
COMPUTER SCIENCE 61A

April 28, 2016

1 Rain, Rain, Go Away

1. For each row below, fill in the blanks in the output displayed by the interactive Python interpreter when the expression is evaluated. Expressions are evaluated in order, and expressions may affect later expressions.

```
>>> cats = [1, 2]
>>> dogs = [cats, cats.append(23), list(cats)]
>>> cats
```

```
>>> dogs[1] = list(dogs)
>>> dogs[1]
```

```
>>> dogs[0].append(2)
>>> cats
```

```
>>> dogs[2].extend([list(cats).pop(0), 3])
>>> dogs[3]
```

```
>>> dogs
```

2 Gift in a Box

1. (Fall 2012) Draw the environment diagram.

```
def box(a):
    def box(b):
        def box(c):
            nonlocal a
            a = a + c
            return (a, b)
        return box
    gift = box(1)
    return (gift(2), gift(3))
box(4)
```

3 The Gift & The Recurse

1. The **quicksort** sorting algorithm is an efficient and commonly used algorithm to order the elements of a list. We choose one element of the list to be the **pivot** element and partition the remaining elements into two lists: one of elements less than the pivot and one of elements greater than the pivot. We recursively sort the two lists, which gives us a sorted list of all the elements less than the pivot and all the elements greater than the pivot, which we can then combine with the pivot for a completely sorted list.

First, implement the `quicksort_list` function. Choose the first element of the list as the pivot. You may assume that all elements are distinct.

```
def quicksort_list(lst):
    """
    >>> quicksort_list([3, 1, 4])
    [1, 3, 4]
    """

    if _____:

        _____

    pivot = lst[0]

    less = _____

    greater = _____

    return _____
```

2. We can also use quicksort to sort linked lists! Implement the `quicksort_link` function, without constructing additional `Link` instances.

You can assume that the `extend_links` function is already defined. It takes two linked lists and mutates the first so that the ending node points to the second. `extend_link` returns the head of the first linked list.

```
>>> l1, l2 = Link(1, Link(2)), Link(3, Link(4))
>>> l3 = extend_links(l1, l2)
>>> l3
Link(1, Link(2, Link(3, Link(4))))
>>> l1 is l3
True
```

```
def quicksort_link(link):
```

```
    """
```

```
    >>> s = Link(3, Link(1, Link(4)))
```

```
    >>> quicksort_link(s)
```

```
    Link(1, Link(3, Link(4)))
```

```
    """
```

```
    if _____:
```

```
        return link
```

```
    pivot, _____ = _____
```

```
    less, greater = _____
```

```
    while link is not Link.empty:
```

```
        curr, rest = link, link.rest
```

```
        if _____:
```

```
            _____
```

```
        else:
```

```
            _____
```

```
        link = _____
```

```
    less = _____
```

```
    greater = _____
```

```
    _____
```

```
    return _____
```

4 Can You Take Me Higher?

1. (Fall 2013) Fill in the blanks in the implementation of `paths`, which takes as input two positive integers `x` and `y`. It returns the number of ways of reaching `y` from `x` by repeatedly incrementing or doubling. For instance, we can reach 9 from 3 by incrementing to 4, doubling to 8, then incrementing again to 9.

```
def inc(x):  
    return x + 1
```

```
def double(x):  
    return x * 2
```

```
def paths(x, y):  
    """Return the number of ways to reach y from x by repeated  
    incrementing or doubling.  
    >>> paths(3, 5) # inc(inc(3))  
    1  
    >>> paths(3, 6) # double(3), inc(inc(inc(3)))  
    2  
    >>> paths(3, 9) # E.g. inc(double(inc(3)))  
    3  
    >>> paths(3, 3) # No calls is a valid path  
    1  
    """  
    if x > y:  
        return _____  
  
    elif x == y:  
        return _____  
  
    else:  
        return _____
```

2. (Fall 2013) Fill in the blanks in the implementation of `pathfinder`, a higher-order function that takes an increasing function `f` and a positive integer `y`. It returns a function that takes a positive integer `x` and returns whether it is possible to reach `y` by applying `f` to `x` zero or more times. For example, 8 can be reached from 2 by applying `double` twice. A function `f` is *increasing* if $f(x) > x$ for all positive integers `x`.

```
def pathfinder(f, y):
    """
    >>> f = pathfinder(double, 8)
    >>> {k: f(k) for k in (1, 2, 3, 4, 5)}
    {1: True, 2: True, 3: False, 4: True, 5: False}
    >>> g = pathfinder(inc, 3)
    >>> {k: g(k) for k in (1, 2, 3, 4, 5)}
    {1: True, 2: True, 3: True, 4: False, 5: False}
    """
    def find_from(x):
        while _____:
            _____
        return _____
```

3. Write a generator function that yields functions that are repeated applications of a one-argument function `f`. The first function yielded should apply `f` 0 times (the identity function), the second function yielded should apply `f` once, etc.

```
def repeated(f):
    """
    >>> [g(1) for _, g in
    ... zip(range(5), repeated(double))]
    [1, 2, 4, 8, 16]
    """
    g = _____
    while True:
        _____
        _____
```

4. Ben Bitdiddle proposes the following alternate solution. Does it work?

```
def ben_repeated(f):
    g = lambda x: x
    while True:
        yield g
        g = lambda x: f(g(x))
```

5 Slim Shady

1. Implement `widest_level`, which takes a `Tree` instance and returns the elements at the depth with the most elements.

```
def widest_level(t):
    """
    >>> sum([[1], [2]], [])
    [1, 2]
    >>> t = Tree(3, [Tree(1, [Tree(1), Tree(5)]),
    ...           Tree(4, [Tree(9, [Tree(2)])])]
    >>> widest_level(t)
    [1, 5, 9]
    """
    levels = []
    x = [t]

    while _____:
        _____
        _____ = sum(_____, [])

    return max(levels, key=_____)
```

6 Scheming With a Broken Heart

1. Consider the following Scheme tree data abstraction.

```
(define (make-tree entry children) (cons entry children))
(define (entry tree) (car tree))
(define (children tree) (cdr tree))
(define tree 'below-example)
;
;           5
;   +-----+-----+
;   |         |         |
;   6         7         2
;   +---+---+   |   +---+---+
;   |     |     |   |   |   |
;   9     8     1   6   4
;
;               |
;               |
;               3
```

Write a procedure `tree-sums` that takes a tree of numbers (like the one above) and outputs a list of sums from following each possible path from root to leaf.

Hint: You may find the `flatten` procedure helpful.

```
(define (flatten lst)
  (cond ((null? lst) nil)
        ((list? (car lst)) (append (flatten (car lst)) (
                                     flatten (cdr lst))))
        (else (cons (car lst) (flatten (cdr lst))))))

(define (tree-sums tree)

  (if _____

      _____

      (map (lambda (x) _____)

           _____)))
```

```
scm> (flatten '(0 (1) ((2)) (3 ((4))))))
(0 1 2 3 4)
scm> (tree-sums tree)
(20 19 13 16 11)
```

7 Stream On

1. Implement the `repeat-nats` procedure, which takes a starting number and returns a stream of natural numbers beginning at the starting value such that each element x is repeated x number of times.

```
(define (repeat-nats start)
```

`take` is a procedure that returns a Scheme list containing the first n elements a stream `s`.

```
(define (take n s)
  (if (or (= n 0) (null? s))
      nil
      (cons (car s) (take (- n 1) (cdr-stream s)))))
```

```
scm> (take 10 (repeat-nats 0))
(1 2 2 3 3 3 4 4 4 4)
```

2. Now implement the `unique-stream` procedure, which takes in a stream and returns new stream that contains each element of the input stream once. Only the first occurrence of each number should be included such that it is in the order that it appears in the original stream. You may want to use `filter-stream` defined below.

```
(define (filter-stream f s)
  (cond
    ((null? s) nil)
    ((f (car s))
     (cons-stream (car s)
                   (filter-stream f (cdr-stream s))))
    (else (filter-stream f (cdr-stream s)))))

(define (unique-stream s)
```

```
(define (lst-to-stream lst)
  (if (null? lst) nil
      (cons-stream (car lst)
                    (lst-to-stream (cdr lst)))))
```

```
scm> (take 10 (unique-stream (lst-to-stream '(1 3 2 5 3 4
2))))
(1 3 2 5 4)
scm> (take 10 (unique-stream (repeat-nats 2)))
(2 3 4 5 6 7 8 9 10 11)
```

8 Turning Tables

1. You're trying to re-organize your music library! The table `tracks` below contains song titles and the corresponding album. Create another table `tracklist` with two columns: the album and a comma-separated list of all songs from that album.

```
create table tracks as
  select "Human" as title, "The Definition" as album union
  select "Simple and Sweet", "The Definition"          union
  select "Paper Planes", "Translations Through Speakers";

create table tracklist as
with
  songs(album, total) as (
    _____
  ),
  _____ as (
    _____
    _____
    _____
  )
select _____
  where _____;
```

```
sqlite3> select * from tracklist order by album;
The Definition|Human, Simple and Sweet
Translations Through Speakers|Paper Planes
```