**CS61A Notes – Week 9: Recursion and tree recursion (solutions)**

1. Write a procedure expt(base, power), which implements the exponent function. For example, expt(3, 2) returns 9, and expt(2, 3) returns 8. Use recursion!  
     
   We know that a0 = 1 for all a != 0, so we can use this fact for our base case. From there, the recursive call just relies on the fact that ab = a\*ab-1.  
     
    def expt(base, power):  
    if power == 0:  
    return 1  
    else:  
    return n \* expt(base, power – 1)
2. Write a procedure merge(s1, s2) which takes two sorted (smallest value first) lists and returns a single list with all of the elements of the two lists, in ascending order. (*Hint*: if you can figure out which list has the smallest element out of both, then we know that the resulting merged list will be that smallest element, followed by the merge of the two lists with the smallest item removed. Don’t forget to handle the case where one list is empty!). Use recursion.  
     
   Since the two sequences we get as arguments are already sorted, we know the smallest element out of both sequences must lie at the front of one of the two. Thus, at each step, we take the smaller of the two first elements, put it at the front of our result, and make a recursive call.  
     
   We stop when either of the sequences becomes empty, at which point we can simply return the other, non-empty sequence. (This also works if both sequences are empty; can you see why?)  
     
    def merge(s1, s2):  
    if not s1: # s1 is empty  
    return s2  
    elif not s2: # s2 is empty  
    return s1  
    elif s1[0] < s2[0]:  
    return [s1[0]] + merge(s1[1:], s2)  
    else:  
    return [s2[0]] + merge(s1, s2[1:])

**Multiple recursive calls at a time: Tree recursion**

1. I want to go up a flight of stairs that has n steps. I can either take 1 or 2 steps each time. How many different ways can I go up this flight of stairs? Write a procedure count\_stair\_ways that solves this problem for me.  
     
   This problem demonstrates the power of recursion. We solve this problem by brute force; we simply try every single method of climbing up the staircase and see how many of them bring us to the top exactly!  
     
   Our first base case is n == 0. If this is the case, then we know we have reached the top of the staircase exactly, which means there is 1 way to climb the staircase. If n < 0, then we have overshot the top of the staircase, which doesn’t count as a way (thus we return 0).  
     
   Now the recursive call simply tries to take one step (recursive call with argument n – 1), then tries to take two steps (recursive call with argument n – 2), and sums the two results together to get our final result.  
     
    def count\_stair\_ways(n):  
    if n == 0:  
    return 1  
    elif n < 0:  
    return 0  
    else:  
    return count\_stair\_ways(n – 1) + count\_stair\_ways(n – 2)
2. Consider the subset\_sum problem: you are given a list of integers and a number k. Is there a subset of the list that adds up to k? For example:  
     
   This is actually a very famous problem. Our approach to this problem is going to be to generate every possible subset and see if any of them ever sum up to k.  
     
   Let’s talk about base cases. If I asked you if whether there is a subset of a list that sums to 0, the answer is **always** yes – all I have to do is take an empty subset, which then sums to 0! This is one of our base cases.  
     
   What if I asked you if an empty list could sum to some k != 0? The answer is clearly no, since the only subset of an empty list is the empty list, and we already know that sums to 0. This is our second base case.  
     
   Note that the ordering of our base cases in the following solution is important, since subset\_sum([], 0) should return True.  
     
   Now we consider the recursive case, looking only at the first item in the list. We can either choose to include this in the sum, which would reduce k by lst[0] in the recursive call, or we leave it out of the sum, which would leave k the same. In both cases, we no longer need to look at the first element, so we can use lst[1:] as our new list in the recursive call. If either of these recursive calls returns true, then we know there is a subset that sums to the value, which is why we combine using or.  
     
   A lot of people asked if you could use k <= 0 as a base case. My answer to this is no, since your list might contain negative integers too! (Consider subset\_sum([8, -2], 6).)  
     
     
    >>> subset\_sum([2, 4, 7, 3], 5) # 2 + 3 = 5  
    True  
    >>> subset\_sum([1, 9, 5, 7, 3], 2)  
    False  
     
    def subset\_sum(lst, k):  
    if k == 0:  
    return True  
    elif not lst: # lst is empty  
    return False  
    else:  
    return subset\_sum(lst[1:], k – ls[0} \  
    or subset\_sum(lst[1:], k)

1. We will now write one of the faster sorting algorithms commonly used, named Mergesort. Merge sort works like this:
   1. If there’s only one (or zero) item(s) in the sequence, it’s already sorted!
   2. If there’s more than one item, then we can split the sequence in half, sort each half recursively, then merge the results (using the merge procedure from earlier in the notes). The result will be a sorted sequence.

Using the described algorithm, write a function mergesort(s) that takes an unsorted sequence s and sorts it.  
  
The code here basically follows the directions, word for word.  
  
 def mergesort(s):  
 if len(s) <= 1:  
 return s  
 else:  
 mid = len(s) // 2  
 return merge(mergesort(s[:mid]), mergesort(s[mid:]))

1. Pascal’s triangle is a useful recursive definition that tells us the coefficients in the expansion of the polynomial *(x + a)n*. Each element in the triangle has a coordinate, given by the row it is on and its position in the row (which you could call a column). Every number in Pascal’s triangle is defined as the sum of the item above it and the item above it and to the left (its position in the row, minus one). If there is a position that does not have an entry, we treat it as if we had a 0 there. Below are the first few rows of this triangle:  
     
    Item: 0 1 2 3 4 5 ...  
    Row 0: 1  
    Row 1: 1 1  
    Row 2: 1 2 1  
    Row 3: 1 3 3 1  
    Row 4: 1 4 6 4 1  
    Row 5: 1 5 10 10 5 1  
    ...  
     
   Define the procedure pascal(row, column) which takes a row and a column, and finds the value at that position in the triangle. Don’t use the closed-form solution, even if you know it!

There are a number of base cases we could’ve used for this problem; here’s one example. The base cases here basically say “if you go out of bounds, return 0, and if you’re in the upper left corner, return 1.” That alone is good enough to make the full program work!  
  
 def pascal(row, column):  
 if row < 0 and column < 0:  
 return 0  
 elif row == 0 and column == 0:  
 return 1  
 else:  
 return pascal(row – 1, column) \  
 + pascal(row – 1, column – 1)