CS 61A Lecture 11

Wednesday, February 18

Announcements

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- Optional Hog Contest due Wednesday 2/18 @ 11:59pm

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- Homework 3 due Thursday 2/19 @ 11:59pm

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- Project 2 due Thursday 2/26 @ 11:59pm

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- Project 2 due Thursday 2/26 @ 11:59pm
    - Bonus point for early submission by Wednesday 2/25 @ 11:59pm!

Box-and-Pointer Notation

The Closure Property of Data Types

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- A method for combining data values satisfies the *closure property* if:

  The result of combination can itself be combined using the same method

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- Hierarchical structures are made up of parts, which themselves are made up of parts, and so on

The Closure Property of Data Types

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- Closure is powerful because it permits us to create hierarchical structures

- Hierarchical structures are made up of parts, which themselves are made up of parts, and so on

Lists can contain lists as elements (in addition to anything else)

Box-and-Pointer Notation in Environment Diagrams

Interactive Diagram

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Lists are represented as a row of index-labeled adjacent boxes, one per element

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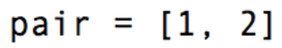

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pair = [1, 2]
```

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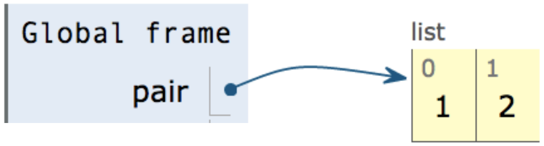

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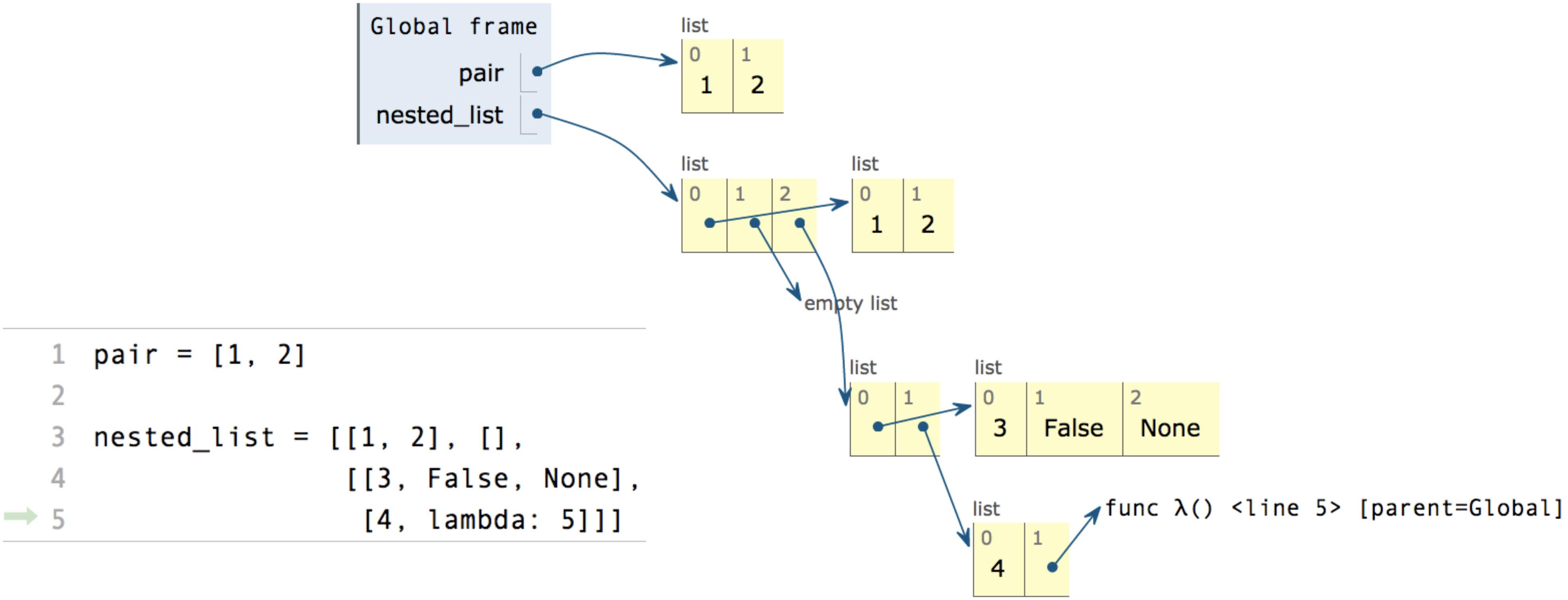

```
pair = [1, 2]

nested_list = [[1, 2], [],
               [[3, False, None],
                [4, lambda: 5]]]
```

Interactive Diagram

Sequence Operations

Membership & Slicing

Python sequences have operators for membership and slicing

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>>> digits = [1, 8, 2, 8]
>>> 2 in digits
True
>>> 1828 not in digits
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```
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Slicing creates a new object

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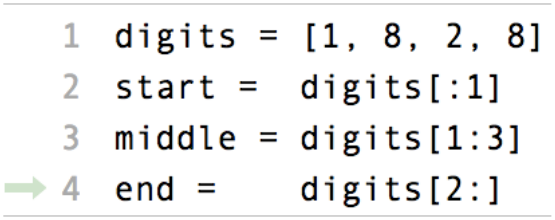

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digits = [1, 8, 2, 8]
start =  digits[:1]
middle = digits[1:3]
end =    digits[2:]
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Membership.

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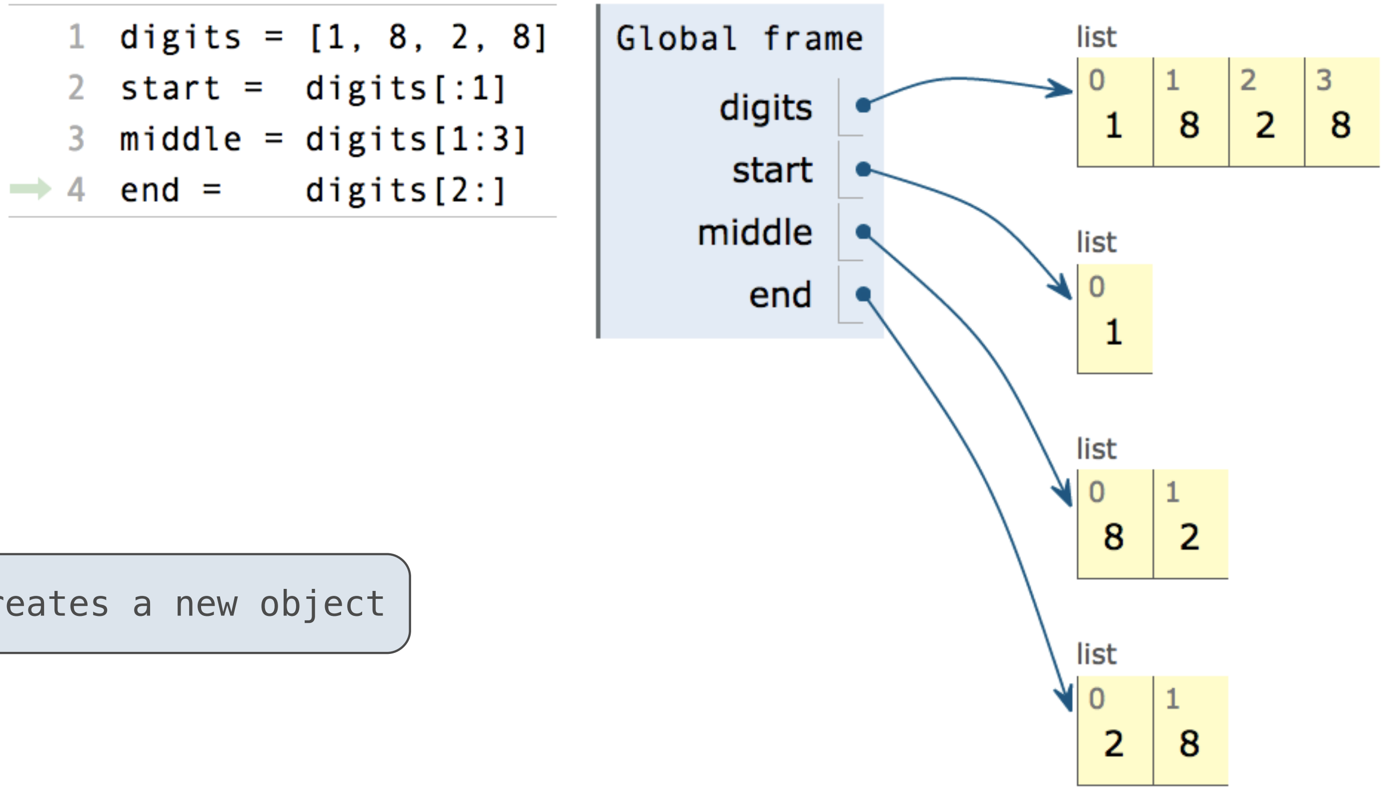

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Slicing.

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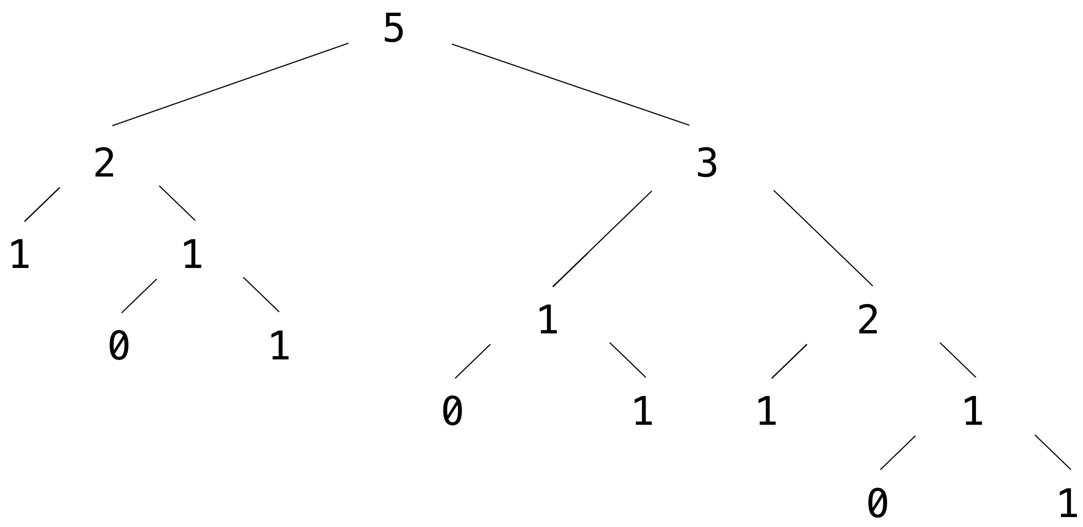
Slicing creates a new object

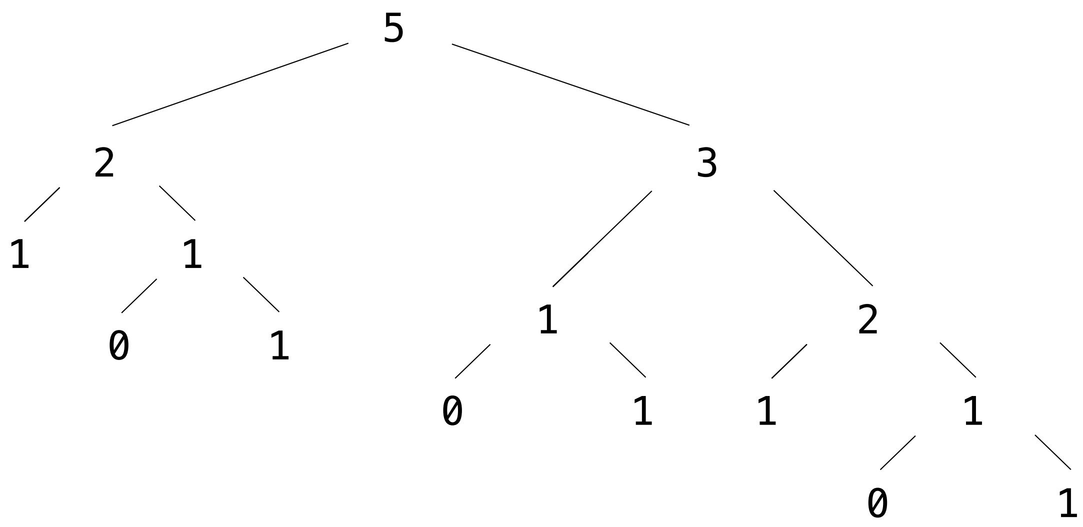
Trees

Tree Abstraction

Tree Abstraction

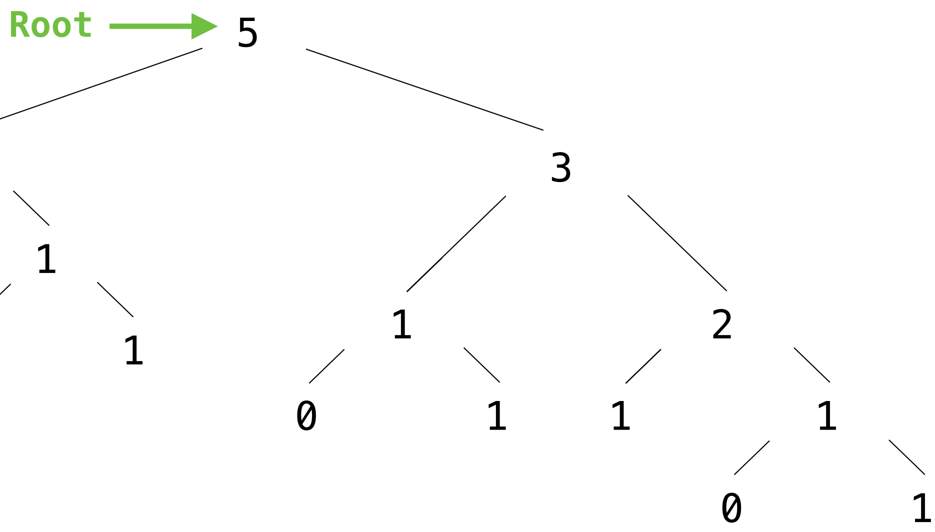


Tree Abstraction



A tree has a root value and a sequence of branches; each branch is a tree

Tree Abstraction



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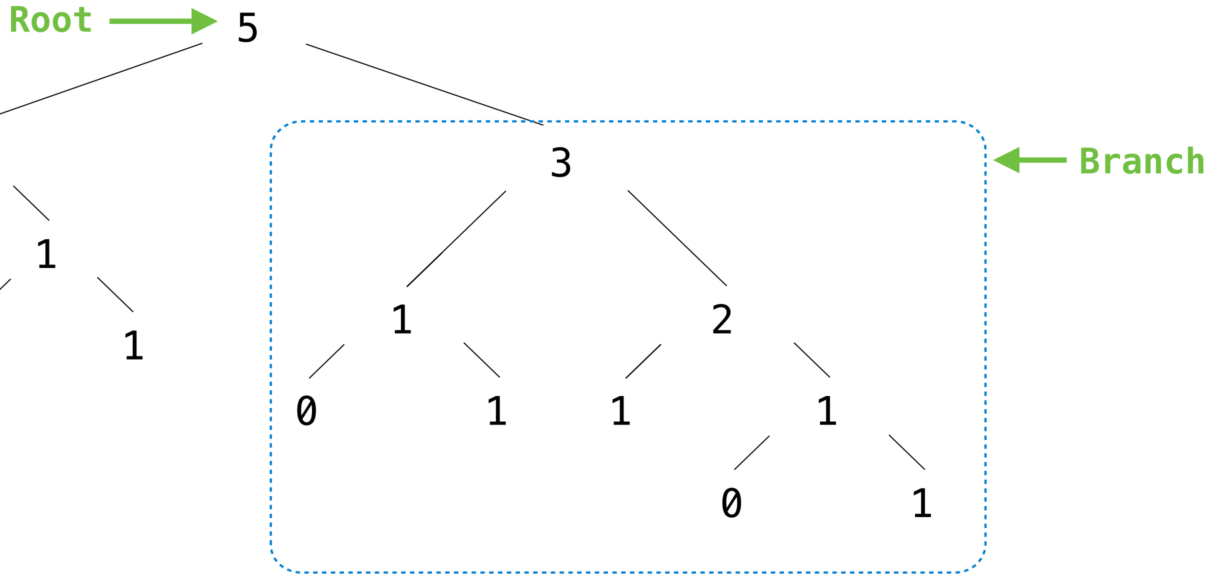

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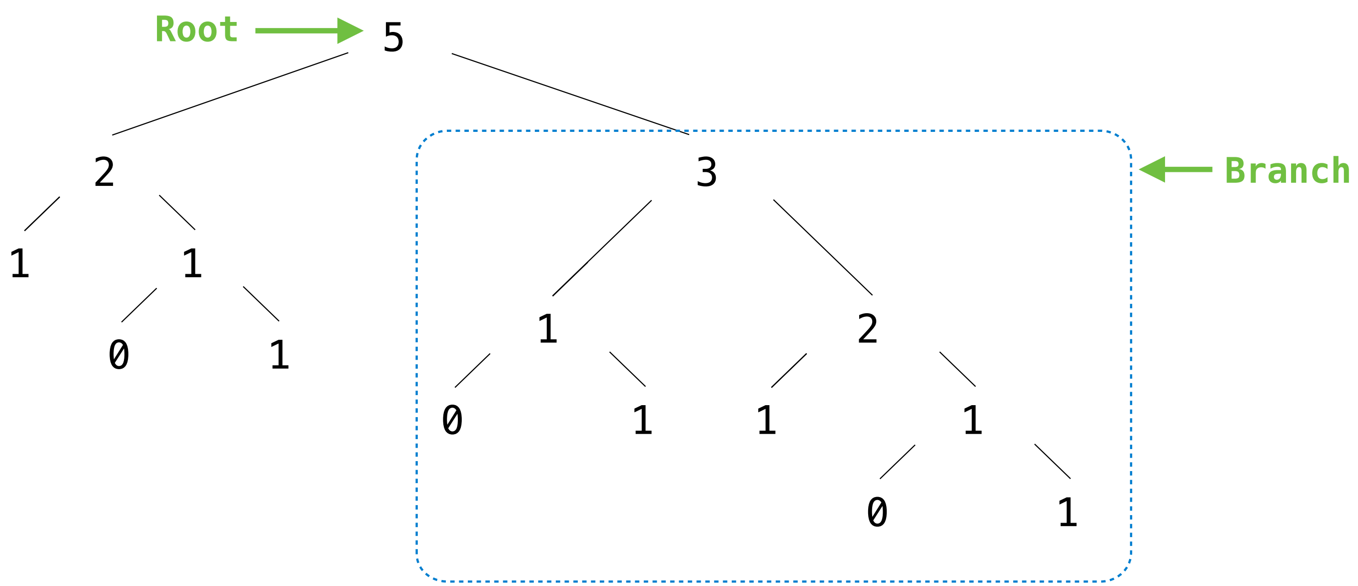
Tree Abstraction



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A tree with zero branches is called a leaf

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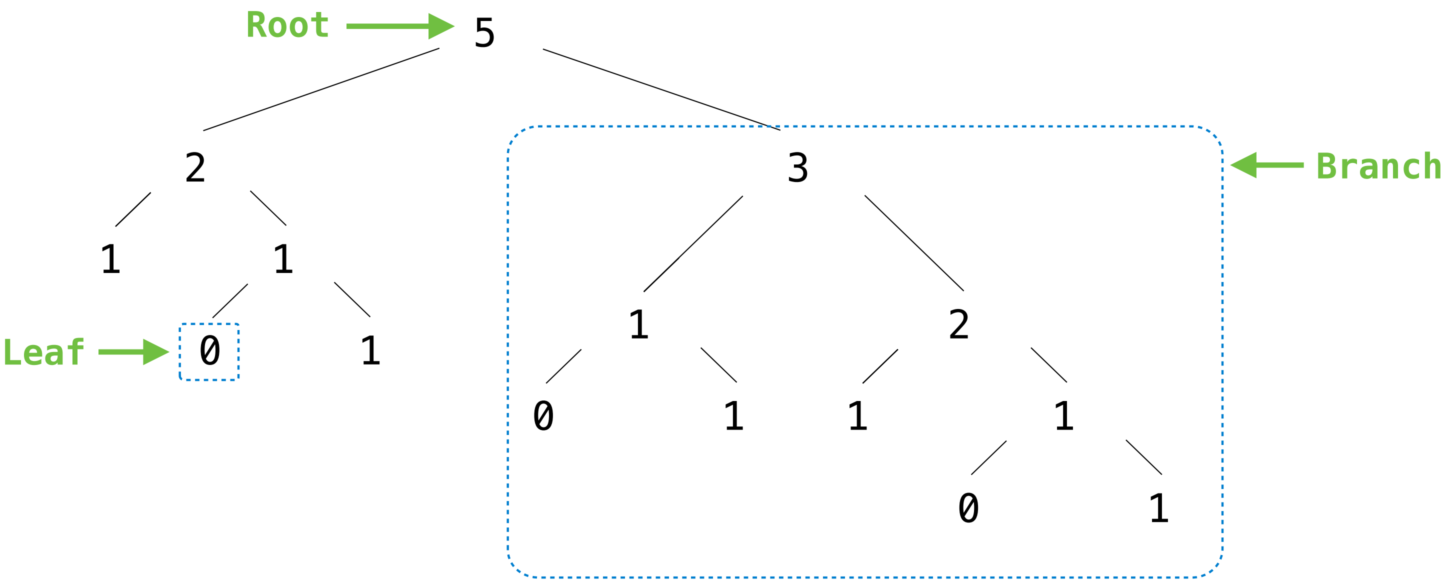

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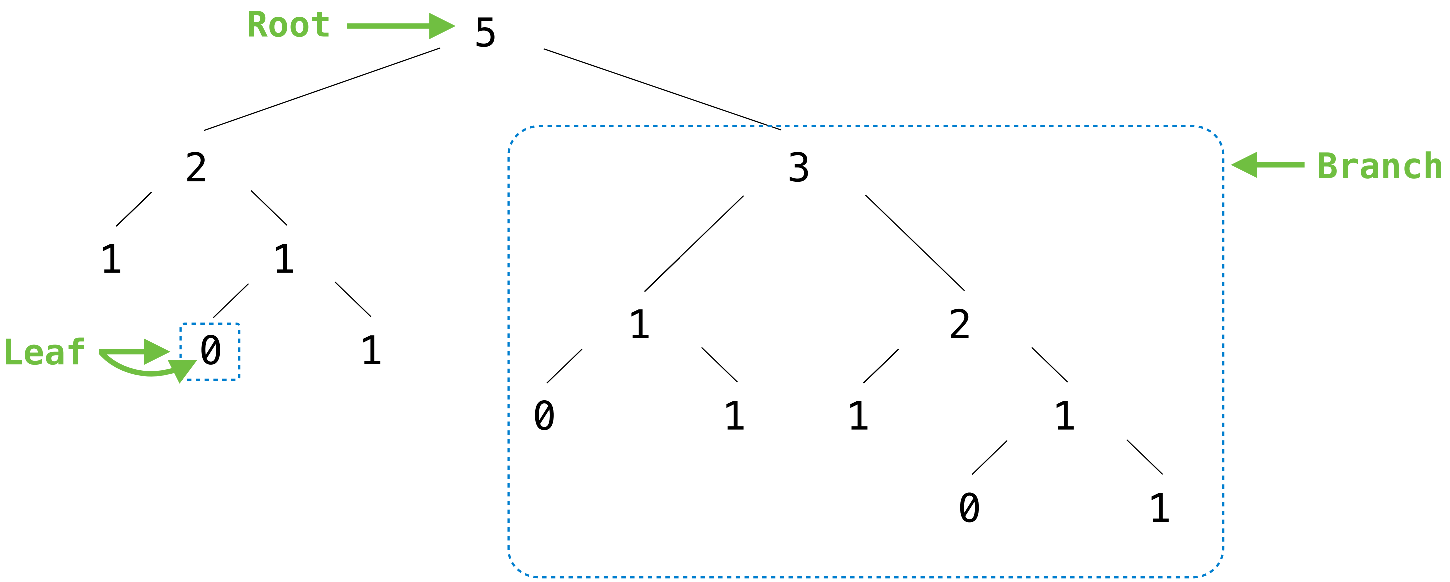

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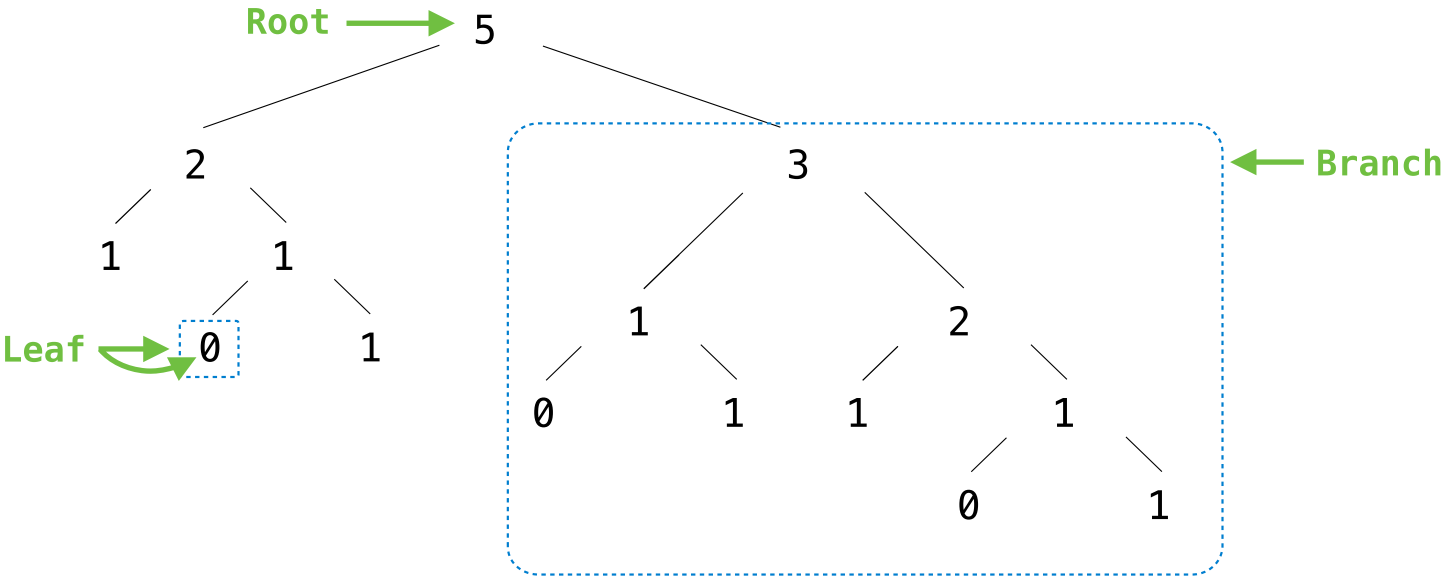

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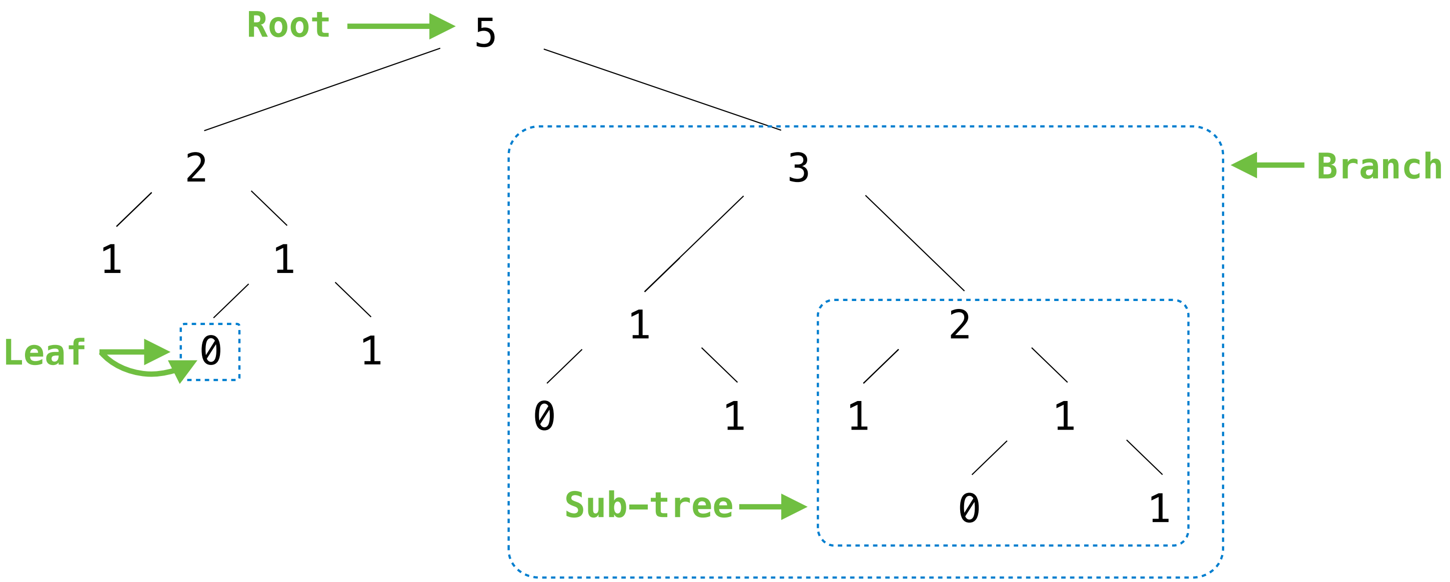

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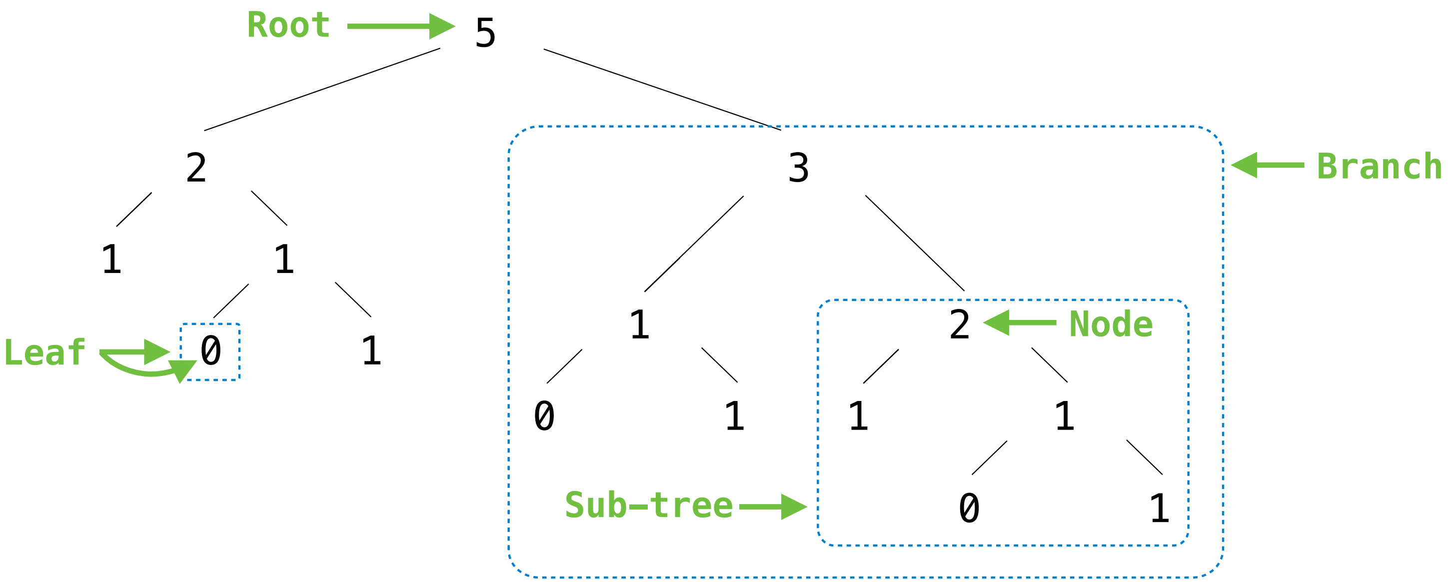

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Implementing the Tree Abstraction

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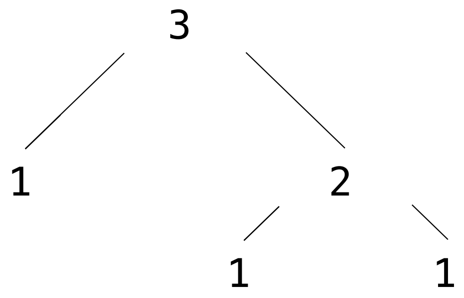

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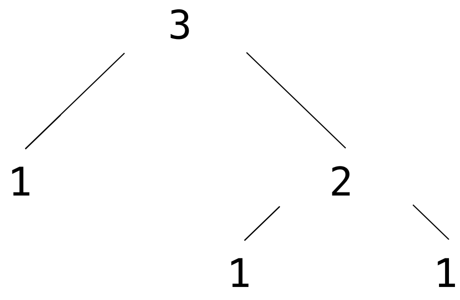

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>>> tree(3, [tree(1),
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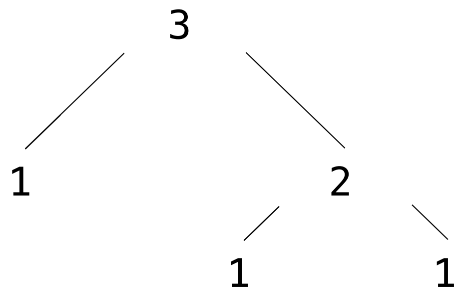

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Implementing the Tree Abstraction

```
def tree(root, branches=[]):
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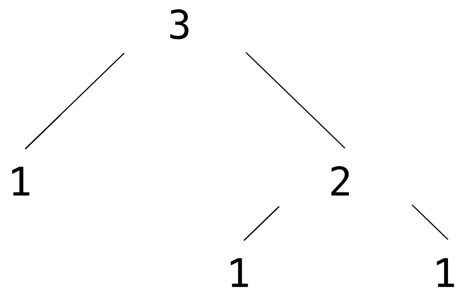

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def tree(root, branches=[]):
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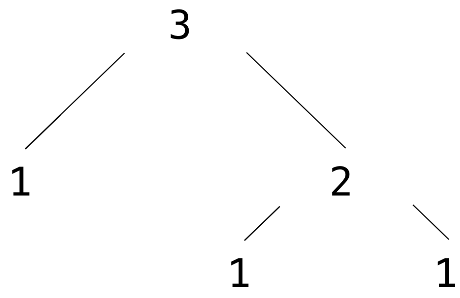

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Implementing the Tree Abstraction

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def tree(root, branches=[]):
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def root(tree):
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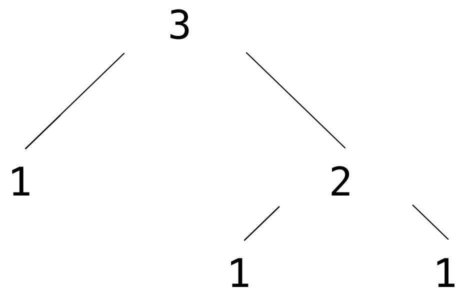

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Implementing the Tree Abstraction

```
def tree(root, branches=[]):
    return [root] + branches

def root(tree):
    return tree[0]
```

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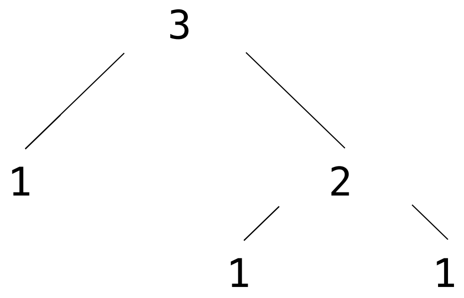

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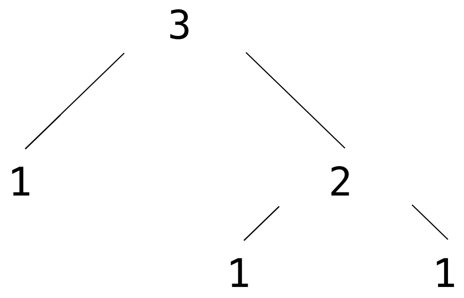

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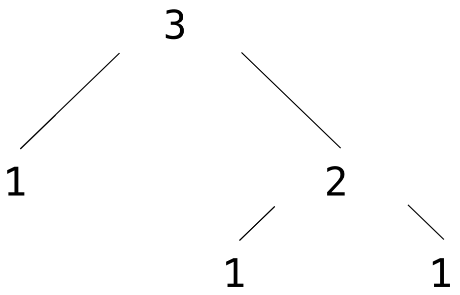

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Implementing the Tree Abstraction

```
def tree(root, branches=[]):
    for branch in branches:
        assert is_tree(branch)
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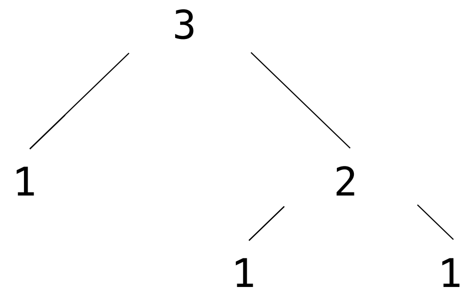

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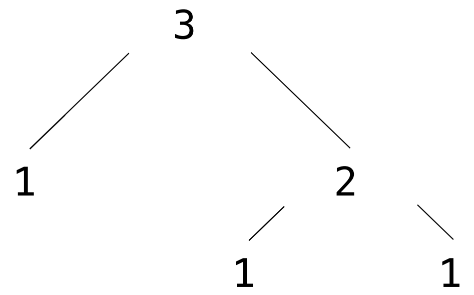
Creates a list from a sequence of branches

A tree has a root value and a sequence of branches; each branch is a tree



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Verifies the tree definition

Creates a list from a sequence of branches

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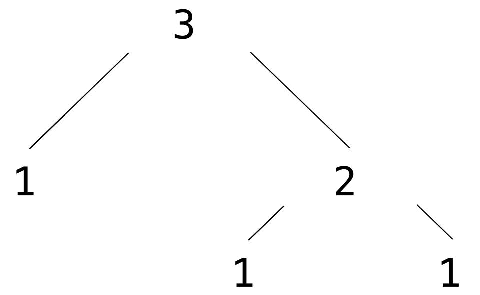

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def root(tree):
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def branches(tree):
    return tree[1:]

def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
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Verifies the tree definition

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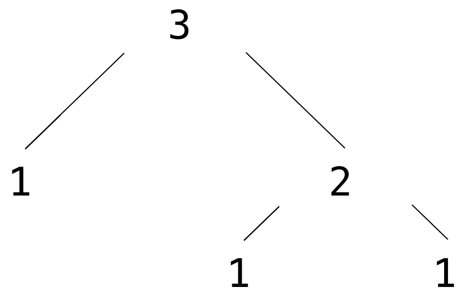

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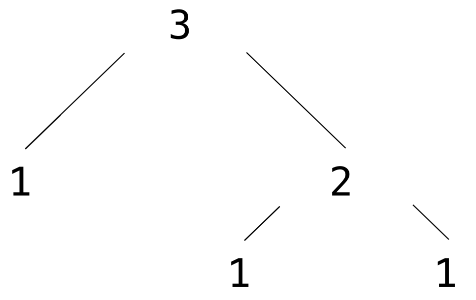
Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list

A tree has a root value and a sequence of branches; each branch is a tree



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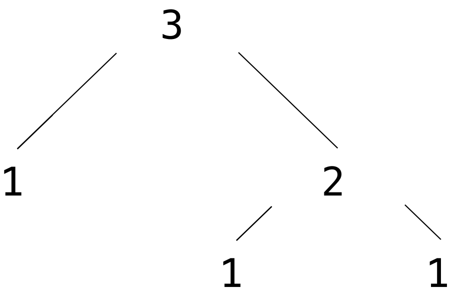

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[3, [1], [2, [1], [1]]]
```

```
def is_leaf(tree):
    return not branches(tree)
```

Implementing the Tree Abstraction

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def tree(root, branches=[]):
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def root(tree):
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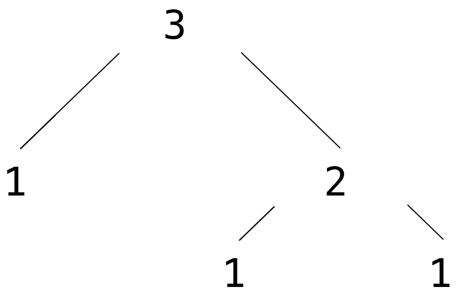

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(Demo)

Tree Processing Uses Recursion

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    """Count the leaves of a tree."""
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Processing a leaf is often the base case of a tree processing function

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The recursive case typically makes a recursive call on each branch, then aggregates

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    else:
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Tree Processing Uses Recursion

Processing a leaf is often the base case of a tree processing function

The recursive case typically makes a recursive call on each branch, then aggregates

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def count_leaves(tree):
    """Count the leaves of a tree."""
    if is_leaf(tree):
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(Demo)

Discussion Question

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Implement `leaves`, which returns a list of the leaf values of a tree

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```
def leaves(tree):
    """Return a list containing the leaves of tree.

    >>> leaves(fib_tree(5))
    [1, 0, 1, 0, 1, 1, 0, 1]
    """
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`Implement leaves, which returns a list of the leaf values of a tree`

`*Hint*: If you sum a list of lists, you get a list containing the elements of those lists`

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    if is_leaf(tree):
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    else:
        return sum([leaves(b) for b in branches(tree)], [])
```

Example: Partition Trees

(Demo)

Interactive Diagram