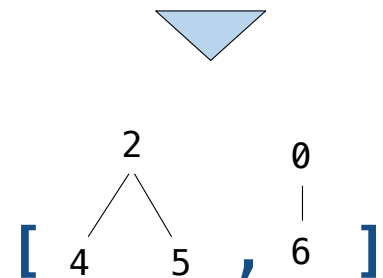
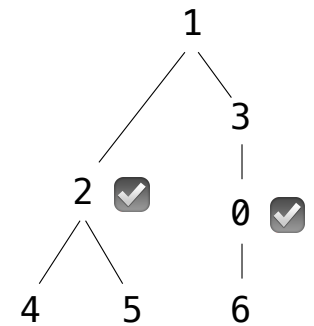
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    """
    result = []
    Signature: Tree -> number
    def process(t): "Find smallest label in t & maybe add t to result"
        if t.is_leaf():
            return _____
        else:
            smallest = _____
            smallest label in a branch of t if _____:
                return min(smallest, t.label)
    process(t)
    return result
```



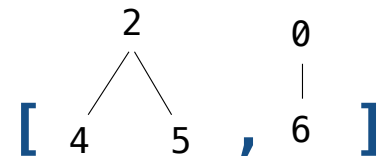
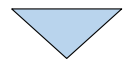
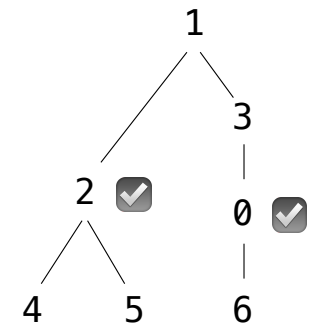
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    """
    result = []
    Signature: Tree -> number
    def process(t): "Find smallest label in t & maybe add t to result"
        if t.is_leaf():
            return t.label
        else:
            smallest = _____
            if t.label < smallest:
                _____
            return min(smallest, t.label)
    process(t)
    return result
```



Designing a Function

Implement `smalls`, which takes a `Tree` instance `t` containing integer labels. It returns the non-leaf nodes in `t` whose labels are smaller than any labels of their descendant nodes.

`def smalls(t):` *Signature: Tree -> List of Trees*

"""Return the non-leaf nodes in t that are smaller than all their descendants."""

```
>>> a = Tree(1, [Tree(2, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(6)])])])
```

```
>>> sorted([t.label for t in smalls(a)])  
[0, 2]
```

"""

`result = []`

Signature: Tree -> number

`def process(t):` *"Find smallest label in t & maybe add t to result"*

`if t.is_leaf():`

`return` _____ `t.label`

`else:`

`smallest =` _____

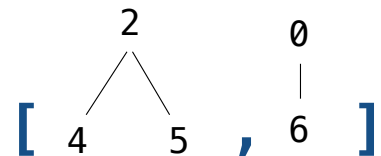
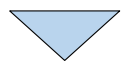
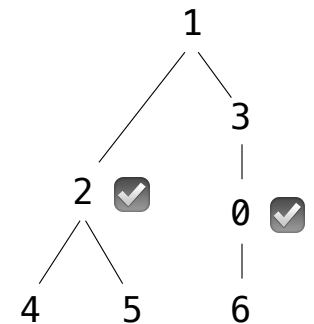
`if` _____ `t.label < smallest`

`result.append(_____)`

`return` `min(smallest, t.label)`

`process(t)`

`return` `result`



Designing a Function

Implement `smalls`, which takes a `Tree` instance `t` containing integer labels. It returns the non-leaf nodes in `t` whose labels are smaller than any labels of their descendant nodes.

`def smalls(t):` *Signature: Tree -> List of Trees*

"""Return the non-leaf nodes in t that are smaller than all their descendants."""

>>> a = Tree(1, [Tree(2, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(6)])])])

>>> sorted([t.label for t in smalls(a)])
[0, 2]

"""

result = []

Signature: Tree -> number

`def process(t):` *"Find smallest label in t & maybe add t to result"*

if t.is_leaf():

 return t.label

else:

 smallest =

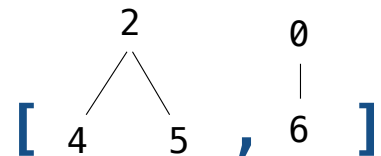
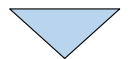
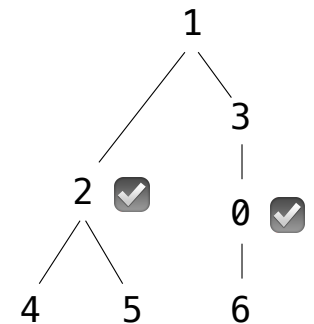
if t.label < smallest

 result.append(t)

 return min(smallest, t.label)

process(t)

return result



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Signature: `Tree -> List of Trees`

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def smalls(t):  
    """Return the non-leaf nodes in t that are smaller than all their descendants.
```

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>>> a = Tree(1, [Tree(2, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(6)])])])  
>>> sorted([t.label for t in smalls(a)])  
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```

```
"""
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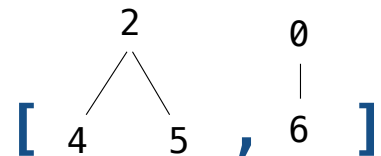
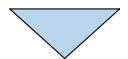
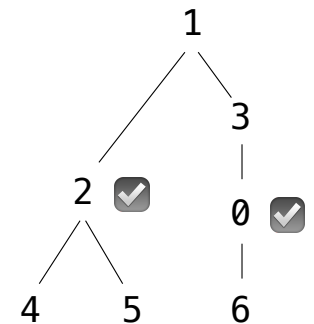
Signature: `Tree -> number`

Find smallest label in t & maybe add t to result

```
def process(t):  
    if t.is_leaf():  
        return t.label  
    else:  
        smallest = min([process(b) for b in t.branches])  
        if t.label < smallest:  
            result.append( t )  
        return min(smallest, t.label)
```

```
process(t)
```

```
return result
```



Interpreters

Interpreter Analysis

What expressions are passed to `scheme_eval` when evaluating the following expressions?

```
(define x (+ 1 2))
```

```
(define (f y) (+ x y))
```

```
(f (if (> 3 2) 4 5))
```

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