CS61A Lecture #38: Cryptography

Announcements:

- HKN surveys on Friday: 5 bonus points for filling out their survey *on Friday* (yes, that means you have to come to lecture).

Cryptography: Purposes

- Source: Ross Anderson, *Security Engineering*.
- Cryptography—the study of the design of ciphers—is a tool used to help meet several goals, among them:
  - Privacy: others can't read our messages.
  - Integrity: others can't change our messages without us knowing.
  - Authentication: we know whom we're talking to.
- Some common terminology: we convert from *plaintext* to *ciphertext* (encryption) and back (decryption).
- Although we typically think of text messages as characters, our algorithms generally process streams of *numbers* or *bits*, making use of standard encodings of characters as numbers.

Substitution

- Simplest scheme is just to permute the alphabet:

```
␣abcdefghijklmnopqrstuvwxyz
tyler␣duniabcfghjkmopqsvwxz
```

- So that

```
"so␣long␣and␣thanks␣for␣all␣the␣fish" =>
"ohtchgutygrtpnygbotdhmtycctpn␣tdion"
```

- Problem: If we intercept ciphertext for which we know the plaintext (e.g., we know a message ends with name of the sender), we learn part of the code.
- Even if we have only ciphertext, we can guess encoding from letter frequencies.

Stream Ciphers

- **Idea:** Use a different encoding for each character position. Enigma was one example.
- Extreme case is the *One-Time Pad:* Receiver and sender share random key sequence at least as long as all data sent. Each character of the key specifies an unpredictable substitution cipher.
- Example:

```
Messages: attack at dawn|oops cancel that order|attack is back on
Key:      vnchkjskruwisn|tjcdktjdjsahtjkdhjrizn|akjqltpotpfhsdjrsqieha...
Cipher:   vfvhmtrkjtzin |gxrvjvjqlwlglqkwgxhlcd|acbqncowkoghuniee
```

(key of 'z' means 'a' ↦ 'z', 'b' ↦ '␣', 'c' ↦ 'a', etc.)

- Unbreakable, but requires lots of shared key information.
- Integrity problems: If I know message is "Pay to Paul N. Hilfinger $100.00" can alter it to "Pay to Paul N. Hilfinger $999.00" [How?]

Aside: A Simple Reversible Combination

- The cipher in the last slide essentially used addition modulo alphabet size as the way to combine plaintext with a key.
- Usually, we use a different method of combining streams: *exclusive or (xor)*, which is the "not equal" operations on bits, defined on individual bits by $x \oplus y = 0$ if x and y are the same, else 1.
Fact: $x \oplus y \oplus x = y$. So,

$$\begin{array}{r} 01100011 \\ \oplus\ 10110101 \\ \hline 11010110 \end{array} \qquad \begin{array}{r} 11010110 \\ \oplus\ 10110101 \\ \hline 01100011 \end{array}$$

- In Python, C, and Java, this operation is written x^y.

Using Random-Number Generators

- Python provides a pseudo-random number generator (used for the Hog project, e.g.): from an initial value, produces any number of "random-looking" numbers.
- Consider a function that creates pseudo-random number generators that produce bits, e.g.:

```
import random
def bit_stream(seed):
    r = random.Random(seed)
    return lambda: r.getrandbits(1)
```

- If two sides of a conversation share the same key to use as a seed, can create the same approximation to a one-time pad, and thus communicate secretly.

Example

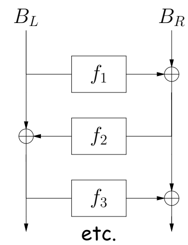
Message	H	e	l	l	o	,		w	o	r	l	d
Message bytes (hex)	48	65	6c	6c	6f	2c	20	77	6f	72	6c	64
Random bytes	5b	49	96	1d	93	eb	6e	2d	a4	1a	52	fb
Encrypted bytes	13	2c	fa	71	fc	c7	4e	5a	cb	68	3e	9f
Encrypted message	?	,	?	q	?	?	N	Z	?	h	>	?

(? in place of non-ASCII)

- Advantage: key can be much shorter than total amount of data.
- Disadvantage: stream of bits isn't really random; may be subject to clever attack (cryptanalysis). This is especially true of standard random number generators like Python's.
- Was used in SSL (Secure Socket Layer) for "secure" web communications.

Block Ciphers

- So far, have encoded bit-by-bit (or byte-by-byte). Another approach is to map blocks of bits at a time, allowing them to be mixed and swapped as well as scrambled.
- Feistel Ciphers: a strategy for generating block ciphers. Break message into $2N$-bit chunks, and break each chunk into N-bit left and right halves, B_L and B_R. Then, put the result through a number of *rounds:*



- Each f_i is some function mapping N-bit blocks to N-bit blocks that is chosen by your key.
- f_i does not have to be invertible.
- Nice feature: to decrypt, run backwards.
- If the f_i are really chosen well enough, these are very good ciphers with enough rounds.

- The Data Encryption Standard (DES) used this strategy with 12 rounds.

Example

- Block size: 32-bits. Number of rounds: 6
- Key: CS61AForever, or in bits (shown in hexadecimal):

K_0	K_1	K_2	K_3	K_4	K_5
4353	3631	4146	6f72	6576	6572
C S	6 1	A F	o r	e v	e r

- $f_i(x) = (W_L + W_R)_R$, where $W = x \cdot K_i$, where E_L and E_R denote the left and right 16 bits of E.
- Message: "Hello, world", or in bits

```
H e l l  o ,  w  o r l d
48656c6c 6f2c2077 6f726c64
```


Encryption, Decryption

i	B_L	B_R	K_i
	H e	l l	
	4865	6c6c	
0	4865	96a4	4353
1	c522	96a4	3631
2	c522	2d2b	4146
3	10ed	2d2b	6f72
4	10ed	79d8	6576
5	ba95	79d8	6572
	ba95	79d8	

i	B_L	B_R	K_i
	o ,	w	
	6f2c	2077	
0	6f2c	8cf7	4353
1	5c32	8cf7	3631
2	5c32	73da	4146
3	13b1	73da	6f72
4	13b1	83b9	6576
5	ca25	83b9	6572
	ca25	83b9	

i	B_L	B_R	K_i
	o r	l d	
	6f72	6c64	
0	6f72	7920	4353
1	67b1	7920	3631
2	67b1	1cf6	4146
3	fb96	1cf6	6f72
4	fb96	942c	6576
5	69d9	942c	6572
	69d9	942c	

i	B_L	B_R	K_i
	ba95	79d8	
5	10ed	79d8	6572
4	10ed	2d2b	6576
3	c522	2d2b	6f72
2	c522	96a4	4146
1	4865	96a4	3631
0	4865	6c6c	4353
	H e	l l	

i	B_L	B_R	K_i
	ca25	83b9	
5	13b1	83b9	6572
4	13b1	73da	6576
3	5c32	73da	6f72
2	5c32	8cf7	4146
1	6f2c	8cf7	3631
0	6f2c	2077	4353
	o ,	w	

i	B_L	B_R	K_i
	69d9	942c	
5	fb96	942c	6572
4	fb96	1cf6	6576
3	67b1	1cf6	6f72
2	67b1	7920	4146
1	6f72	7920	3631
0	6f72	6c64	4353
	o r	l d	

Chaining

- It's possible to abuse a good cipher, making messages vulnerable.
- If you simply break a message into pieces and then encrypt each piece, an eavesdropper (traditionally named Eve) can tell that two messages you send are the same, even if she doesn't know what the messages are.
- E.g., in advance of the Battle of Midway (WWII), the Allies determined that the target of the Japanese operation was, in fact, Midway by arranging to have the Japanese intercept and retransmit in coded form a message containing the word "Midway." This allowed them to determine what island other encoded Japanese communications were referring to.
- One fix is *chaining*: before encrypting a block, xor it with the encoding of the previous block. Start the process off with a throw-away random block.

Public Key Cryptography

- So far, our ciphers have been *symmetric:* both sides of a conversation share the same secret information (a key).
- If I haven't contacted someone before, how can we trade secret keys so as to use one of these methods?
- One idea is to use *public keys* so that everyone knows enough to communicate with us, but not enough to listen in when others communicate with us..
- Here, information is *asymmetric:* we publish a *public key* that everyone can know, and keep back a *private key.*
- Rely on it being easy to decipher messages knowing the private key, but impractically difficult without it.
- Unfortunately, we haven't actually proved that any of these *public-key systems* really are essentially impractical to crack, and quantum computing (if made to work at scale) would break the most common one.
- But for now, all is well.

Example: Diffie-Hellman key exchange

- Assume that everyone has agreed ahead of time about a large public prime number p and another number $g < p$.
- Every person, Y, now chooses a secret number, s_y, and publishes the value $K_Y = g^{S_Y} \bmod p$ next to his name.
- If A (Alice) wants to communicate with B (Bob), she can look up Bob's published number, K_b, and use $(K_b)^{s_a} \bmod p$ as the encrypting key.
- Bob, seeing a message from Alice, computes $(K_a)^{s_b} \bmod p$.
- But $K_b^{s_a} \equiv (g^{s_b})^{s_a} \equiv g^{s_b \cdot s_a} \equiv (g^{s_a})^{s_b} \equiv (K_a)^{s_b} \bmod p$, so both Bob and Alice have the same key!
- Nobody else knows this key, because of the difficulty of finding x such that $a^x = b \bmod p$ (for large p and x).

Example

p	101	Public
g	17	
Alice's secret key	19	Private
Alice's published key	$17^{19} \bmod 101 = 6$	
Bob's secret key	33	
Bob's published key	$17^{33} \bmod 101 = 65$	
Alice's computed key	$65^{19} \bmod 101 = 14$	Private to Alice and Bob.
Bob's computed key	$6^{33} \bmod 101 = 14$	

Other Public-Key Methods

- General idea with public-key methods is that everyone publishes a public key, K_p, while retaining a secret private key, K_s.
- Typically these keys are very large numbers (hundreds of bits).
- A common method, RSA encryption, uses a public key consisting of the product pq of two large prime numbers and a value e that has no factors in common with $p-1$ and $q-1$. The private key is the two numbers p and q.
- It is very hard to compute p and q from the product pq.
- To encrypt message M, compute $C = M^e \bmod pq$.
- It is very hard to compute M from C unless you know p and q (not just pq). But it is "easy" (with a computer) if you do know them.
- The method uses Euler's generalization of Fermat's (Little) Theorem, but we'll let you wait until the CS170 series to find out how [plug].

Signatures

- Suppose I receive a message, M, that supposedly comes from you. How do I know it does?
- Using public-key methods, this is relatively easy.
- One approach (no details here) is that you first compute a condensation of M, $h(M)$, where it is very hard to find another message, M' such that $h(M) = h(M')$ and $h(M)$ is a (big) integer in some limited range (say 128 bits).
- Now append to your message a value $S = f(h(M), K_s)$, where f is a "signing function".
- We choose f so that it has the property that there is an easily computed function f' such that $f'(S, K_p) = h(M)$.
- So I, by computing $h(M)$ and comparing it to $f'(S, K_p)$, can tell whether you signed the message.

Special Effects: Playing Cards Over the Phone?

- How do I play a card game over the phone, so that neither side can (undetectably) cheat?
- To keep it simple, assume we have a two-person game between Alice and Bob where all cards get revealed.
- For each game, let each side choose a secret encryption key, and assume an algorithm that is *commutative:* if a message is encrypted by secret key A and then by key B, it can be decrypted by the two keys in either order.

Playing Cards Over the Phone: Method

- Alice shuffles and encrypts a deck of cards, and sends them to Bob.
- Bob encrypts the encrypted cards, shuffles them, and sends them back to Alice (doubly encrypted).
- Alice deals cards to Bob by selecting and decrypting them, and sending them to Bob, who can decrypt them.
- Alice deals cards to herself by sending them to Bob, having him decrypt them and send them (now singly encrypted) back to Alice.
- At the end of the game, all information can be revealed, and both sides can check for consistency.

Zero-Knowledge Proofs

- Suppose I possess the answer to a puzzle, and want to convince you that I have the answer *without* revealing anything about what it is.
- This is an example of a *zero-knowledge proof* (Abadi, Goldwasser, and Rackoff).
- Many uses, such as authentication (I want to prove who I am), or enforcing honesty while maintaining privacy.
- Example: Prove that I know how to 3-color a graph.
- Given a graph (a network of nodes connected by edges) a *3-coloring* is an assignment of colors to nodes (from a palette of three) such that no nodes joined by an edge have the same color.
- Don't always exist, and hard to find when they do.
- Can I provde to you that I know how to color a particular large graph without letting you know how?
- Demo: `http://web.mit.edu/~ezyang/Public/graph/svg.html`