Lecture #21: Search and Sets

Announcements

- My office hours this Thursday (only) are 3–4PM.
- Homework 5 to be released later today. Many problems on it were just the optional ones from this week's lab.

Container Objects and Searching

- Lists, linked lists, trees, and dictionaries are various objects whose principle purpose is to *contain values* and present them in various ways.
- We've principally considered operations that involve retrieving all values and doing something with them.
- But a central activity of many programs and algorithms is *finding* a value that meets certain criteria *in* one of these containers.
- Several Python data structures provide methods for finding:

```
x in aList     # Is x in aList?
x in aDict     # Is x a key in aDict?
aDict[x]       # What is V if aDict contains the entry (x, V)?
"61A" in text # Does substring '61A' appear in string text?
```

Sets

- Current versions of Python also have *sets*, which are intended to behave like mathematical sets.
- Examples:

```
A = { 1, 3, 2 }     # Definition by extension
B = set([1, 3, 5]) # Contents can come from an iterable
set()               # The empty set
{}                  # The empty dictionary (sorry)
{ x for x in L if x % 2 == 1 }
                    # Set generator:  odd members of L
                    # Like {x|x ∈ L and x is odd }
A | B  == { 1, 2, 3, 5 } == A.union(B)
                    # A ∪ B
A & B == { 1, 3 } == A.intersection(B)
A - B == { 2 } == A.difference(B) == { x for x in A if x not in B }
A < (A | B) == True # A ⊂ A ∪ B
3 in A == True       # 3 ∈ A
len(A) == 3          # |A| or size of A
```

Sets are Iterables

- Like other container types, one can iterate over sets.
- Python sets are *unordered:* ordering of iterator results is undefined.

```
>>> for x in { 5000, 3000, 100 }: print(x, end=" ")
3000 5000 100
>>> list( { 5000, 3000, 100 } )
[3000, 5000, 100]
```

Example

How can I test whether a list contains duplicates?

```
def hasDuplicates(L):
    """Return true iff list L contains duplicated values."""
```

Implementing Sets: Unordered Lists

- Clearly, lists also contain collections of values, so we could use them to implement sets.
- Must be careful to avoid duplicate elements (important when iterating).
- The algorithm for "member of" (`x in S`) is familiar:

```
def contains(S, x):
    """True iff list S (considered as a set) contains x."""
    for y in S:
        if x == y:
            return True
    return False
```

- If N is the length of `S`, what is the worst-case time bound?

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- If N is the length of `S`, what is the worst-case time bound? *Answer:* $\Theta(N)$

Implementing Sets: Insertion/Formation w/ Unordered List

What's the time required for this? Assume appending to a list takes $O(1)$ time (which is true on average).

```
def toSet(L):
    """Returns an unordered list containing all values in L without
    duplicates."""
    result = []
    for x in L:
        if not contains(result, x):
            result.append(x)
    return result
```

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Answer: $\Theta(N^2)$

Implementing Sets: Ordered Lists

- If we keep list sorted (say in ascending order), can use *binary search*:

```
def contains(S, x):
    """Returns true if X is in S, a list sorted in ascending order."""
    L, U = 0, len(S)-1
    while L <= U:
        M = (L + U) // 2
        if x == S[M]:
            return True
        elif x < S[M]:
            U = M - 1
        else:
            L = M + 1
    return False
```

- What's the execution time here (if N is `len(S)`)?

Implementing Sets: Ordered Lists

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        else:
            L = M + 1
    return False
```

- What's the execution time here (if N is `len(S)`)? **Answer:** $\Theta(\lg N)$

Implementing Sets: Insertion/Formation w/ Ordered List

What's the time required for this? Assume appending to a list takes $O(1)$ time (which is true on average).

```
def toSet(LST):
    """Returns an ordered list containing all values in LST without
    duplicates."""
    result = []
    for x in lst:
        L, U = 0, len(result)-1
        while L <= U:
            M = (L + U) // 2
            if x == result[M]:
                break
            elif x < result[M]:
                U = M - 1
            else:
                L = M + 1
        if L > U:
           result.insert(L, x)
```

Implementing Sets: Insertion/Formation w/ Ordered List

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Answer: $\Theta(N^2)$

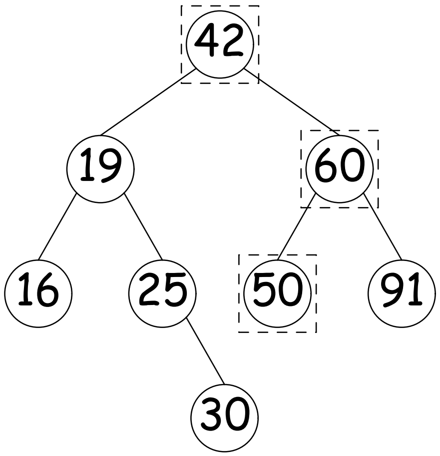
Binary Search Trees

Binary Search Property:

- In a *binary tree*, each inner node has two children (called “left” and “right”, typically), but trees are allowed to be *empty* (no label, no children).
- A *binary search tree* (BST) satifies two other properties:
- All nodes in left subtree of a node have *smaller* keys.
- All nodes in right subtree of node have *larger* keys.
- This allows binary search, but in a tree.

Finding

- Searching for 50 and 49:




```
def contains(S, x):
    """Returns true iff BST S contains x."""
    if S == BinTree.empty:
        return False
    if S.label == x:
        return True
    elif S.label < x:
        return contains(S.right, x)
    else:
        return contains(S.left, x)
```

- Dashed boxes show which node labels we look at.
- Number looked at proportional to height of tree.
- What is worst-case time?
- If tree is "bushy," what is worst-case time?

Finding

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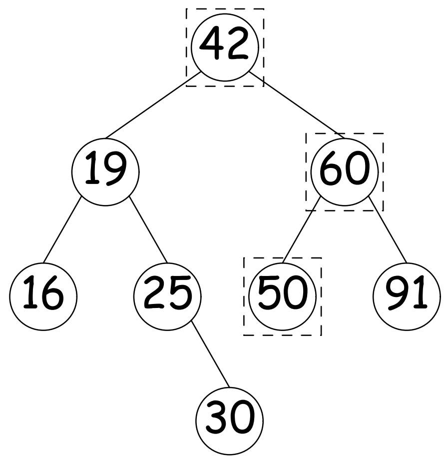


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- Dashed boxes show which node labels we look at.
- Number looked at proportional to height of tree.
- What is worst-case time? Answer: $\Theta(N)$
- If tree is "bushy," what is worst-case time?

Finding

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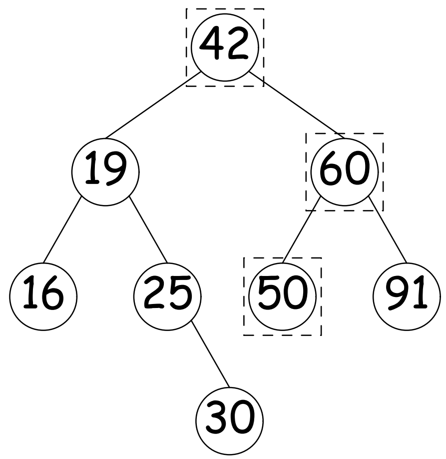


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```
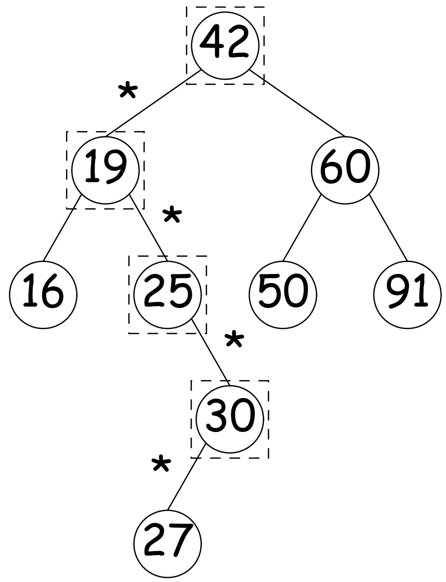

- Dashed boxes show which node labels we look at.
- Number looked at proportional to height of tree.
- What is worst-case time? *Answer*: $\Theta(N)$
- If tree is "bushy," what is worst-case time? *Answer*: $\Theta(\lg N)$

Inserting

- Inserting 27




```
def add(S, x):
    """Add X to binary search tree S destructively,
    if not already present, returning new tree."""
  if S == BinTree.empty:
      return BinTree(x)
  elif S.label < x:
      S.right = add(S.right, x)
  else:
      S.left = add(S.left, x)
  return S
```

- Starred edges are set (to themselves, unless initially null).
- Again, time proportional to height.

What Does Python Do?

- Python uses a different method to store sets (also dictionaries).
- In effect, instead of a binary search tree, uses an *n-ary tree with height 2*.
- Instead of using $<$, $>$, uses a more general *hashing function*.
- *Usually*, this gives $\Theta(1)$ for searches.
- Take CS61B for details.