

## 61A Lecture 30

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Monday, April 13

## Announcements

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- If you want the first early submission point, you need to:
  - Pass the tests for the designated questions
  - Run `python3 ok --submit`
  - Log on to <http://ok.cs61a.org> and create a group with your partner

## Ray Tracing

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A technique for displaying a 3D scene on a 2D screen by tracing a path through every pixel

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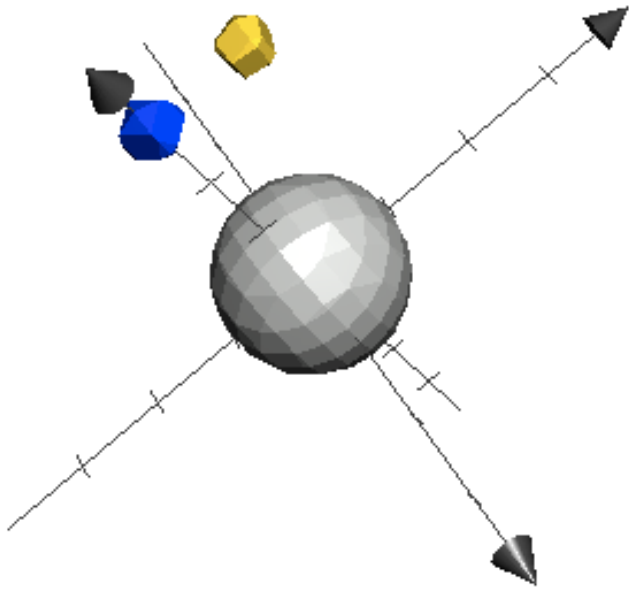
**The Scene:**

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### The Scene:

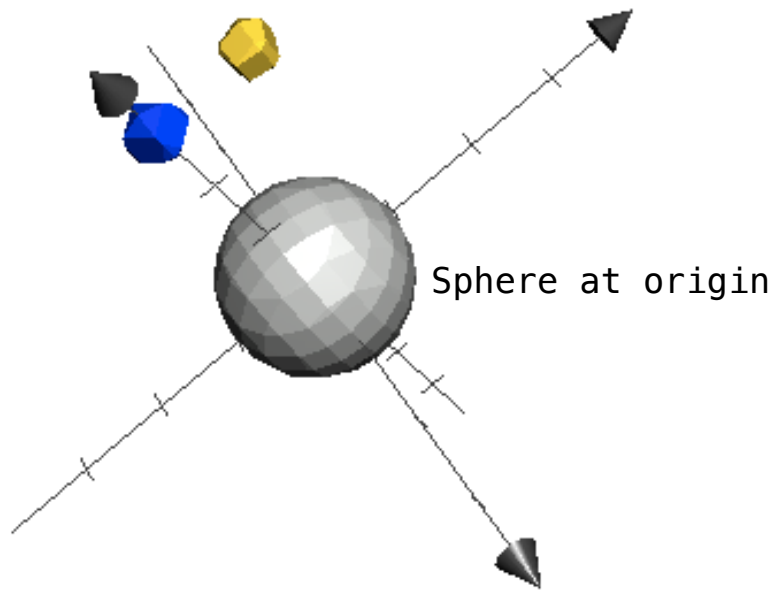


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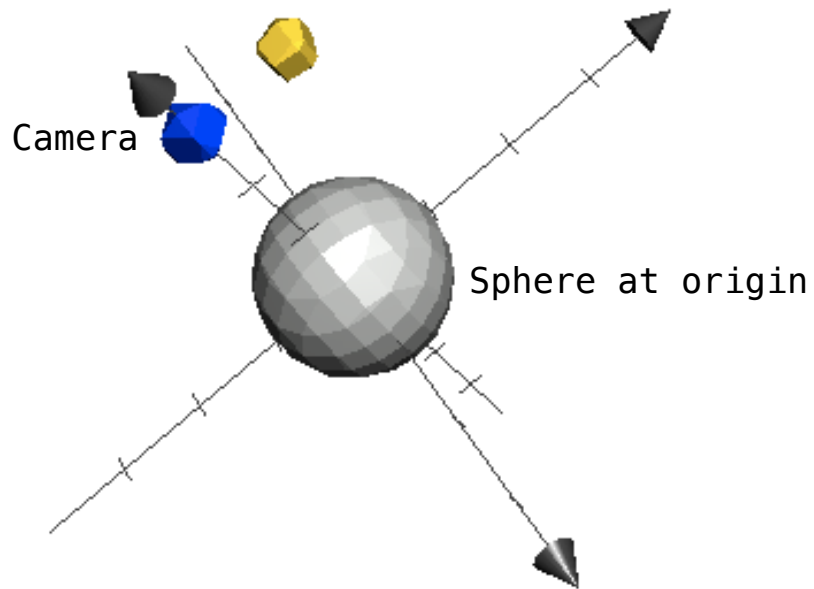


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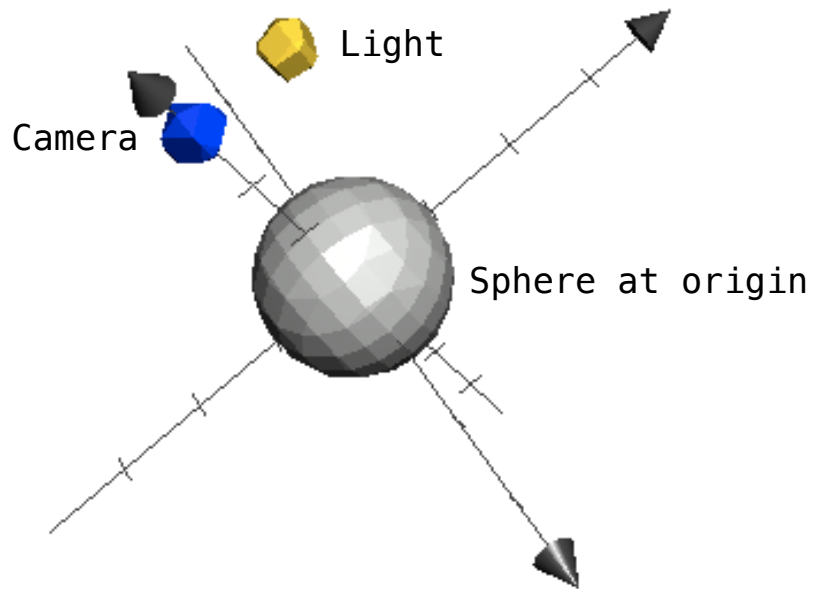


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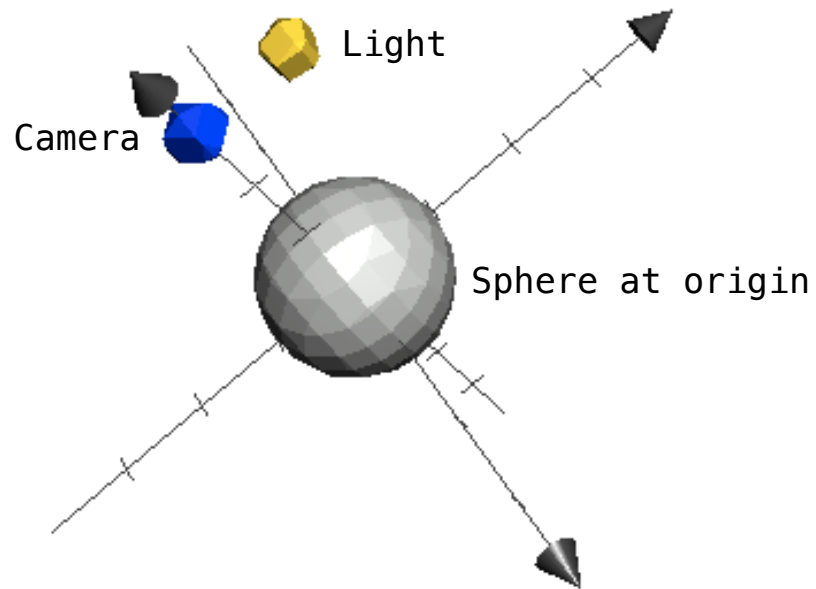
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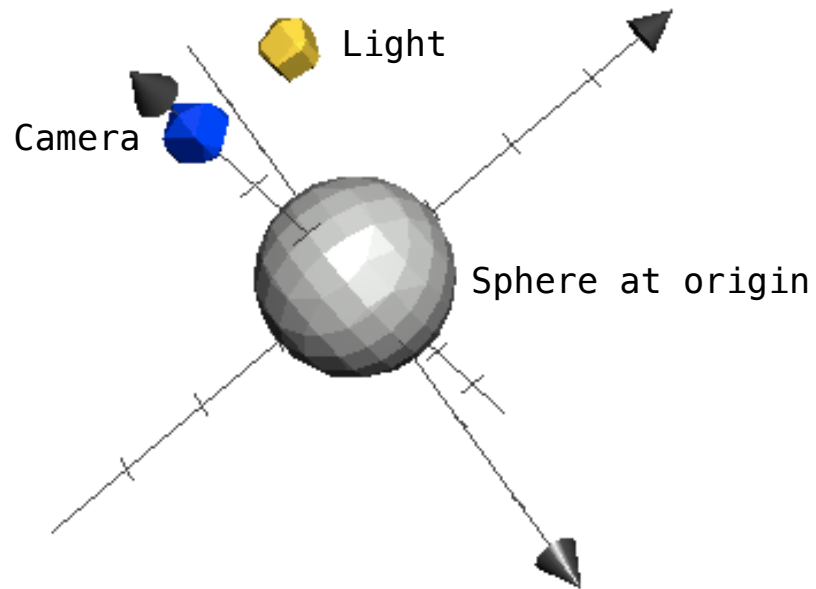
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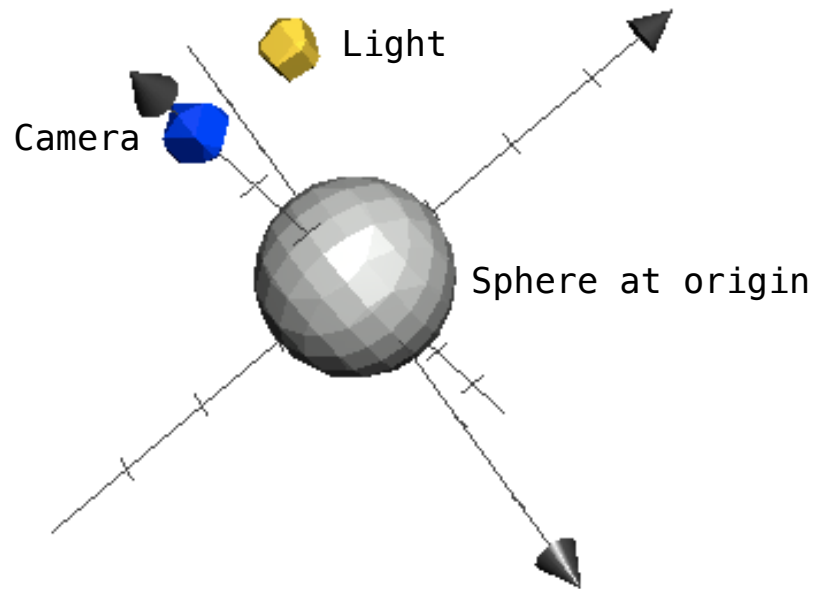
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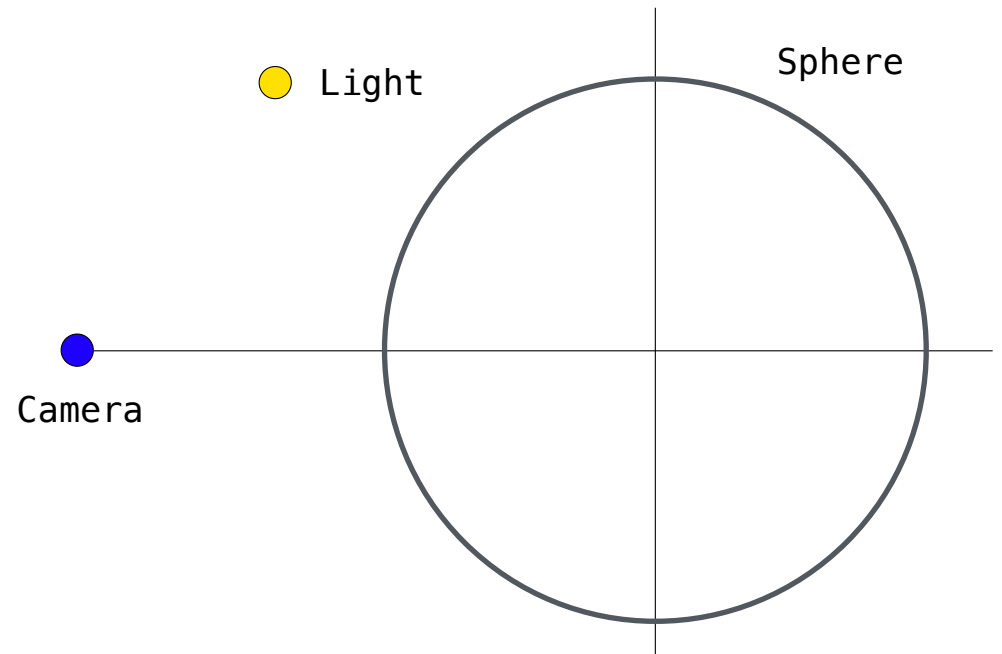
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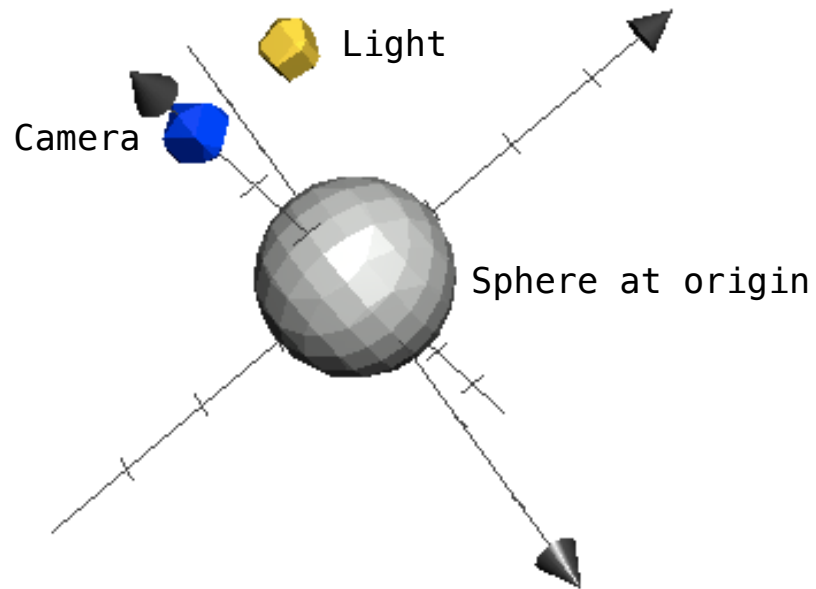


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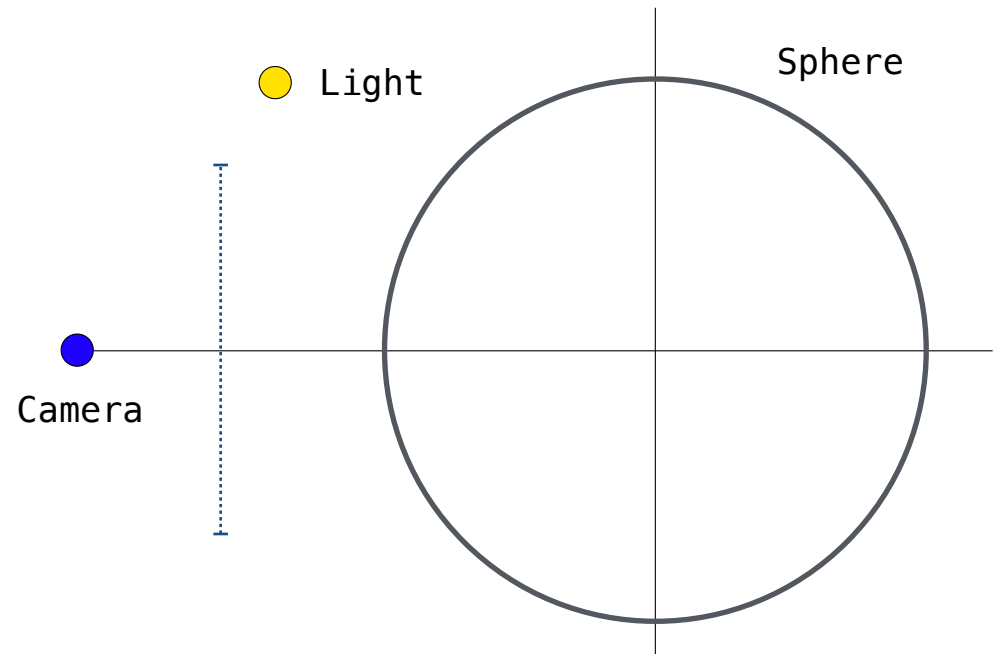
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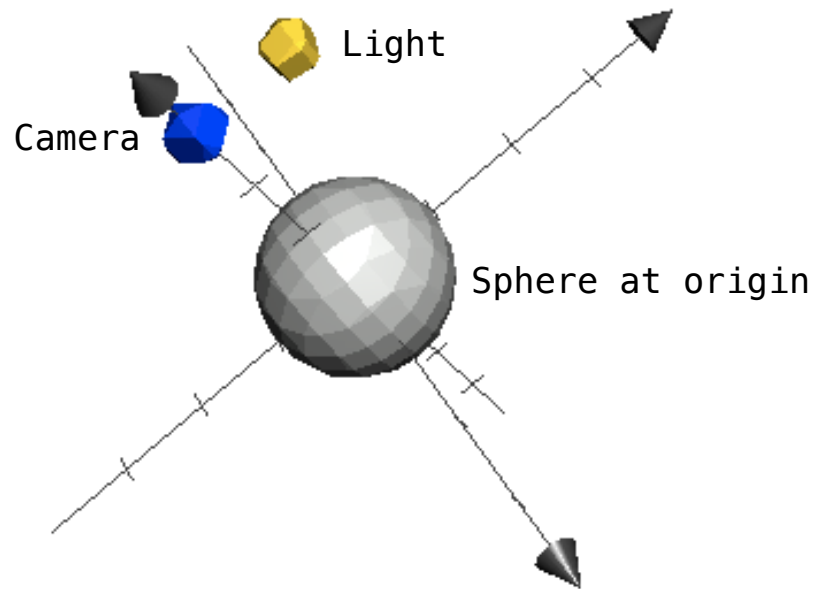


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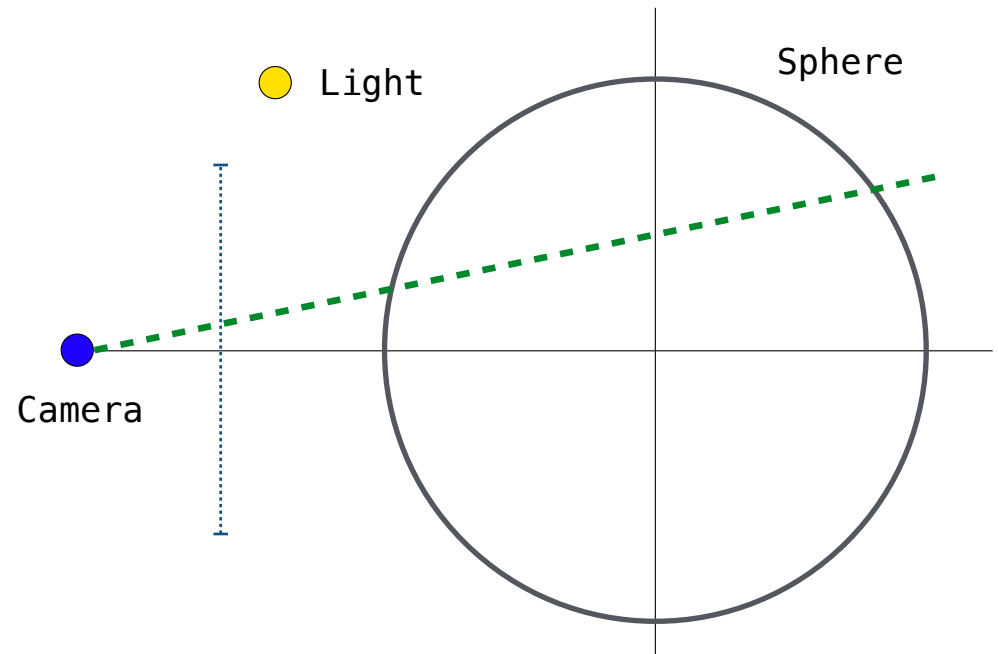
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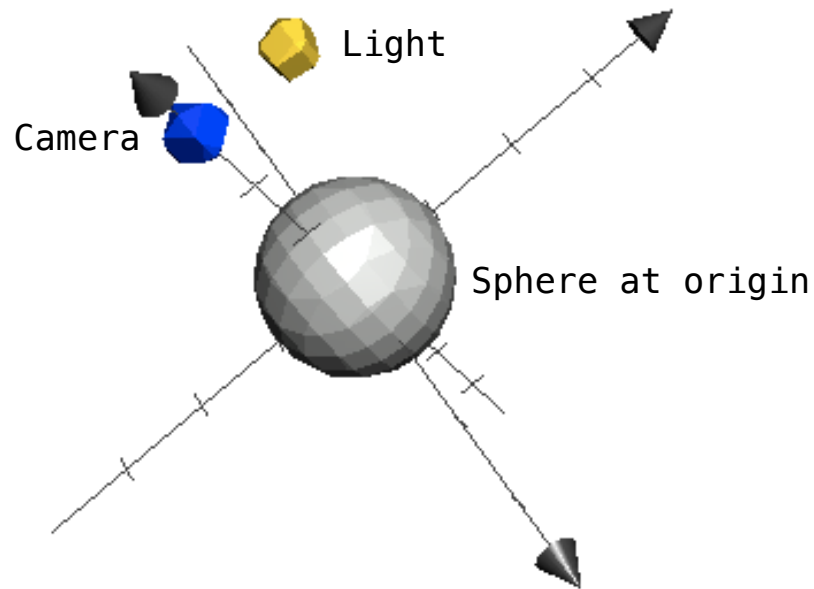


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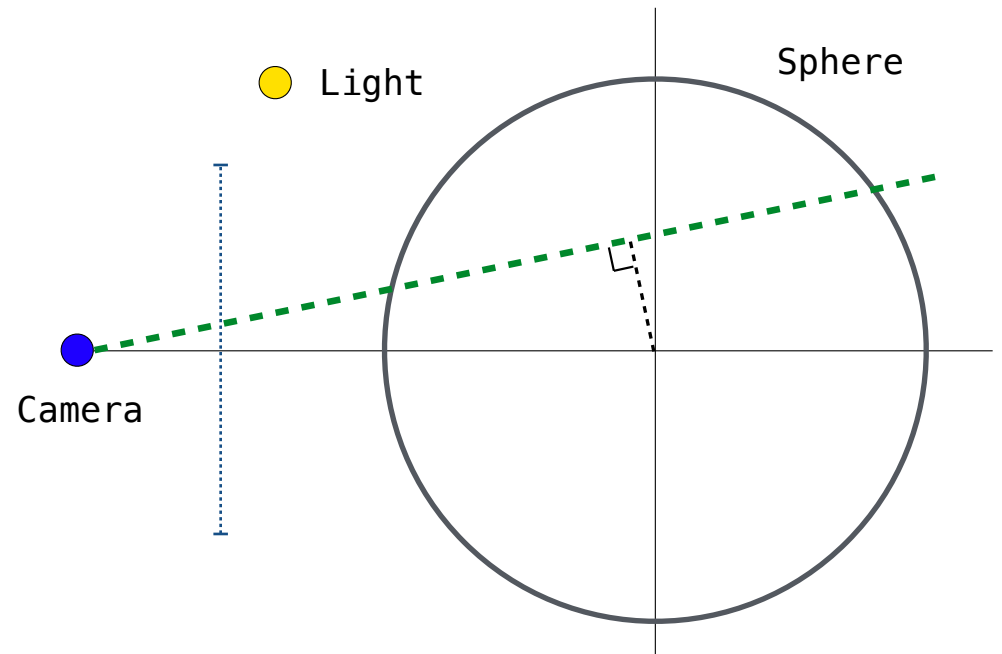
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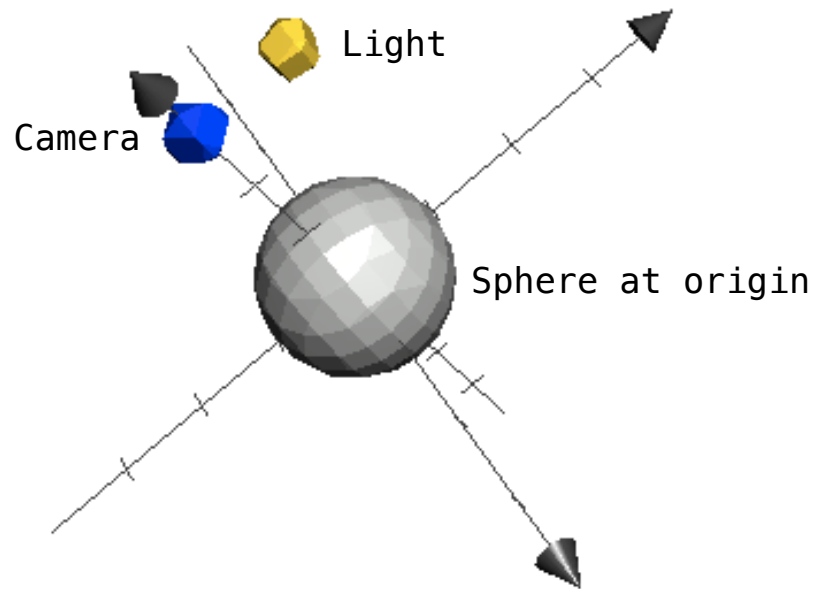


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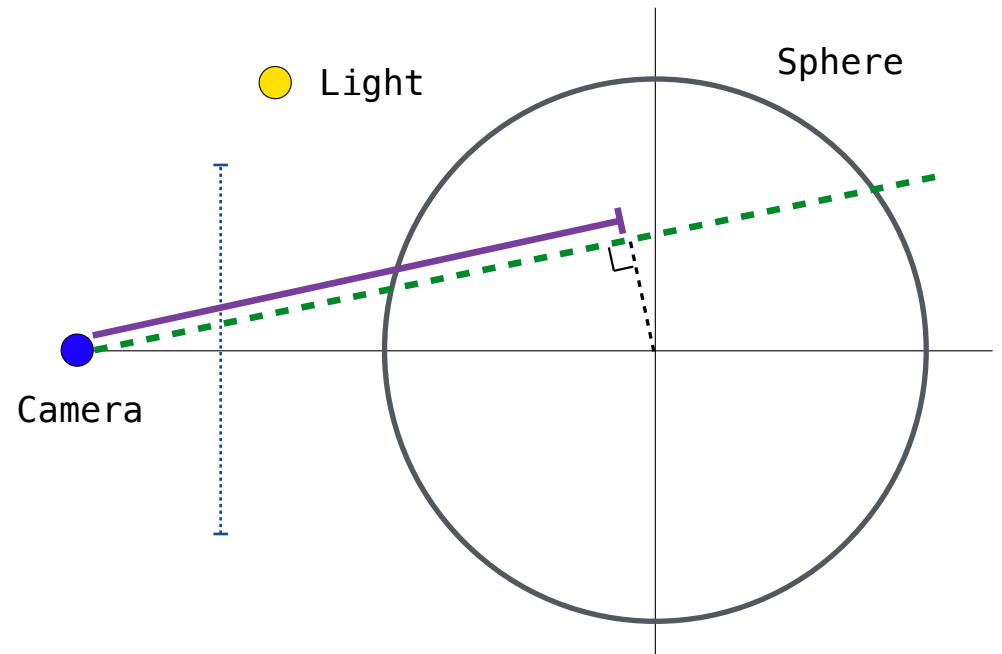
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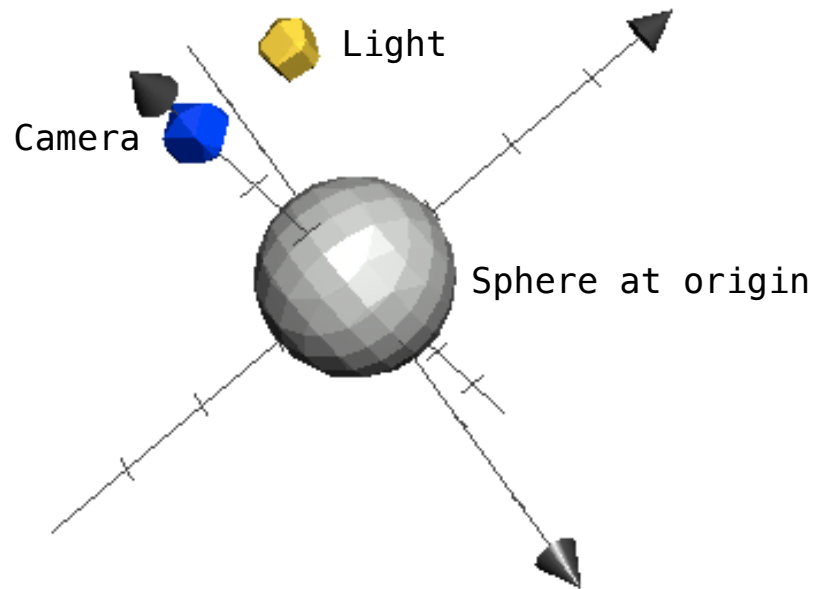


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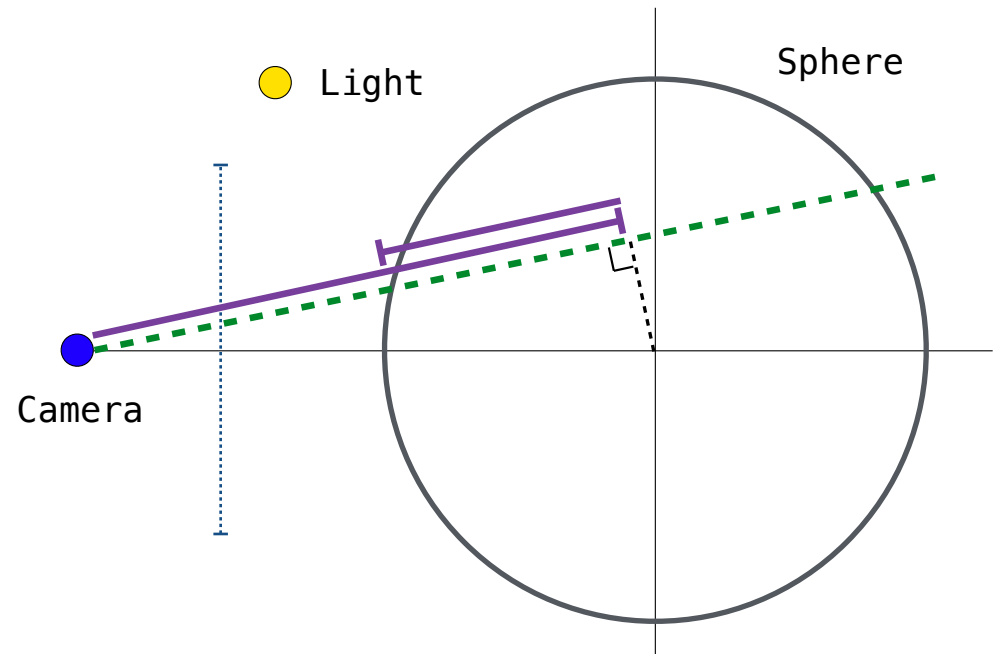
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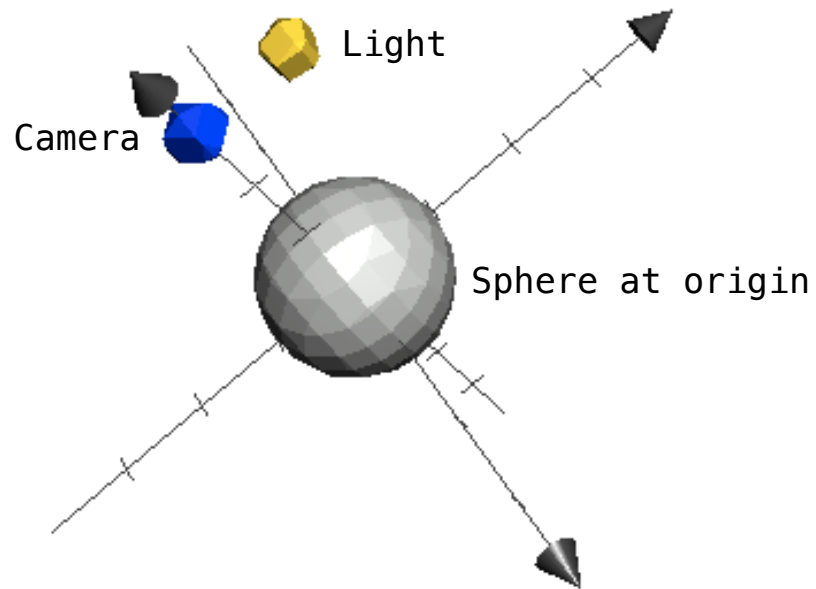


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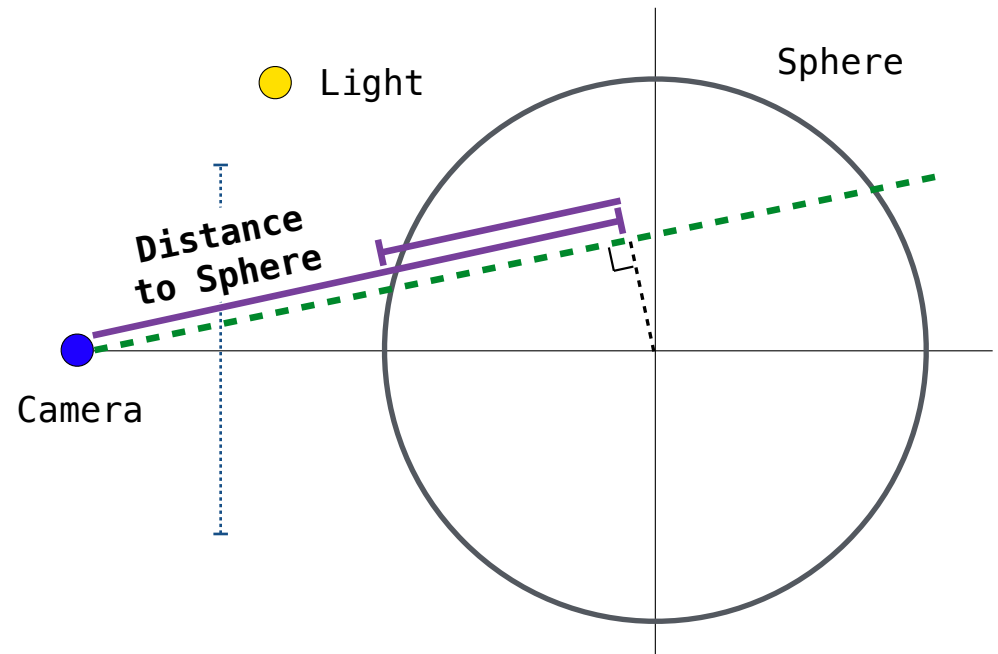
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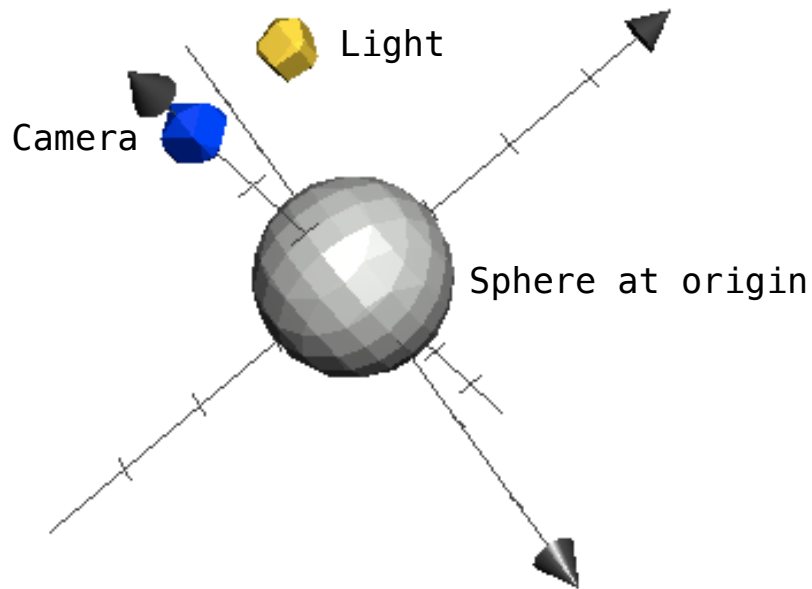


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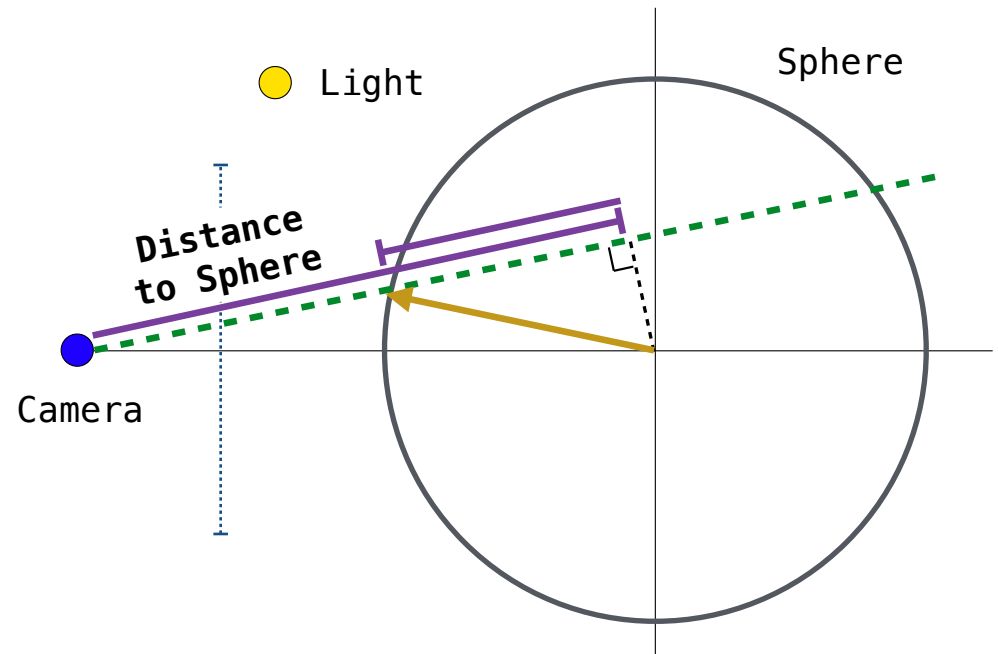
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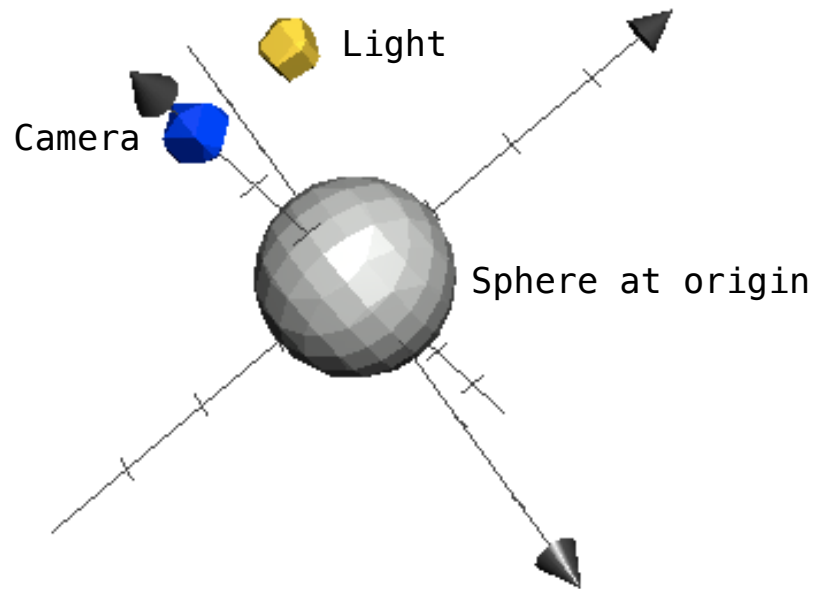


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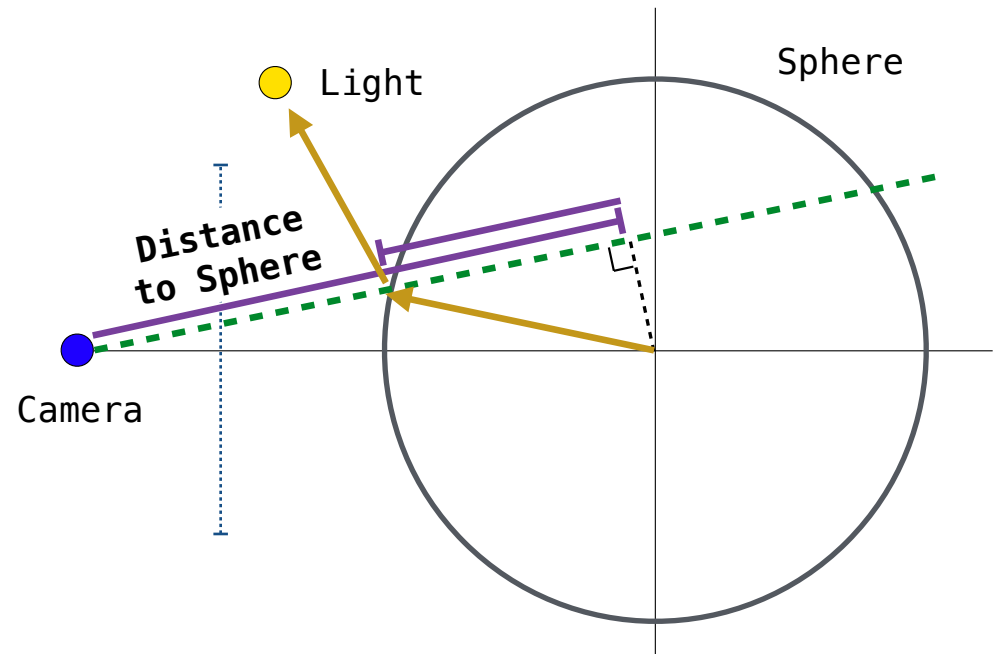
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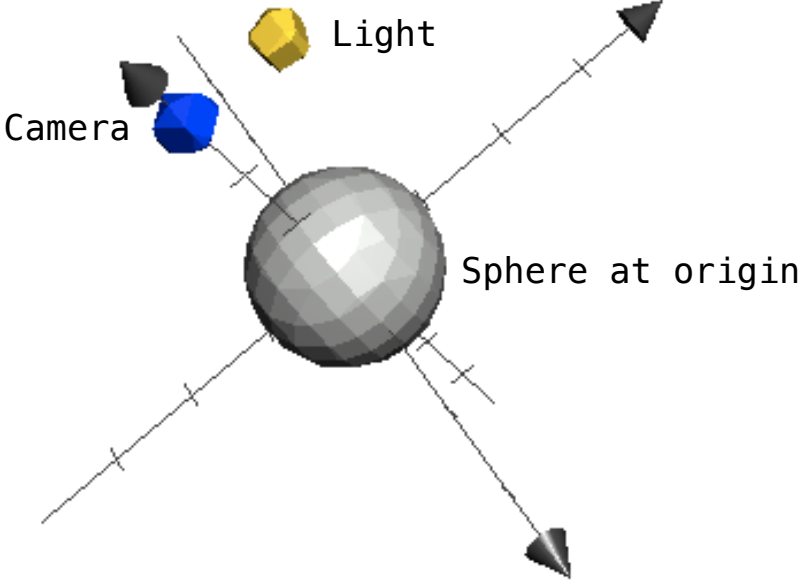


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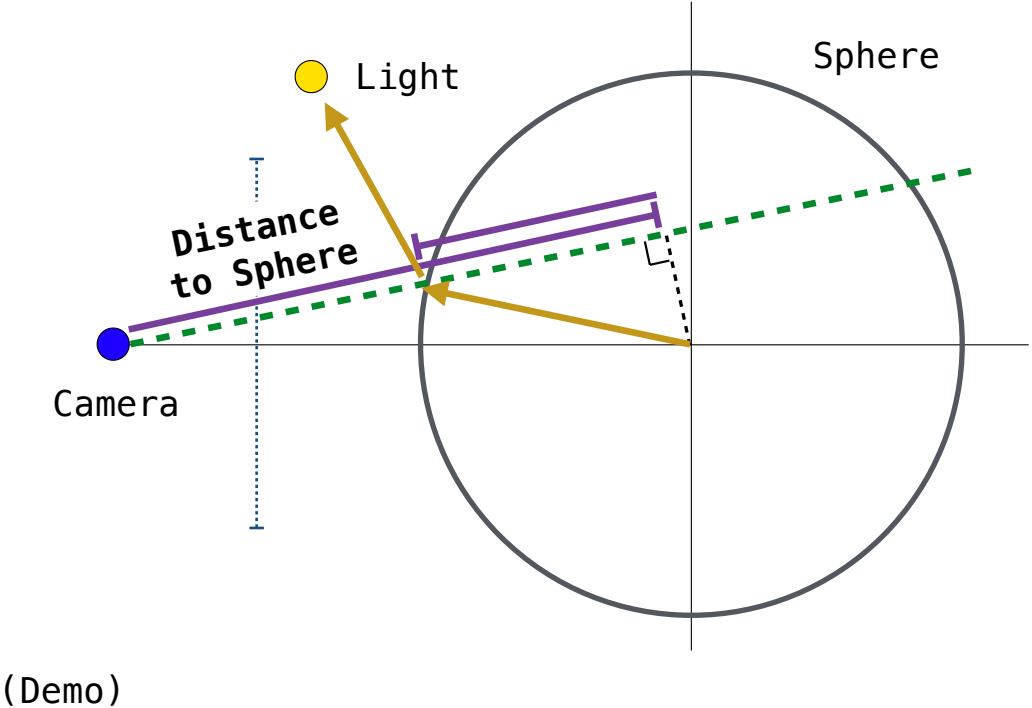
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## Information Hiding

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class FibIter:
    """An iterator over Fibonacci numbers."""
    def __init__(self):
        self._next = 0
        self._addend = 1

    def __next__(self):
        result = self._next
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Starting a name with *two underscores* enforces restricted access from outside the class

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def fib_generator():
    """A generator function for Fibonacci numbers.

    >>> fibs = fib_generator()
    >>> [next(fibs) for _ in range(10)]
    [0, 1, 1, 2, 3, 5, 8, 13, 21, 34]
    """
    yield 0
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    """  
    yield 0  
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There is no way to access values bound to "previous" and "current" externally



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# Streams



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A stream is a linked list, but the rest of the list is computed on demand

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Link( \_\_\_\_\_ , \_\_\_\_\_ )

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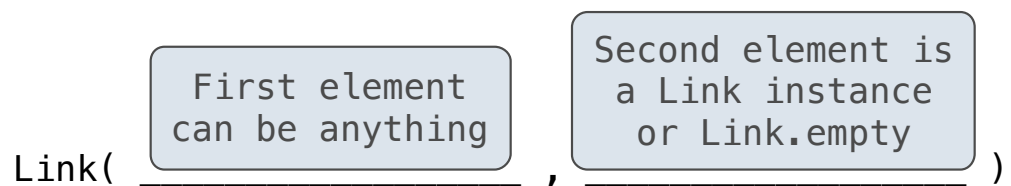
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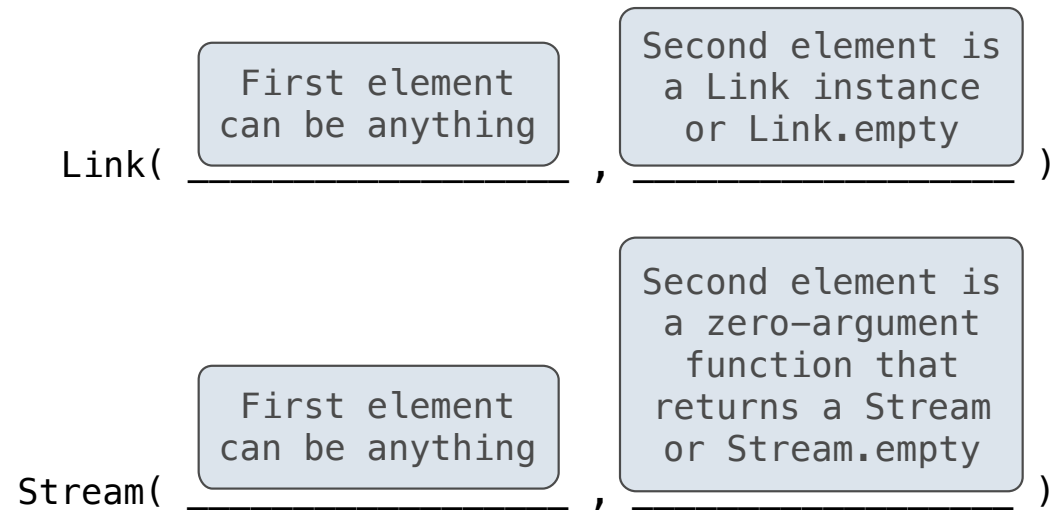
Link( First element  
can be anything , Second element is  
a Link instance  
or Link.empty )

Stream( First element  
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## Streams are Lazy Linked Lists

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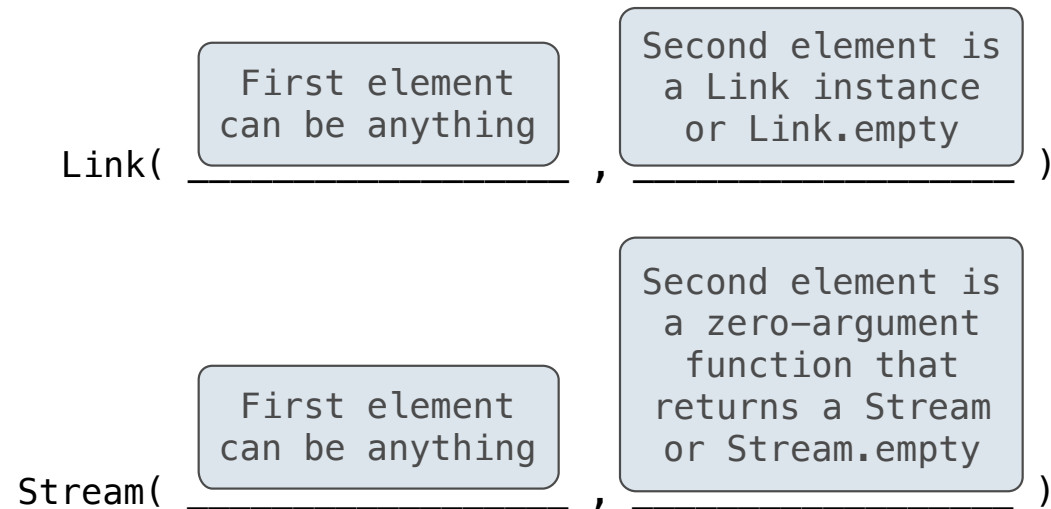
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Once created, Streams and Links can be used interchangeably using `first` and `rest` methods





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def integer_stream(first=1):  
    """Return a stream of consecutive integers, starting with first.  
  
    >>> s = integer_stream(3)  
    >>> s.first  
    3  
    >>> s.rest.first  
    4  
    """  
  
    def compute_rest():  
        return integer_stream(first+1)  
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(Demo)

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```
def f(x=1):  
    s = Stream([x], lambda: s)  
    return s
```

```
def f(x=[]):  
    x.append(1)  
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---

Which definition will produce which row of elements after executing `s = f()`?

	<b>s.first</b>	<b>s.rest.first</b>
<pre>def f(x=1):     return Stream([x], lambda: f([x]))</pre>	[1]	[1, 1]
<pre>def f(x=[1]):     return Stream(x, lambda: f(x+[1]))</pre>	[1, 1]	[1, 1]
<pre>def f(x=1):     s = Stream([x], lambda: s)     return s</pre>	[1]	[1]
<pre>def f(x=[]):     x.append(1)     return Stream(x, lambda: f(x))</pre>	[1]	[[1]]

# Stream Processing

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(Demo)

## Stream Implementation

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    def __init__(self, first, compute_rest=lambda: Stream.empty):
        assert callable(compute_rest), 'compute_rest must be callable.'
        self.first = first
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        self.first = first
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    @property
    def rest(self):
        """Return the rest of the stream, computing it if necessary."""
        if self._compute_rest is not None:
            self._rest = self._compute_rest()
            self._compute_rest = None
        return self._rest
```

## Higher-Order Functions on Streams

## Mapping a Function over a Stream

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def map_stream(fn, s):
    """Map a function fn over the elements of a stream s."""
    if s is Stream.empty:
        return s
    def compute_rest():
        return map_stream(fn, s.rest)
    return Stream(fn(s.first), compute_rest)
```

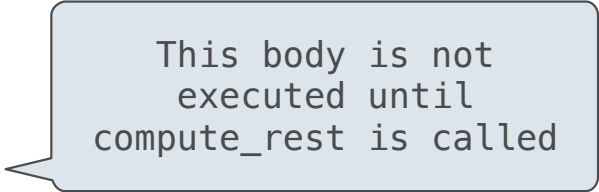


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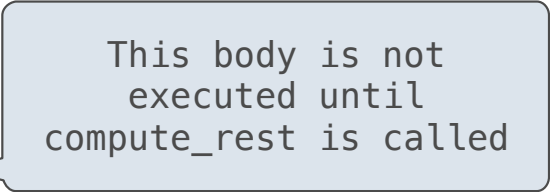
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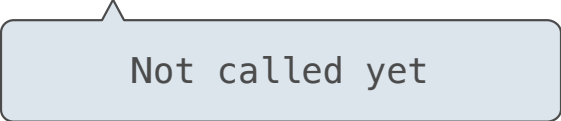
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```
>>> s = integer_stream(3)  
>>> s  
Stream(3, <...>)  
>>> m = map_stream(lambda x: x*x, s)  
>>> first_k(m, 5)  
[9, 16, 25, 36, 49]
```

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def filter_stream(fn, s):
    """Filter stream s with predicate function fn."""
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Actually compute the rest

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(Demo)