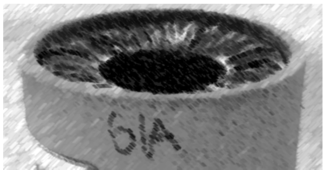

# CS61A Lecture 25

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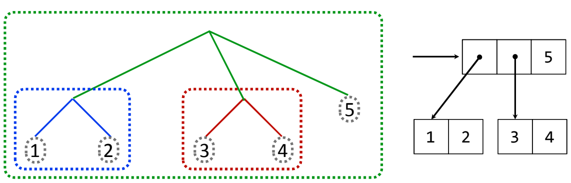
## Announcements

- HW8 due tonight at 7pm
- Midterm 2 Thursday at 7pm
  - See course website for more information

## Tree Structured Data

Nested Sequences are Hierarchical Structures.

```
((1, 2), (3, 4), 5)
```




*In every tree, a vast forest*

Example: http://goo.gl/0h6n5

## Recursive Tree Processing

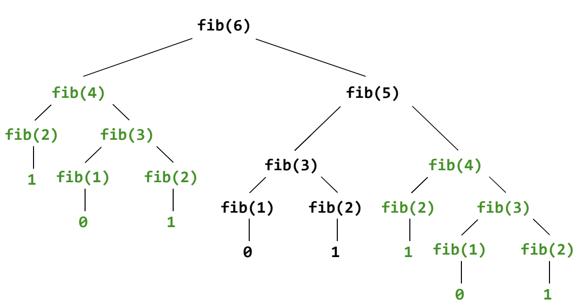
Tree operations typically make recursive calls on branches

```
def count_leaves(tree):
    if type(tree) != tuple:
        return 1
    return sum(map(count_leaves, tree))


def map_tree(tree, fn):
    if type(tree) != tuple:
        return fn(tree)
    return tuple(map_tree(branch, fn)
                 for branch in tree)
```

## Trees with Internal Node Values

Trees can have values at internal nodes as well as their leaves.




## Trees with Internal Node Values

Trees can have values at internal nodes as well as their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right

def fib_tree(n):
    if n == 1:
        return Tree(0)
    if n == 2:
        return Tree(1)
    left = fib_tree(n - 2)
    right = fib_tree(n - 1)
    return Tree(left.entry + right.entry, left, right)
```

## Memoization

Tree recursive functions can compute the same thing many times

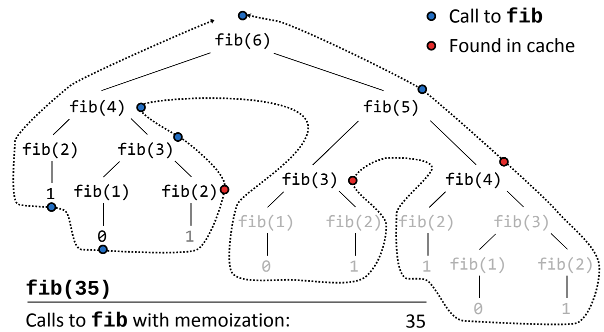
**Idea:** Remember the results that have been computed before

```
def memo(f):
    cache = {}
    def memoized(n):
        if n not in cache:
            cache[n] = f(n)
        return cache[n]
    return memoized
```

Keys are arguments that map to return values

Same behavior as **f**, if **f** is a pure function

## Memoized Tree Recursion




**fib(35)**

Calls to **fib** with memoization: 35

Calls to **fib** without memoization: 18,454,929

## Orders of Growth

Iterative, recursive, and memoized implementations are not the same.

```
def fib_iter(n):
    prev, curr = 1, 0
    for _ in range(n - 1):
        prev, curr = curr, prev + curr
    return curr
```

Time: $\Theta(n)$ Space: $\Theta(1)$

```
def fib(n):
    if n == 1:
        return 0
    if n == 2:
        return 1
    return fib(n - 2) + fib(n - 1)
```

Time: $\Theta(\phi^n)$ Space: $\Theta(n)$

```
fib = memo(fib)
```

Time: $\Theta(n)$ Space: $\Theta(n)$

## Sets

One more built-in Python container type

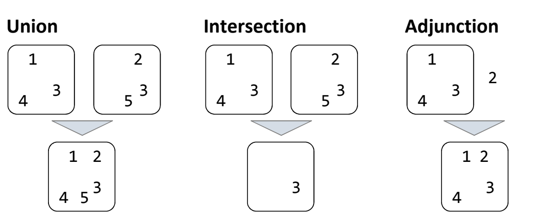
- Set literals are enclosed in braces
- Duplicate elements are removed on construction
- Sets are unordered, just like dictionary entries

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
>>> 3 in s
True
>>> len(s)
4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```

## Implementing Sets

What we should be able to do with a set:

- Membership testing: Is a value an element of a set?
- Union: Return a set with all elements in *set1* **or** *set2*
- Intersection: Return a set with any elements in *set1* **and** *set2*
- Adjunction: Return a set with all elements in *s* and a value *v*



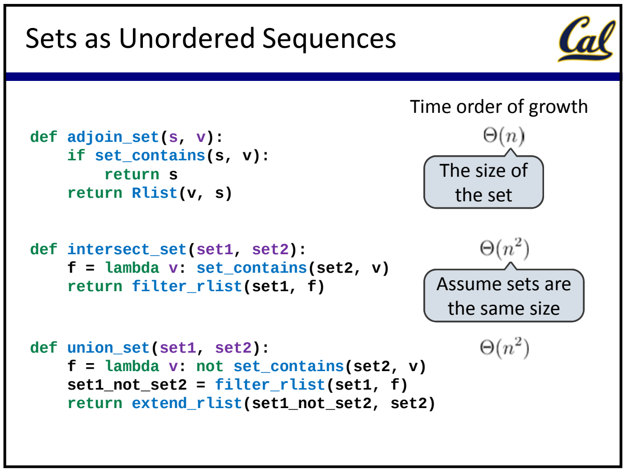

## Sets as Unordered Sequences

**Proposal 1**: A set is represented by a recursive list that contains no duplicate items

This is how we implemented dictionaries

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
        return False
    elif s.first == v:
        return True
    return set_contains(s.rest, v)
```


Sets as Unordered Sequences
Time order of growth
```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    return Rlist(v, s)
```
$\Theta(n)$
The size of the set
```
def intersect_set(set1, set2):
    f = lambda v: set_contains(set2, v)
    return filter_rlist(set1, f)
```
$\Theta(n^2)$
Assume sets are the same size
```
def union_set(set1, set2):
    f = lambda v: not set_contains(set2, v)
    set1_not_set2 = filter_rlist(set1, f)
    return extend_rlist(set1_not_set2, set2)
```
$\Theta(n^2)$