Lecture #11: Sequences to Trees

Announcements

- Room assignments for Test #1 will be mailed out.
- Next test last week of March (after spring break).
- HKN review session 3–6PM on Saturday in 306.
- Official TA review session 5–7PM tonight (155 Dwinelle).

Review: Sequence Comprehension

- Syntax:

```
[ <expr> for <var> in <sequence expr> ]
[ <expr> for <var> in <sequence expr> if <boolean expression> ]
```

- Examples:

```
[ 2**x for x in range(5) ] == [1, 2, 4, 8, 16 ]
L = [5, 7, 8, 10, 6, 8, 7, 4, 9, 8]
[ x for x in L if x % 2 == 1 ] == [ 5, 7, 7, 9 ]
```


Representing Multi-Dimensional Structures

- How do we represent a two-dimensional table (like a matrix)?
- Answer: use a *sequence of sequences* (typically a list of lists or tuple of tuples).
- The same approach is used in C, C++, and Java.
- Example:

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 8 \end{bmatrix}$$

becomes

```
(( 1, 2, 0, 4 ), ( 0, 1, 3, -1), (0, 0, 1, 8))
     # or
[[ 1, 2, 0, 4 ], [ 0, 1, 3, -1], [0, 0, 1, 8]]
     # or (for old Fortran hands):
[[ 1, 0, 0 ], [ 2, 1, 0 ], [ 0, 3, 1 ], [ 4, -1, 8 ]]
```


Problem: Creating A Two-Dimensional Array

```
def multiplication_table(rows, cols):
    """A ROWS x COLS multiplication table wwhere row x column y
    (element [x][y]) contains xy.  Example:
    >>> multiplication_table(4, 3)
    [[0, 0, 0], [0, 1, 2], [0, 2, 4], [0, 3, 6]]
    """
    return _____
```


Problem: Creating a Triangular Array

- There's no reason the rows in a 2D list must have the same length.

```
def triangle(rows):
    """A ROWSxROWS lower-triangular array
    containing "*"s."""
```

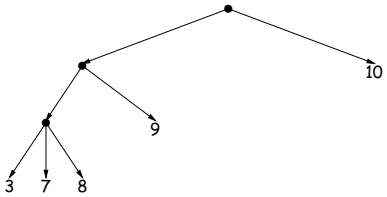

Variation: Creating a Numbered Triangular Array

- This time, use numbers instead of asterisks.

```
def numbered_triangle(rows):
    """A ROWSxROWS lower-triangular array whose elements
    are integers, starting at 0 going left-to-right,
    up-to-down.
    >>> numbered_triangle(3)
    [ [ 0 ], [ 1, 2 ], [ 3, 4, 5 ] ]"""
```

And Why Stop There? Trees

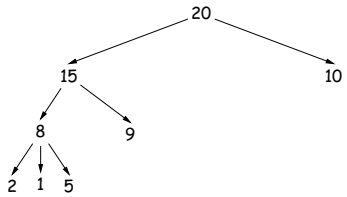
- We can have rows of rows, and rows of rows of rows, but we needn't stop at an arbitrary limit.
- Result what is a form of *tree*:



- The dots and numbers are generally called *vertices* or *nodes*, connected by *edges*.
- Top node is the *root*, bottom ones are *leaves*, others are *inner nodes*.
- Each node is itself the root of a *subtree*; those immediately below are its *children*.

Trees With Labels

- Generally, there can be data at each node, called *labels*:



- How can we represent this structure?

Tree Interface

- Evidently, trees have labels and children, suggesting an API like this:

```
def make_tree(label, kids = [])
    """A (sub)tree with given LABEL at its root, whose children
    are KIDS."""

def label(tree):
    """The label on TREE."""

def children(tree):
    """The immediate descendants of TREE (each a tree)."""

def isleaf(tree):
    """True if TREE is a leaf node."""
```

- Representation?

Tree Representation

```
def make_tree(label, kids = [])
    """A (sub)tree with given LABEL at its root, whose children
    are KIDS."""
    return [ label ] + kids

def label(tree):
    """The label on TREE."""
    return tree[0]

def children(tree):
    """The immediate descendants of TREE (each a tree)."""
    return tree[1:]

def isleaf(tree):
    """True if TREE is a leaf node."""
    return len(tree) == 1
```

Alternatives?

Tree Representation (II)

```
def make_tree(label, kids = [])
    """A (sub)tree with given LABEL at its root, whose children
    are KIDS."""
    return (label, kids)

def label(tree):
    """The label on TREE."""
    return tree[0]

def children(tree):
    """The immediate descendants of TREE (each a tree)."""
    return tree[1]

def isleaf(tree):
    """True if TREE is a leaf node."""
    return len(children(tree)) == 0
```


Algorithms on Trees

- Trees have a recursive structure. A tree is:
  - A label and
  - Zero or more children, each a tree.
- Recursive structure implies recursive algorithm.

Counting Leaves

```
def count_leaves(tree):
    """The number of leaf nodes in TREE."""
```