61A Lecture 21

Friday, March 13

Announcements

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- No discussion sections next Thursday 3/19 or Friday 3/20
- Lecture next Friday 3/20 is a video (but a great one)

Sets

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One more built-in Python container type

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>>> s
{1, 2, 3, 4}
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>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
>>> 3 in s
True
```

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>>> len(s)
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>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
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4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
```

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>>> s
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>>> 3 in s
True
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>>> s.union({1, 5})
{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```

Implementing Sets

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What we should be able to do with a set:

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- **Membership testing:** Is a value an element of a set?

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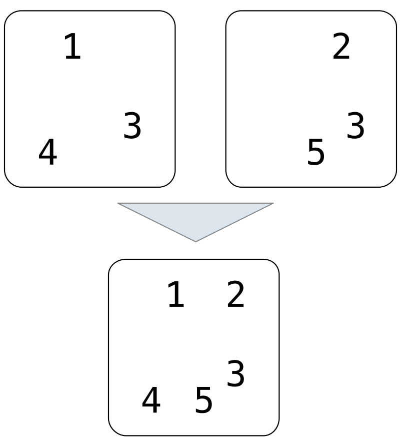

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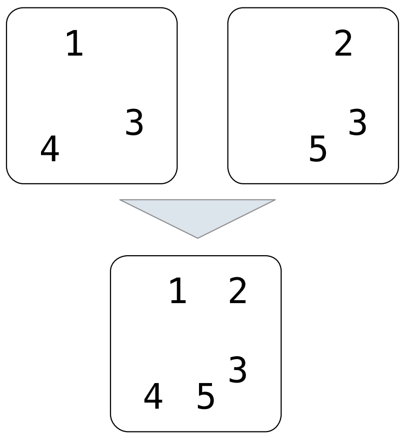

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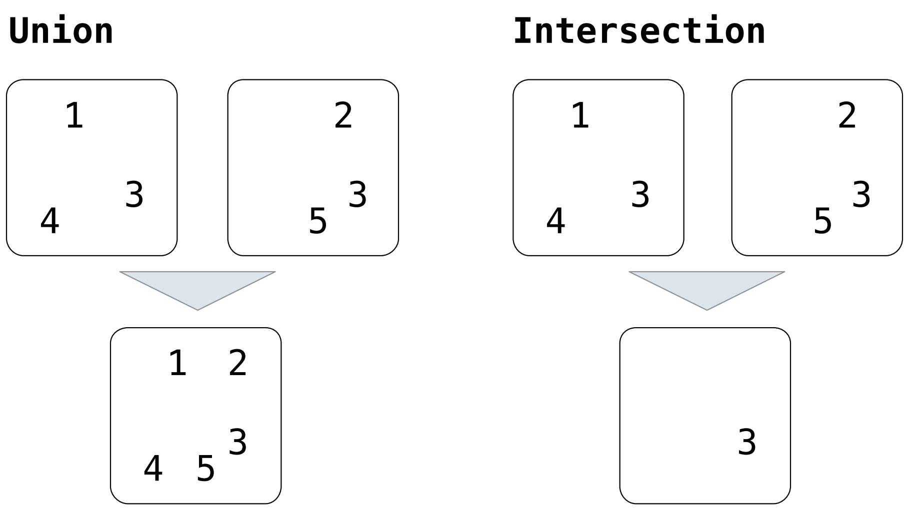

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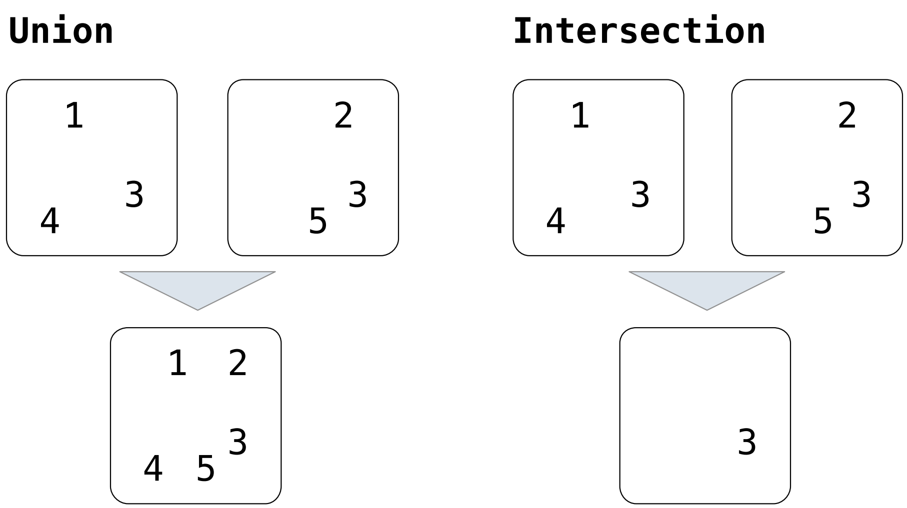

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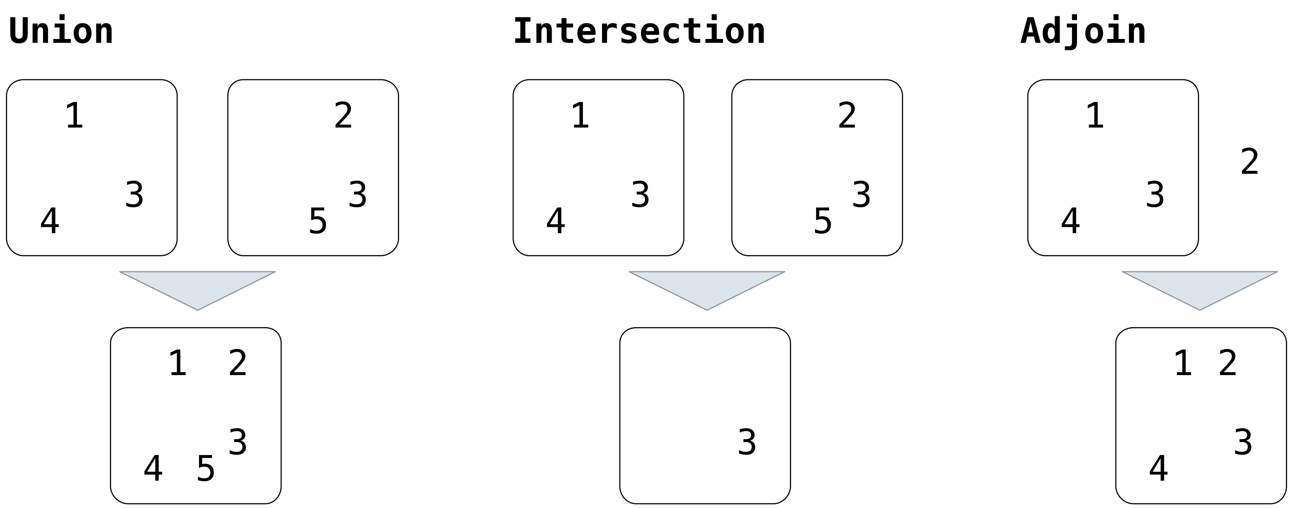

Sets as Unordered Sequences

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Proposal 1: A set is represented by a linked list that contains no duplicate items.

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    >>> s = Link(1, Link(2, Link(3)))
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(Demo)

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def adjoin_set(s, v):
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The size of the set

Sets as Unordered Sequences

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def adjoin_set(s, v):
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def intersect_set(set1, set2):
    in_set2 = lambda v: set_contains(set2, v)
    return keep_if(set1, in_set2)
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def union_set(set1, set2):
    not_in_set2 = lambda v: not set_contains(set2, v)
    set1_not_set2 = keep_if(set1, not_in_set2)
    return extend(set1_not_set2, set2)
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(Demo)

Sets as Ordered Sequences

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Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

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Parts of the program that...	Assume that sets are...	Using...

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Use sets to contain values		

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Parts of the program that...	Assume that sets are...	Using...
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Sets as Ordered Sequences

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Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	empty, set_contains, adjoin_set, intersect_set, union_set

Sets as Ordered Sequences

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Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	empty, set_contains, adjoin_set, intersect_set, union_set
Implement set operations		

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Implement set operations	Ordered linked lists	

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Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	`empty`, `set_contains`, `adjoin_set`, `intersect_set`, `union_set`
Implement set operations	Ordered linked lists	`first`, `rest`, `<`, `>`, `==`

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Different parts of a program may make different assumptions about data

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Order of growth?

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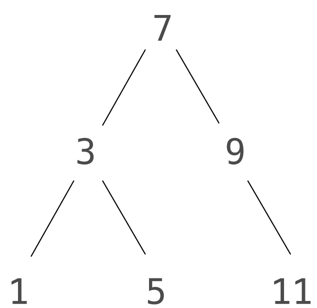

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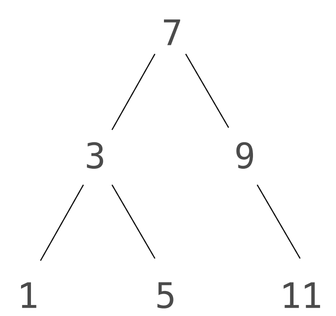

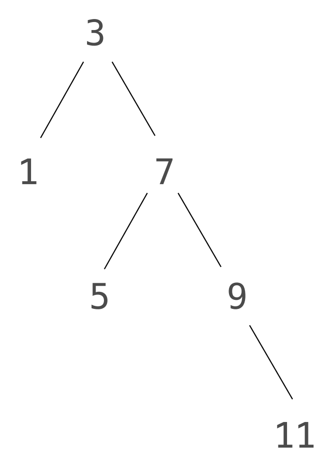

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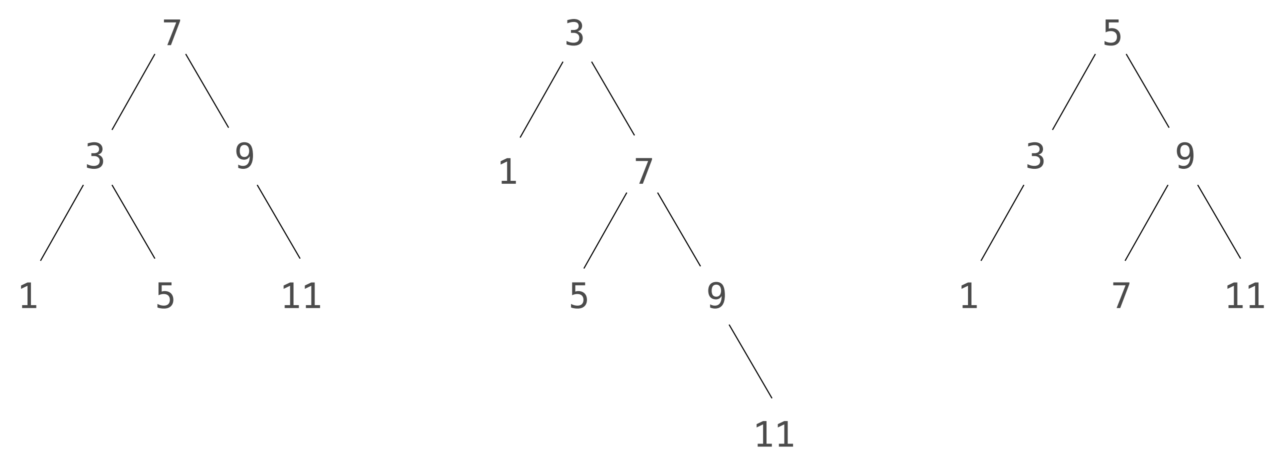

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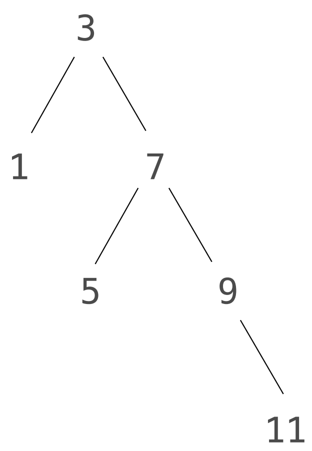

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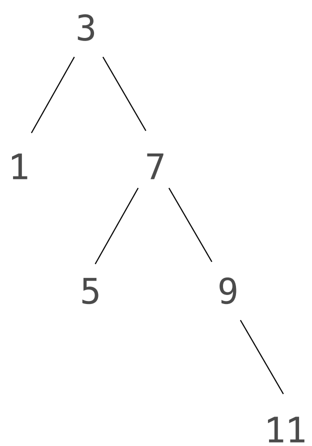

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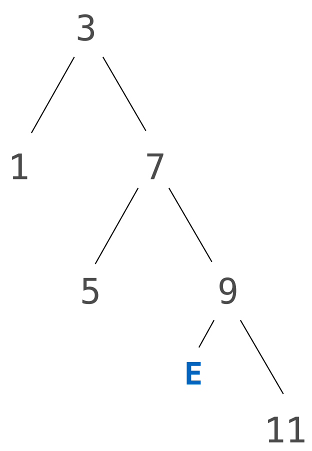

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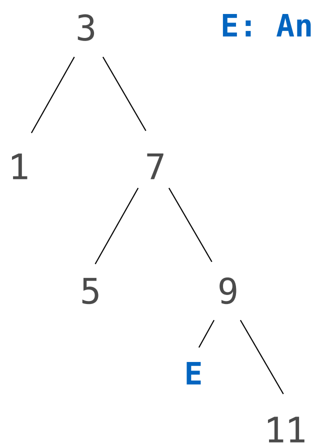

E: An empty tree

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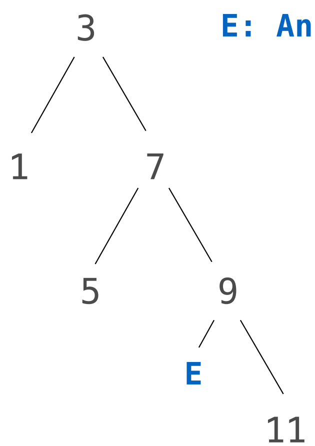

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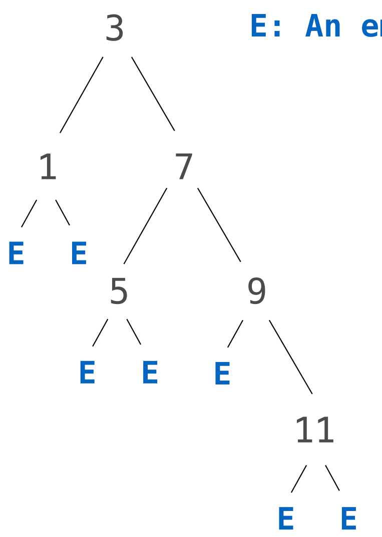

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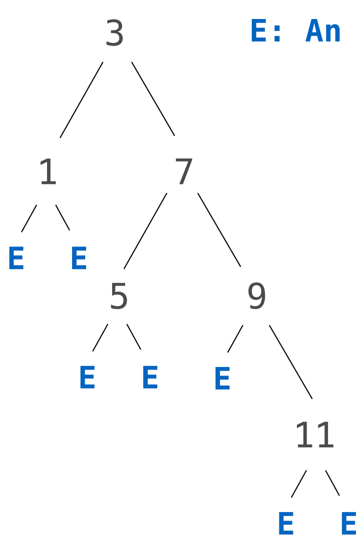

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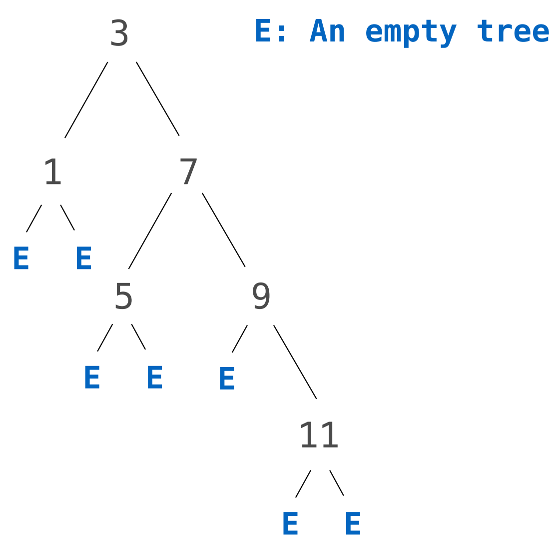


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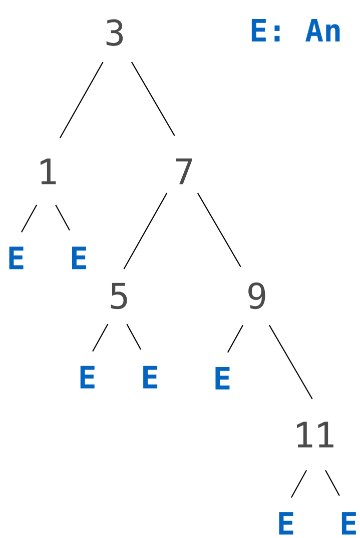


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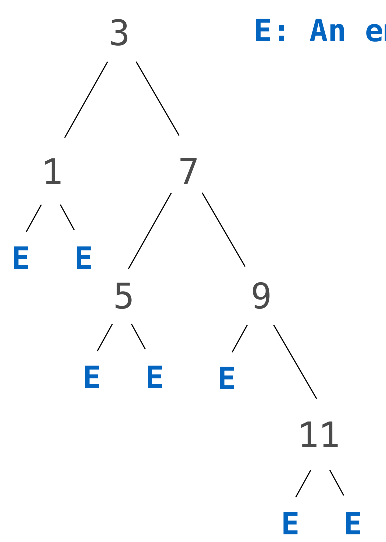

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    def left(self):
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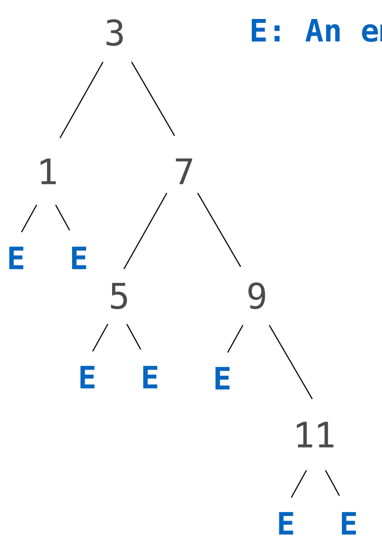

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    def left(self):
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    @property
    def right(self):
        return self.branches[1]
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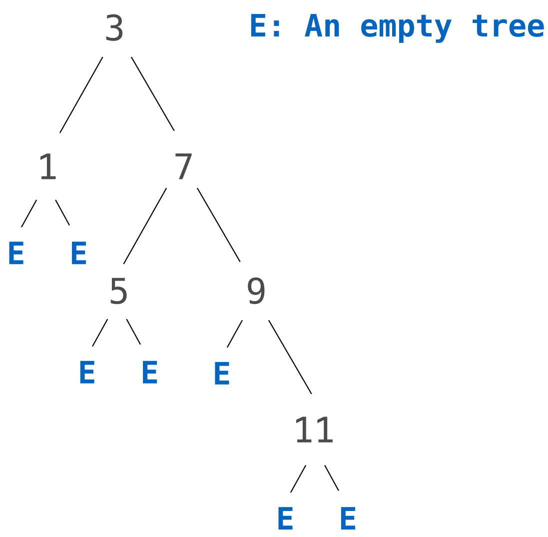


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Bin = BinaryTree
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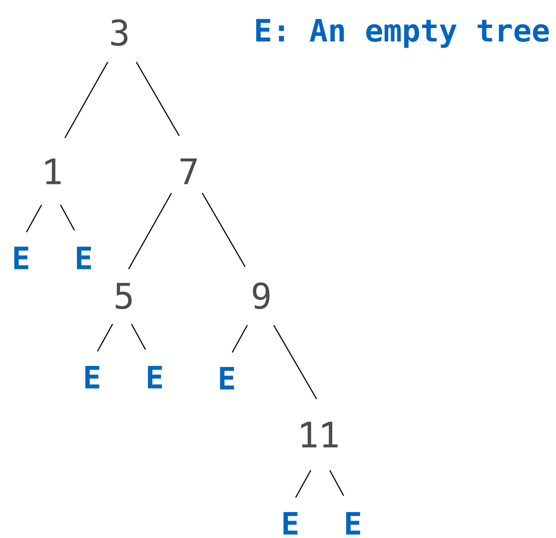


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Bin = BinaryTree
t = Bin(3, Bin(1),
           Bin(7, Bin(5),
                  Bin(9, Bin.empty,
                         Bin(11)))))
```

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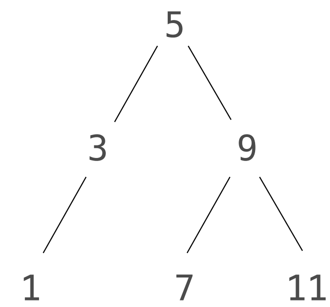

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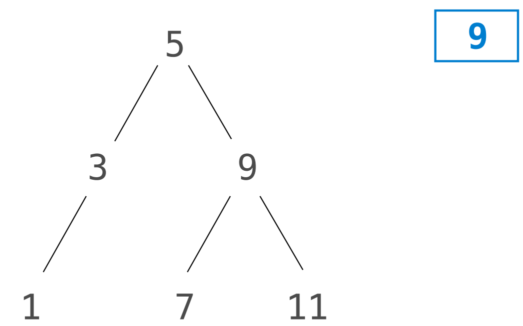

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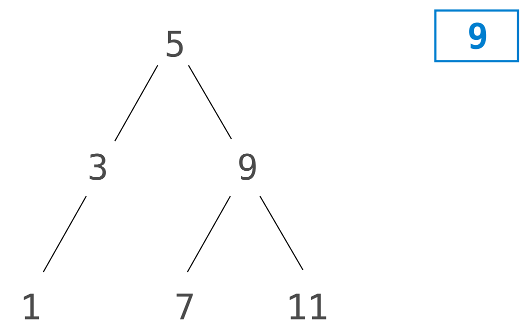

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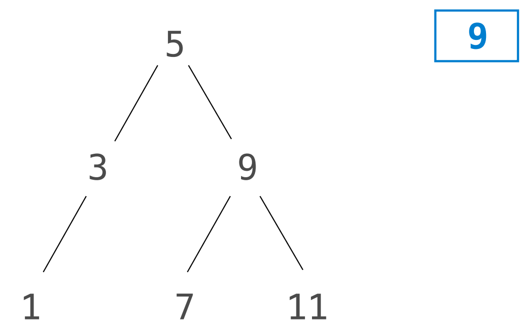

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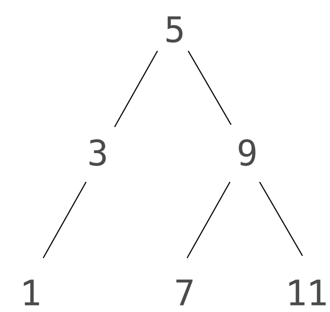

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    if s.is_empty:
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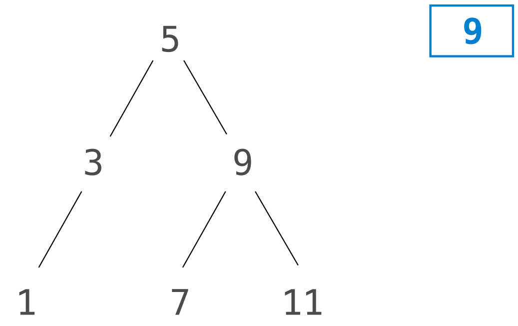
        return set_contains(s.right, v)
```



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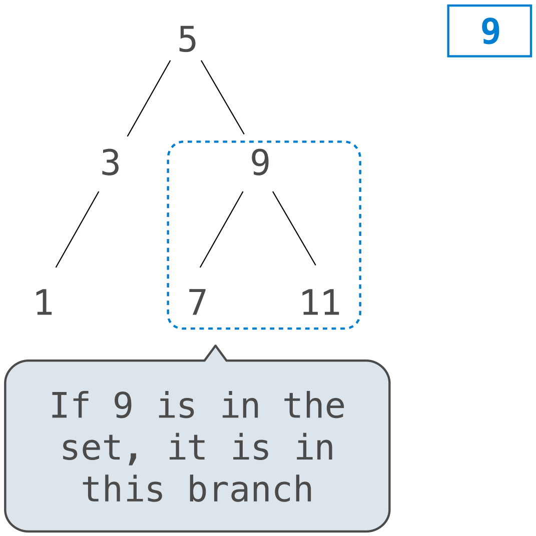

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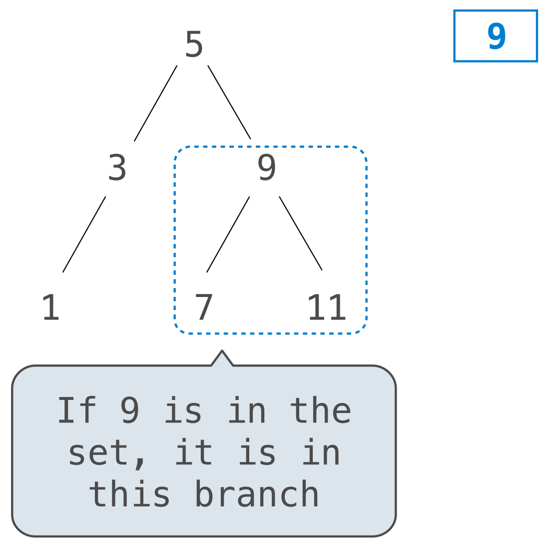

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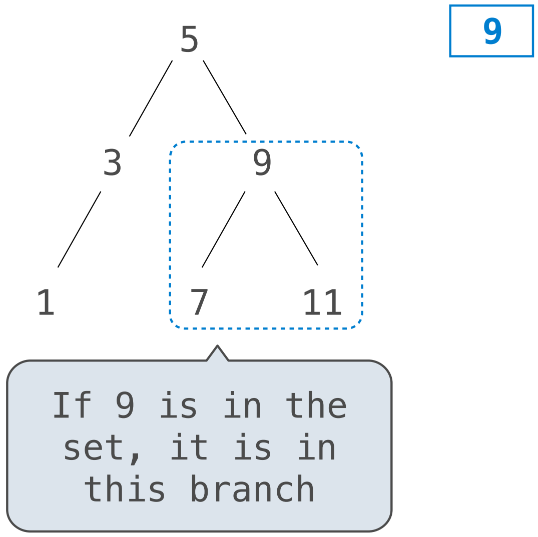

Order of growth?

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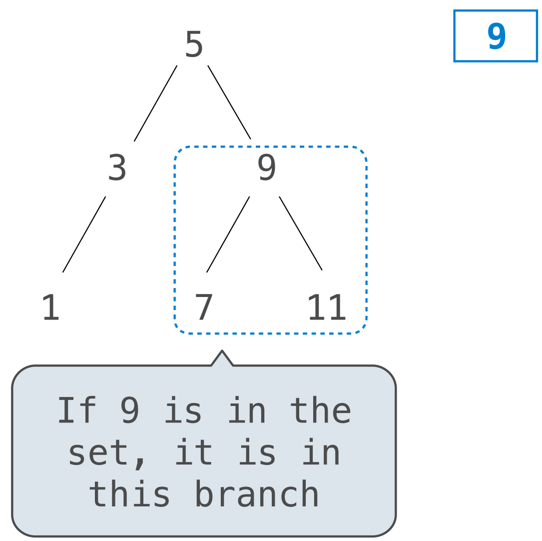

Order of growth? $\Theta(h)$ on average

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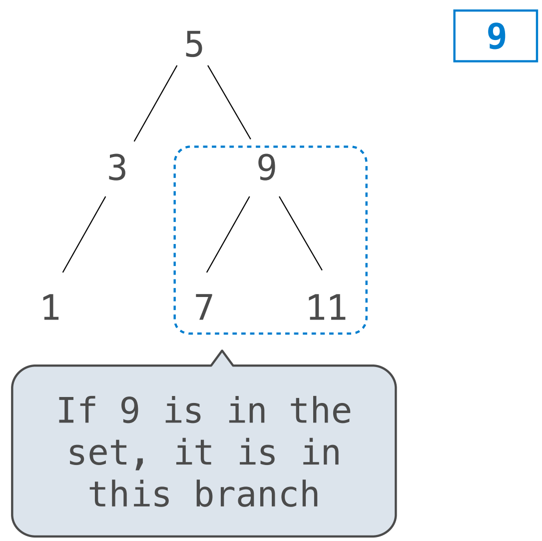

Order of growth? $\Theta(h)$ on average $\Theta(\log n)$ on average for a balanced tree

Adjoining to a Tree Set

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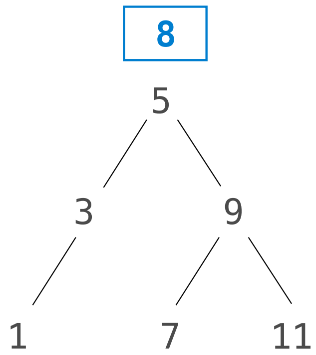

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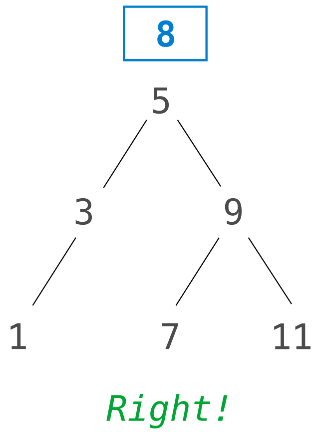

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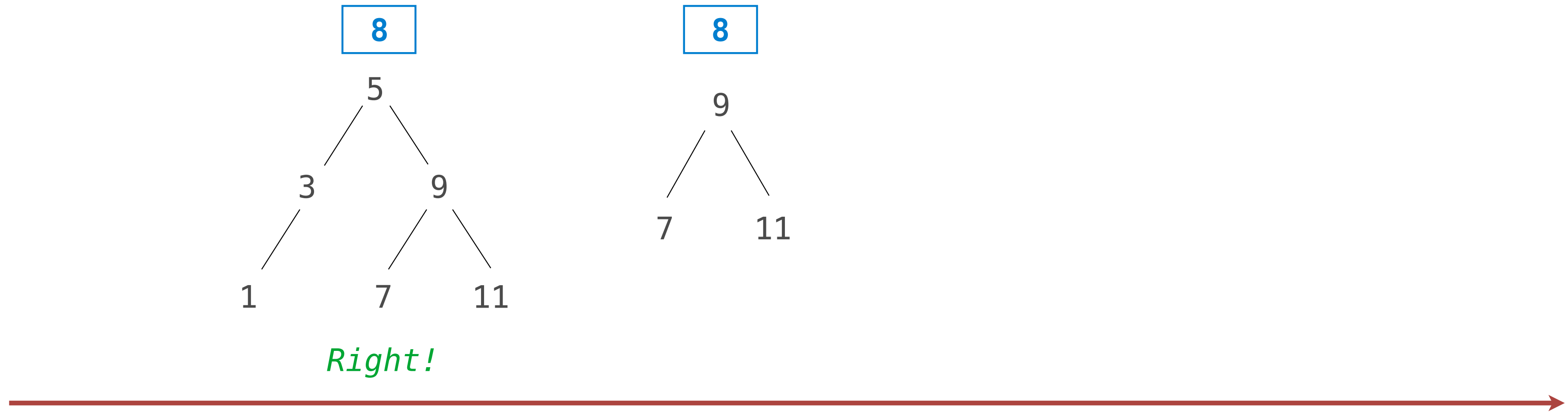

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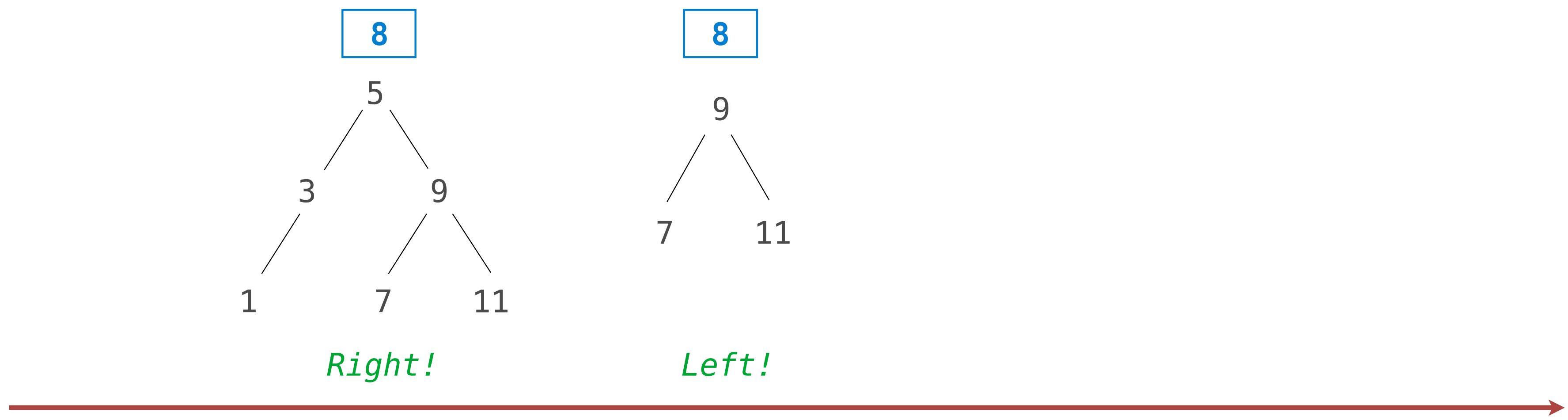

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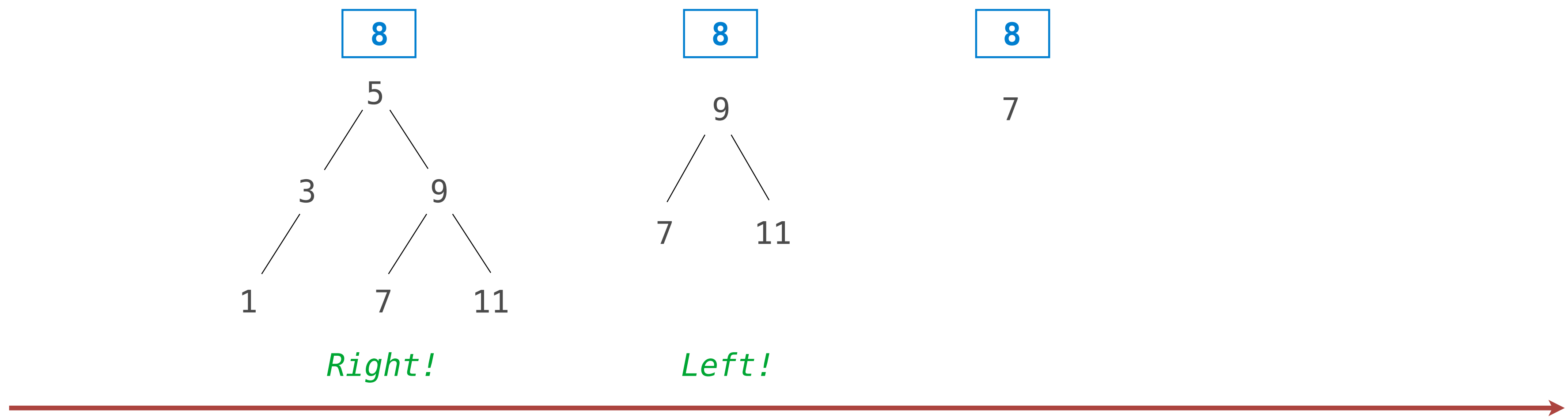

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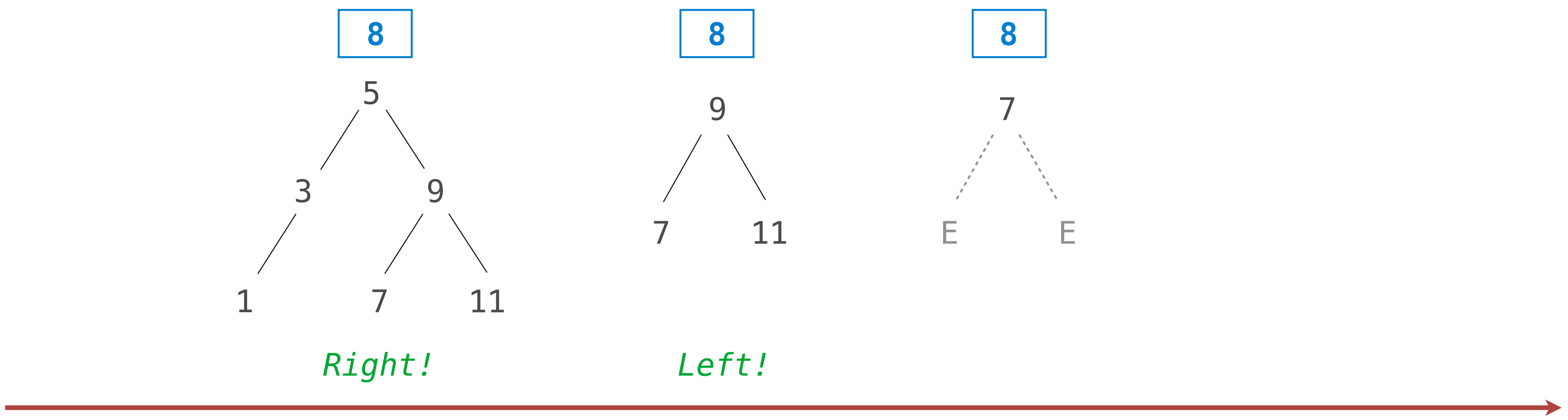

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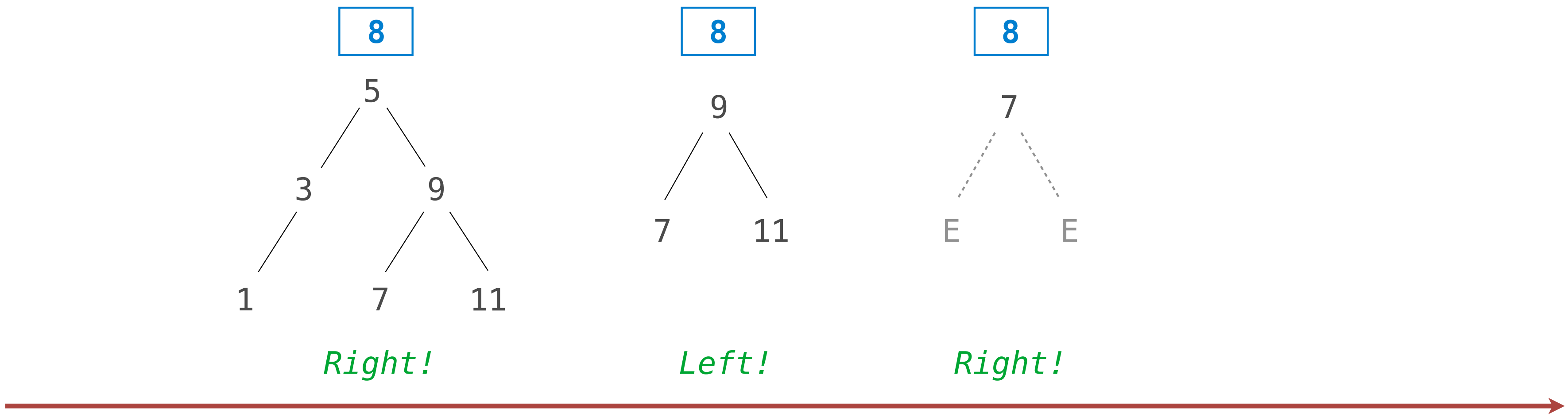

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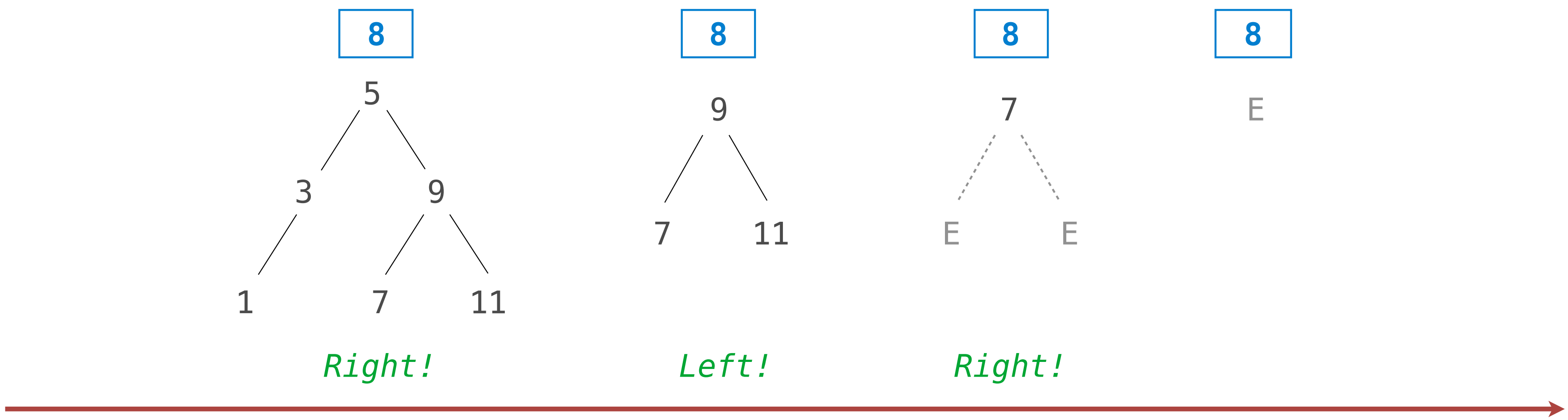
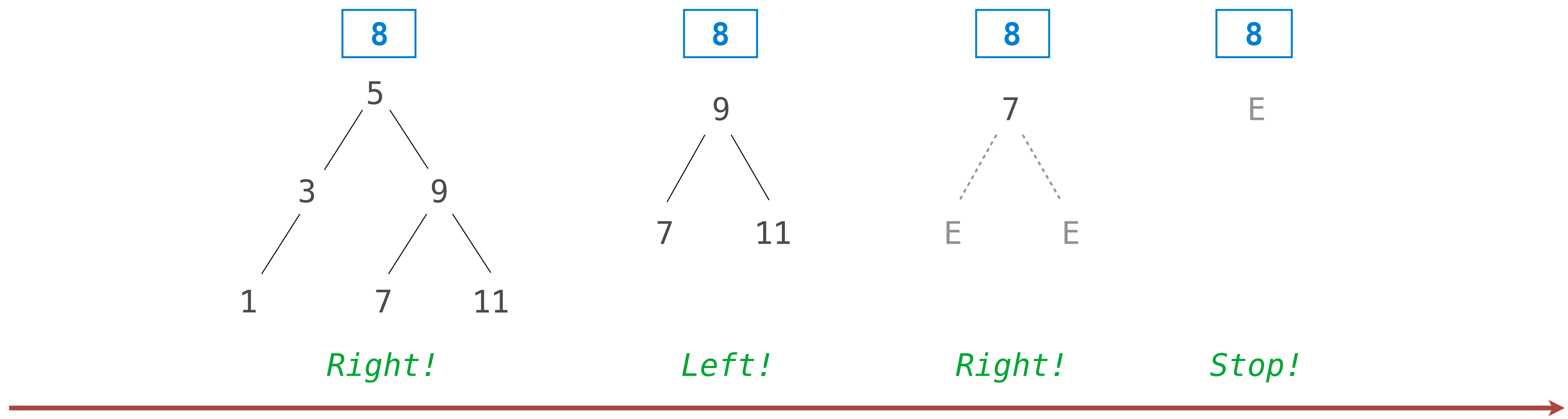


Adjoining to a Tree Set



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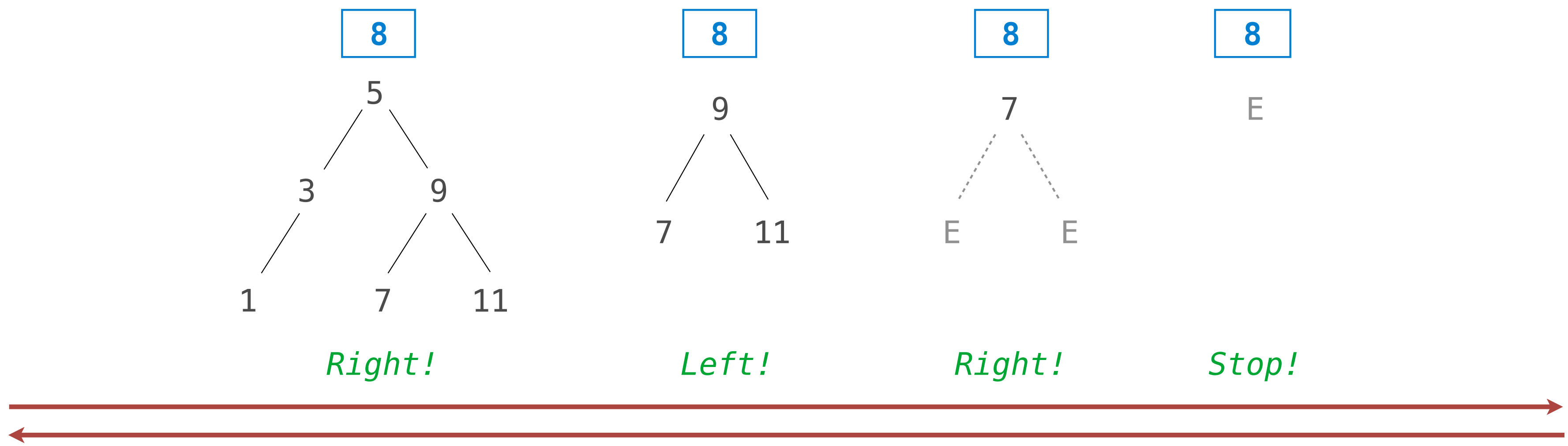
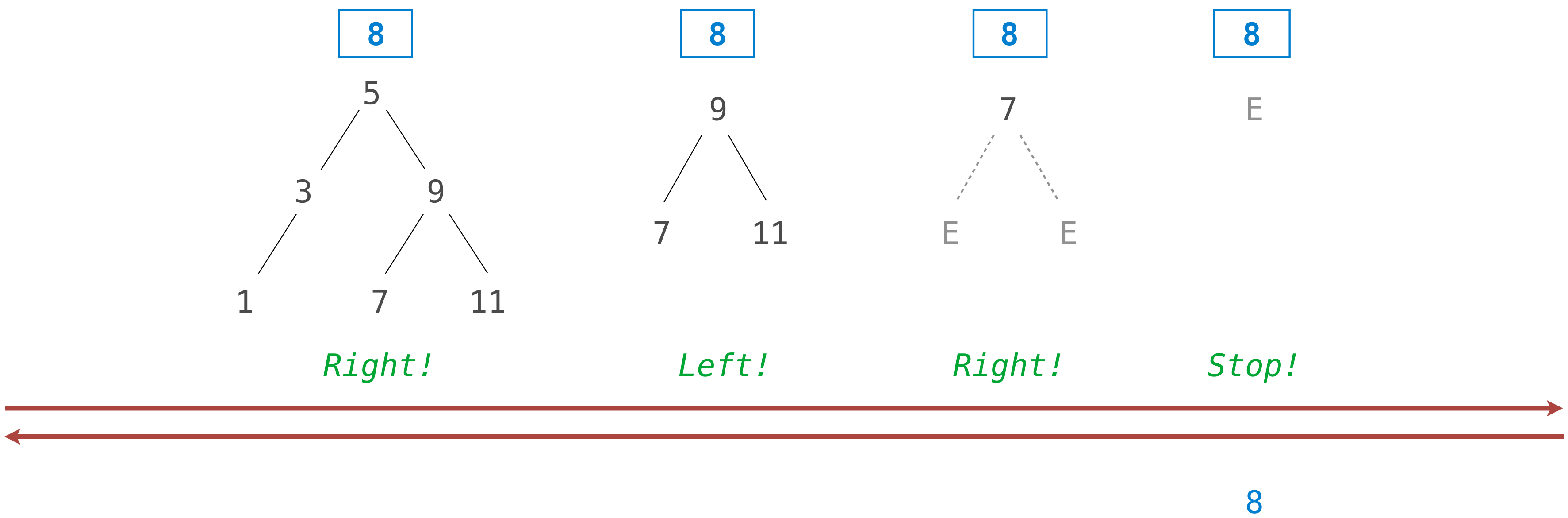


Adjoining to a Tree Set



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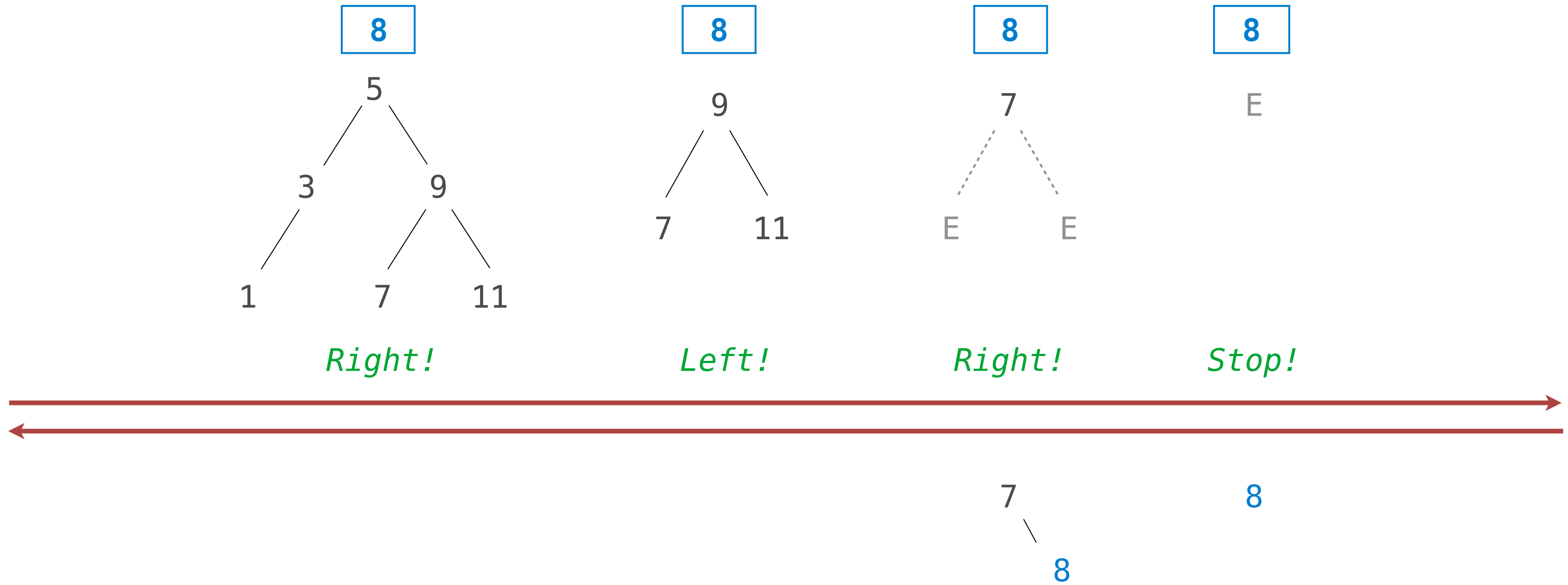

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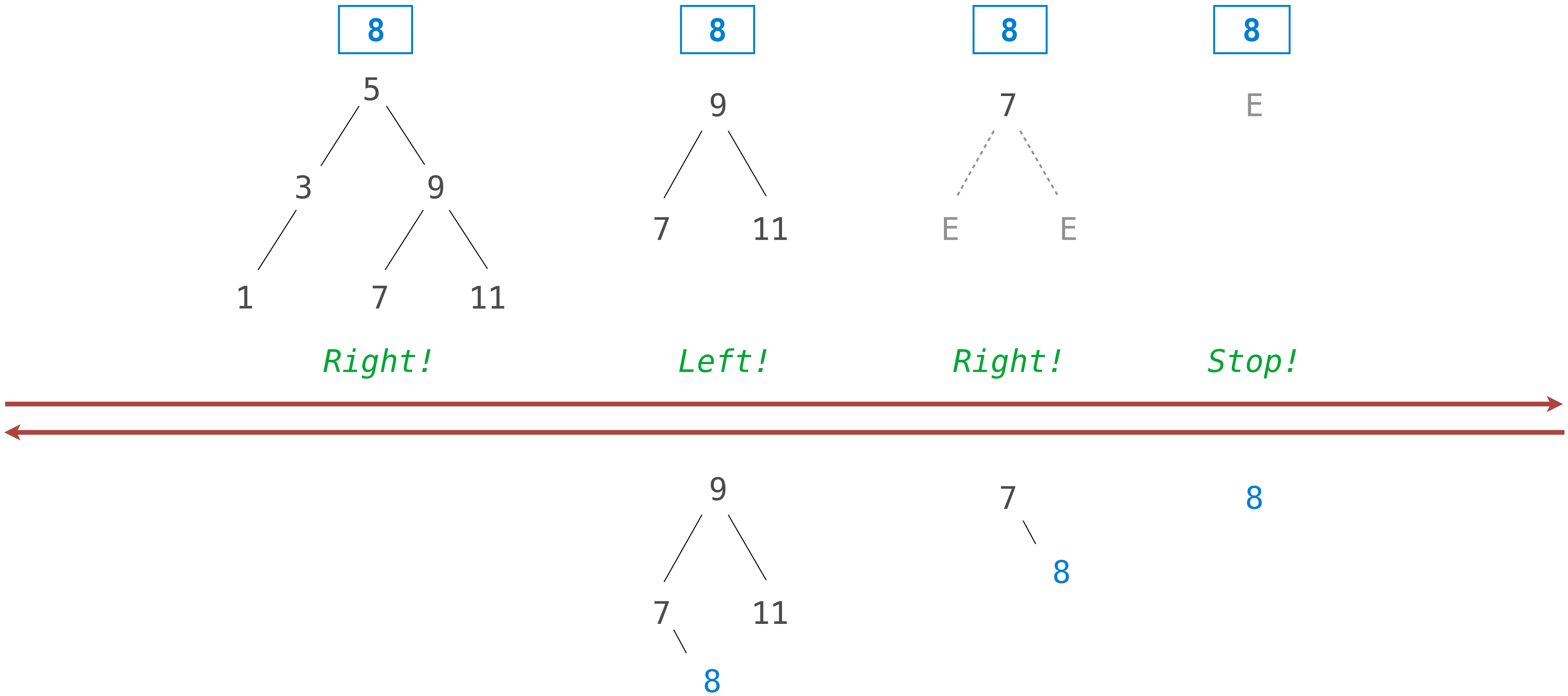

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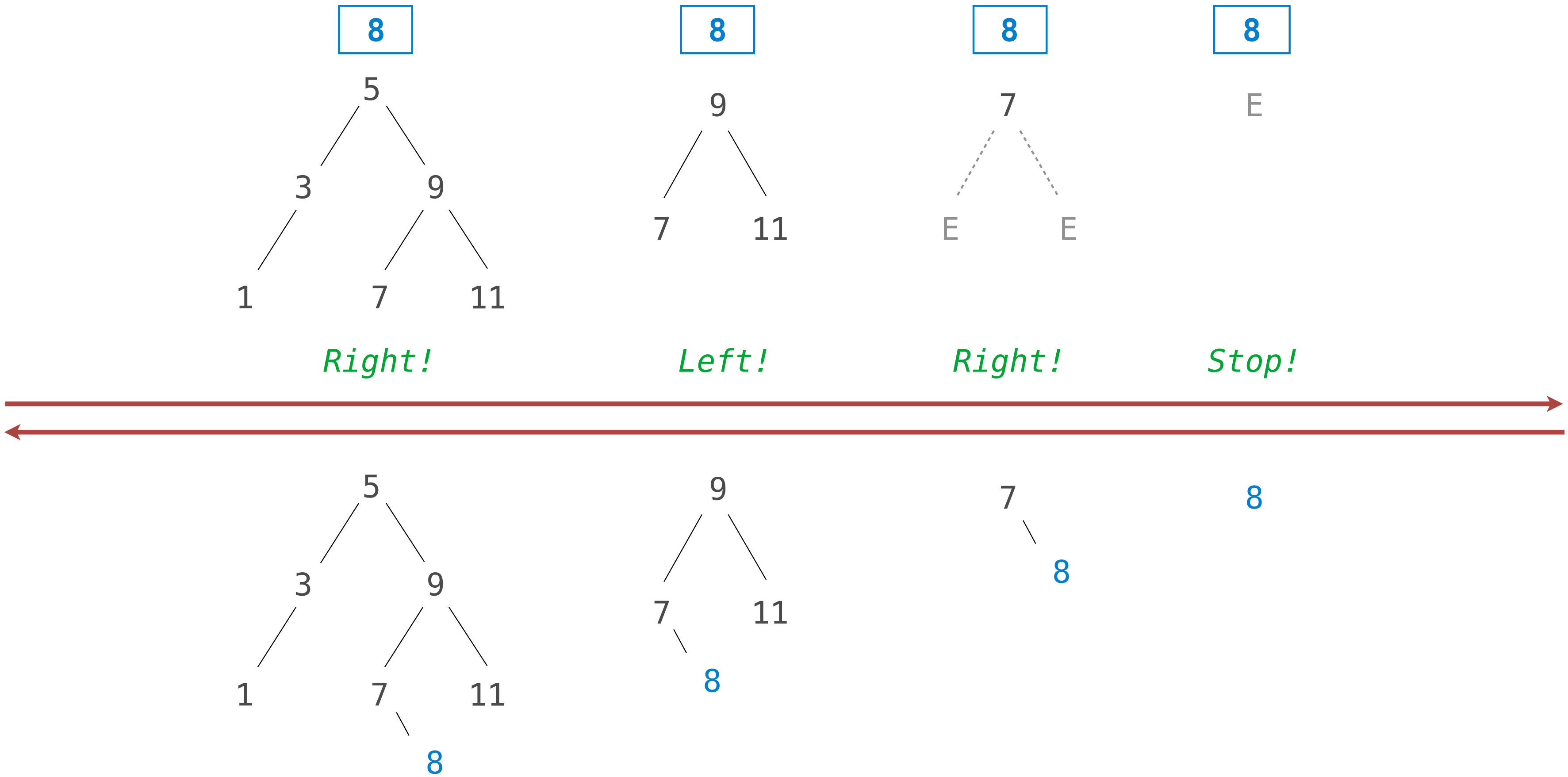
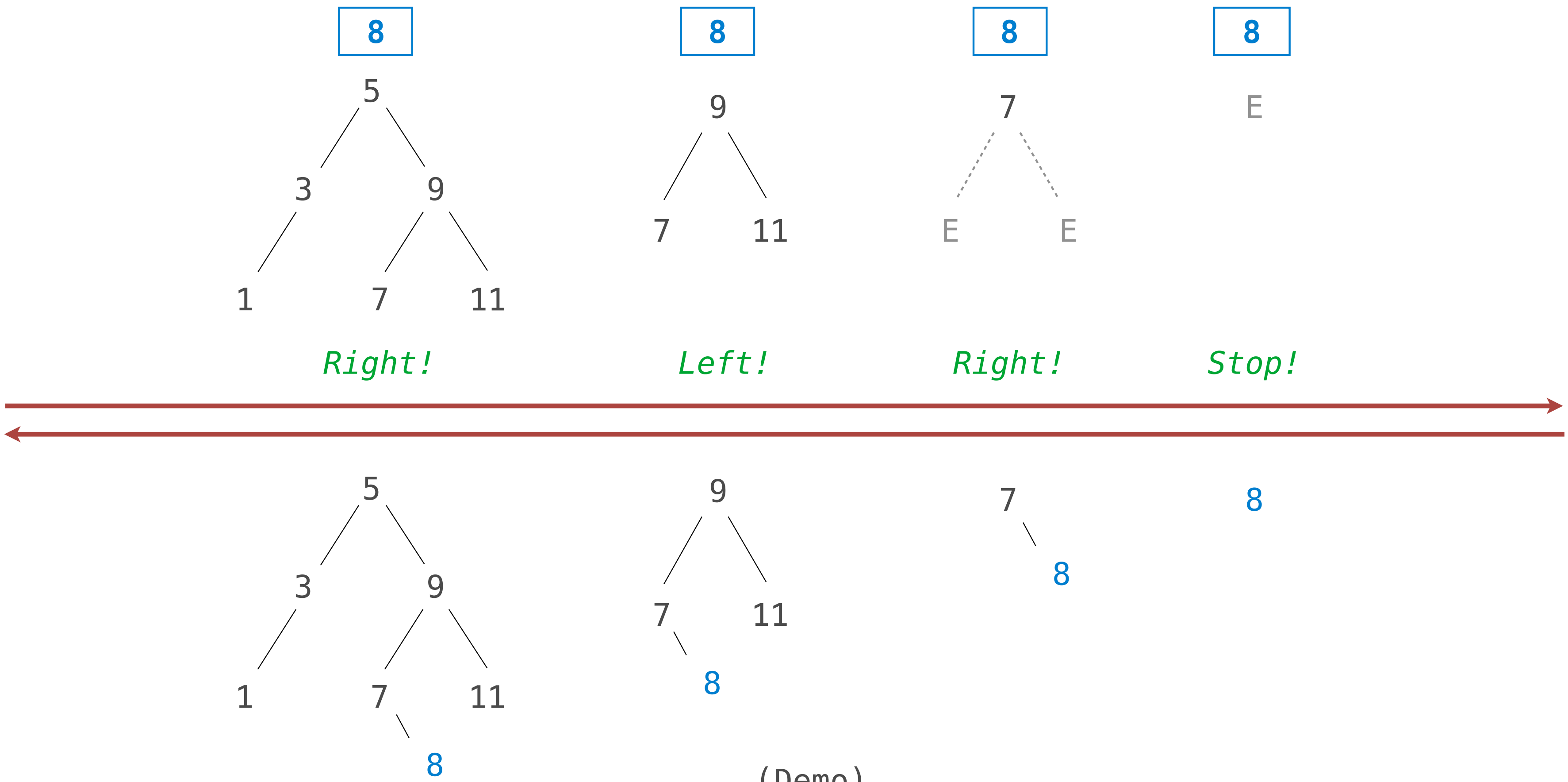


Adjoining to a Tree Set



(Demo)