Higher-Order Functions

Announcements

Designing Functions

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square returns the square of x

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(Demo)

Generalization

Generalizing Patterns with Arguments

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```
Regular geometric shapes relate length and area.
```

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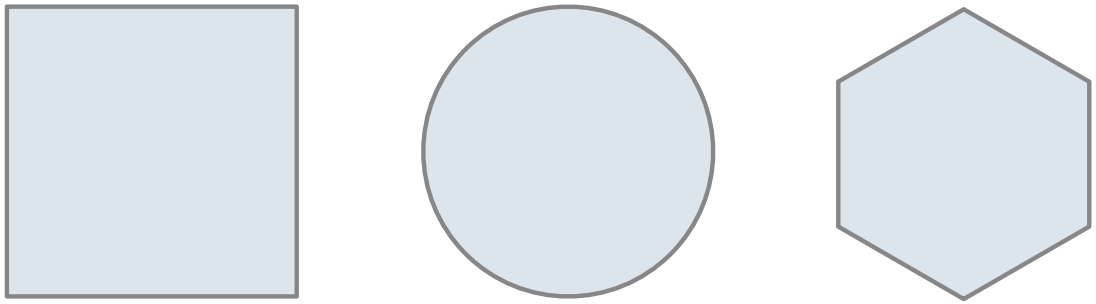
Shape:



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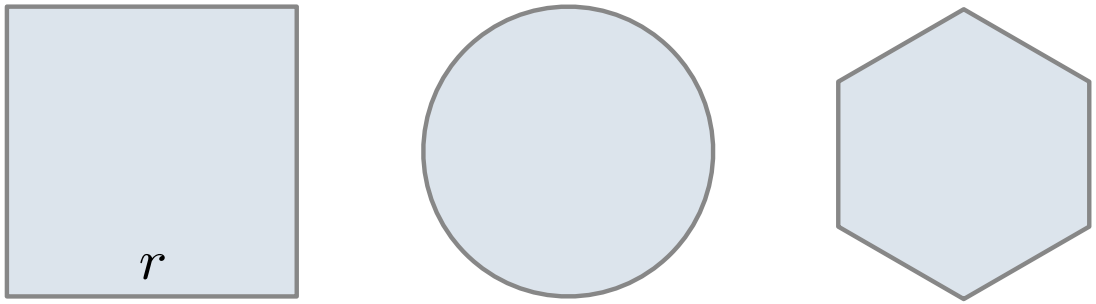

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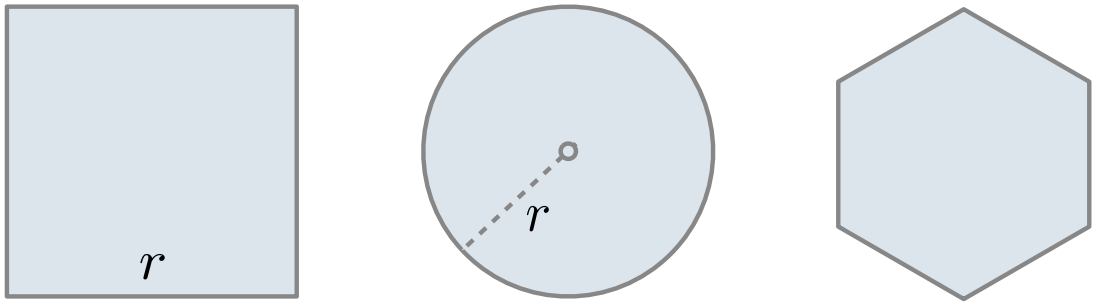

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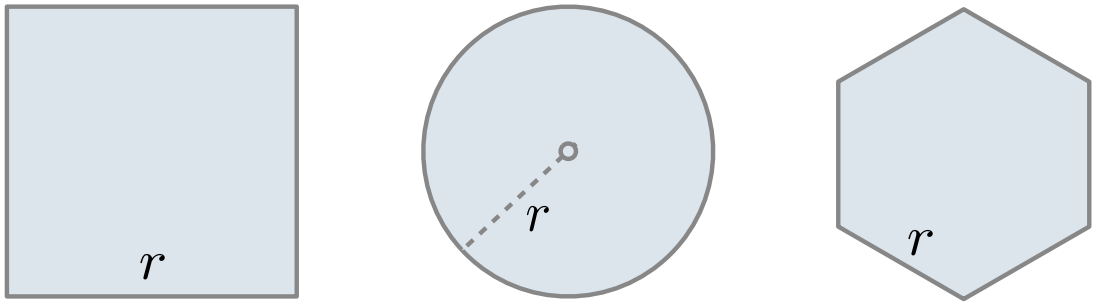

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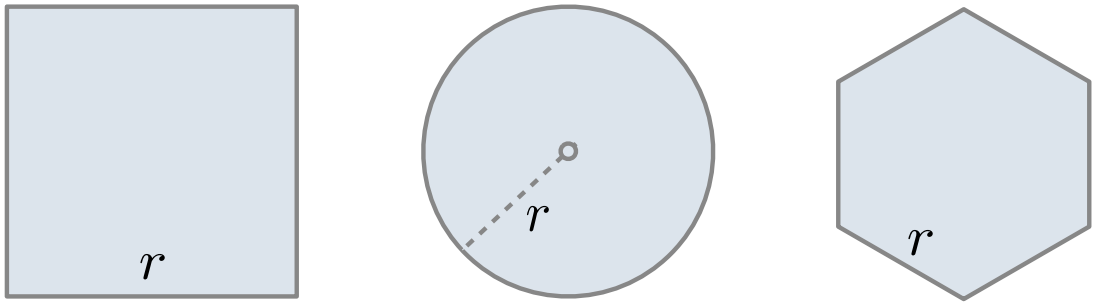

Area:

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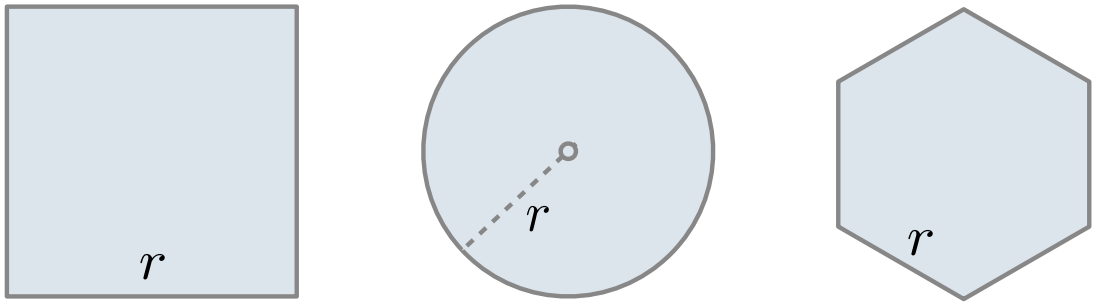

Area: r^2

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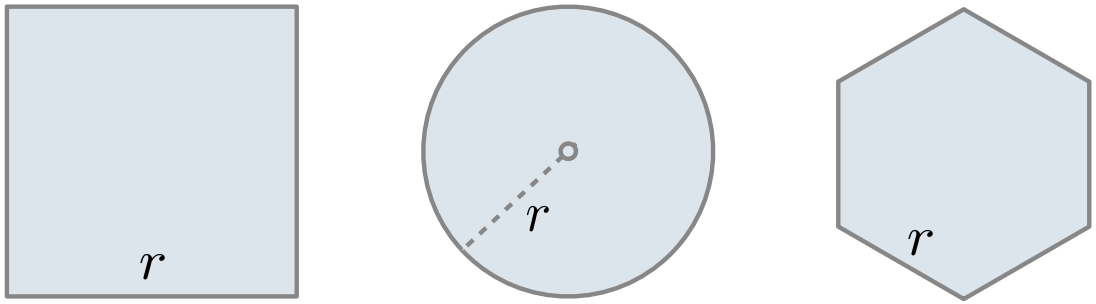

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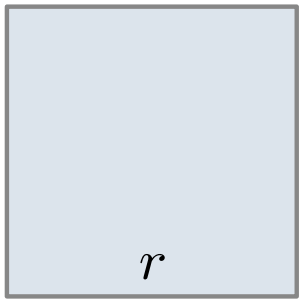
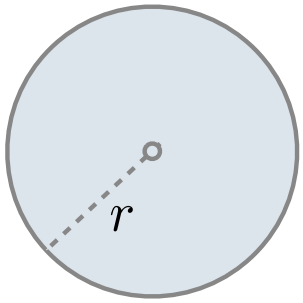
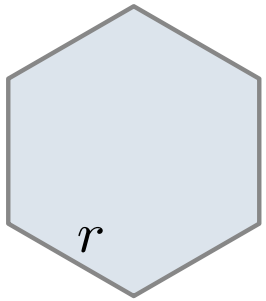
Shape:			
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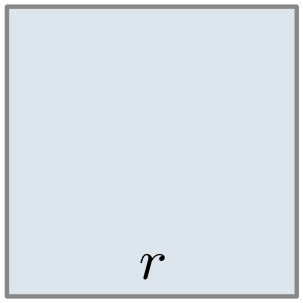
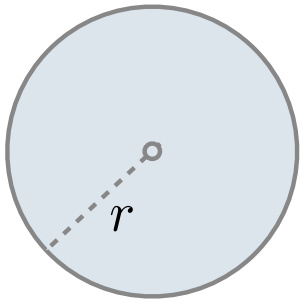
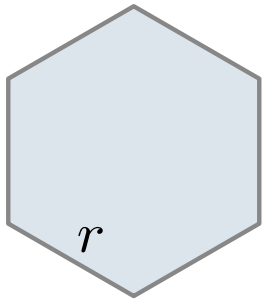
Shape:			
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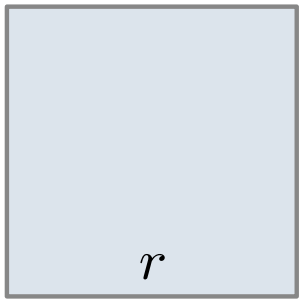
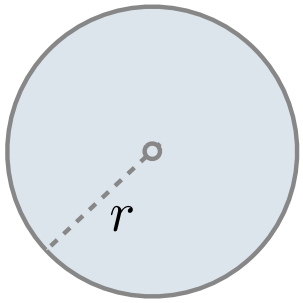
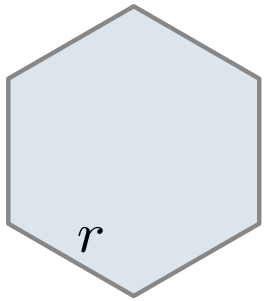
Shape:			
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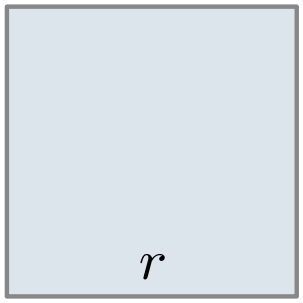
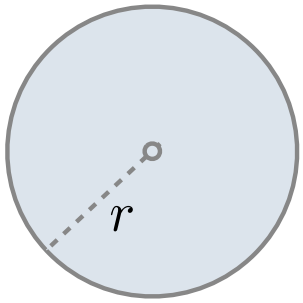
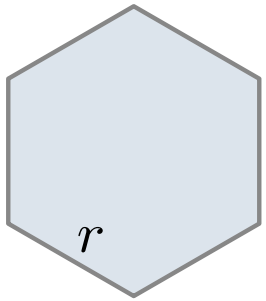
Shape:			
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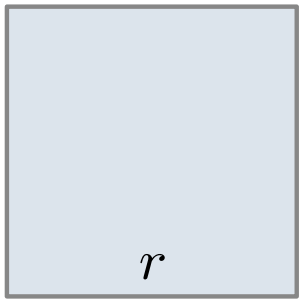

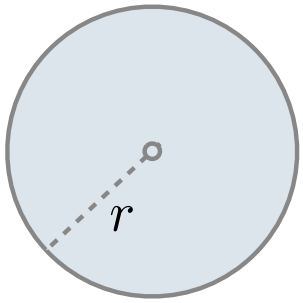

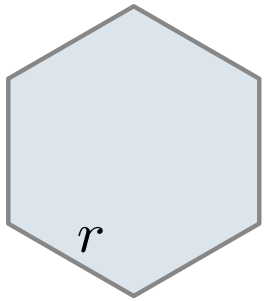

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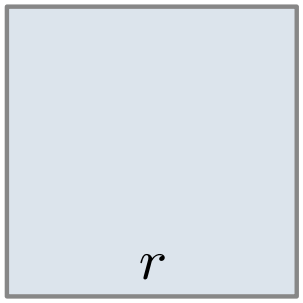

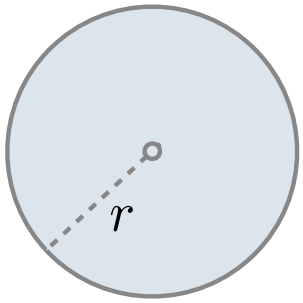

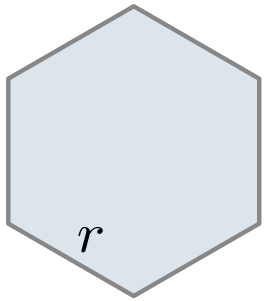

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Finding common structure allows for shared implementation

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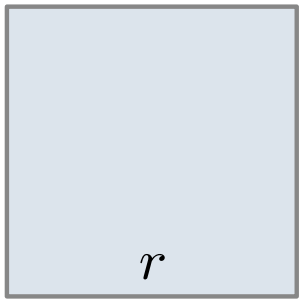

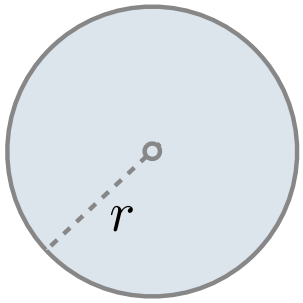

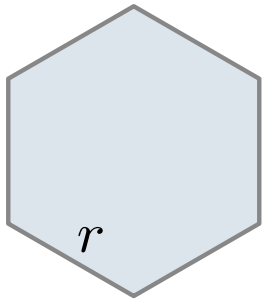

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(Demo)

Higher-Order Functions

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$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

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(Demo)

Summation Example

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def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
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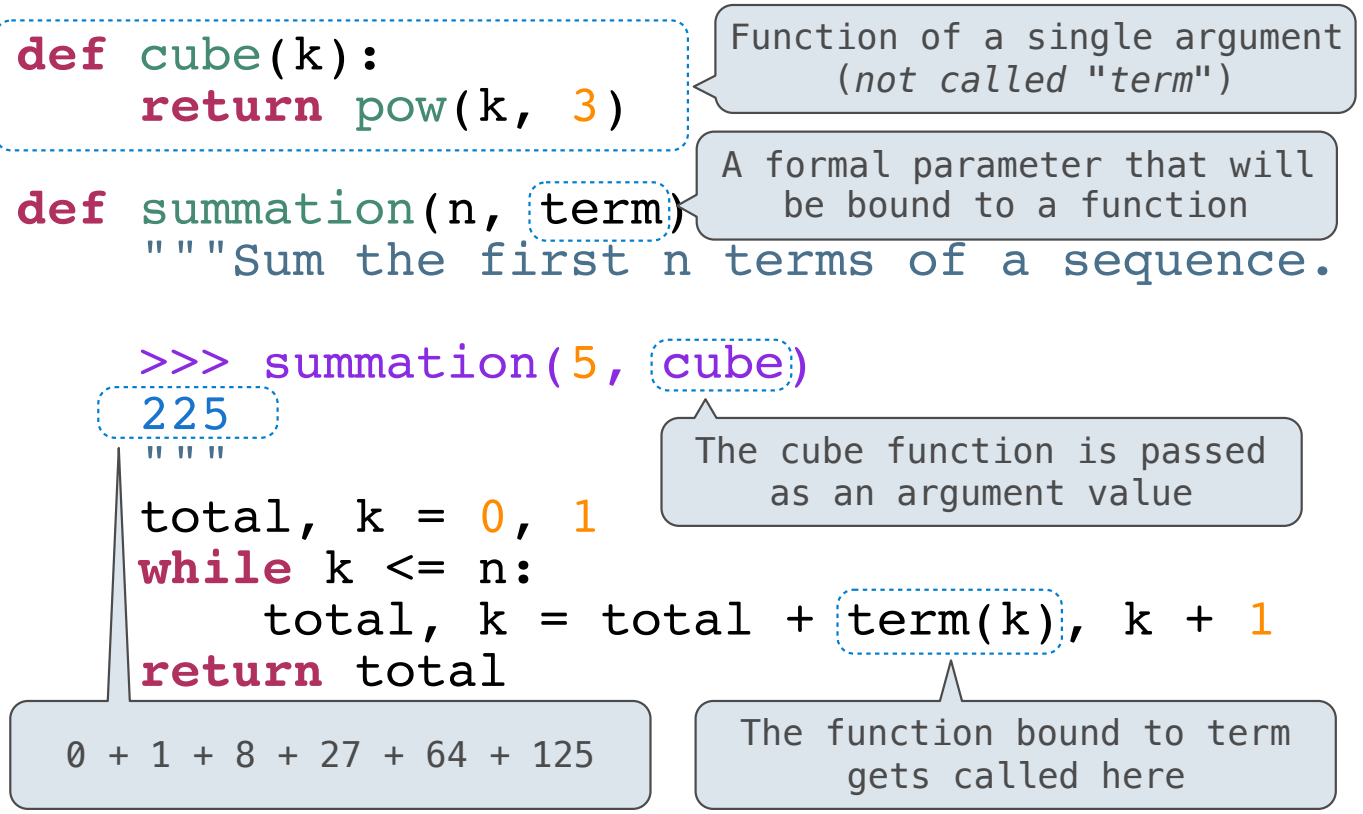

Functions as Return Values

(Demo)

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A def statement within another def statement

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    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

The name add_three is bound to a function

A def statement within another def statement

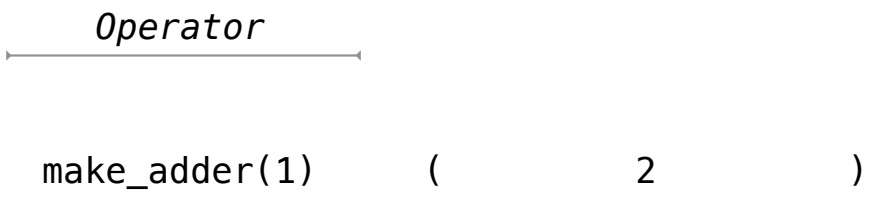
Can refer to names in the enclosing function

Call Expressions as Operator Expressions

Call Expressions as Operator Expressions

```
make_adder(1)     (         2         )
```

Call Expressions as Operator Expressions



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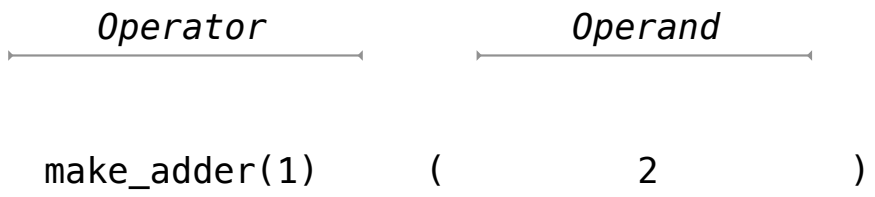

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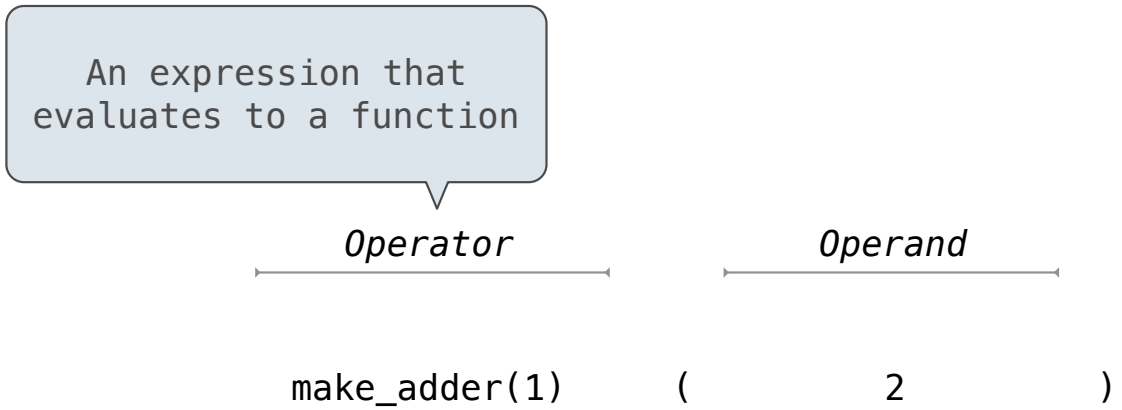

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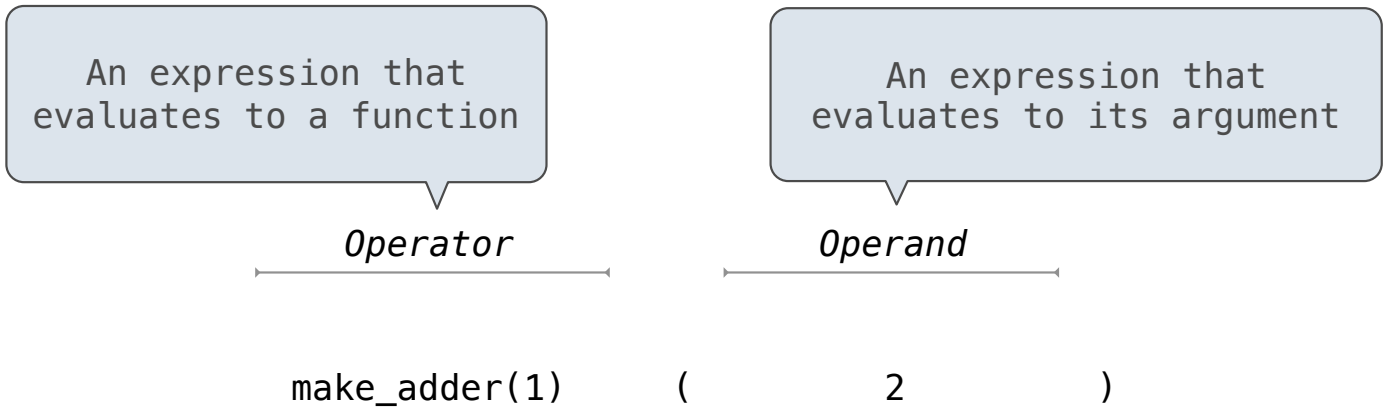

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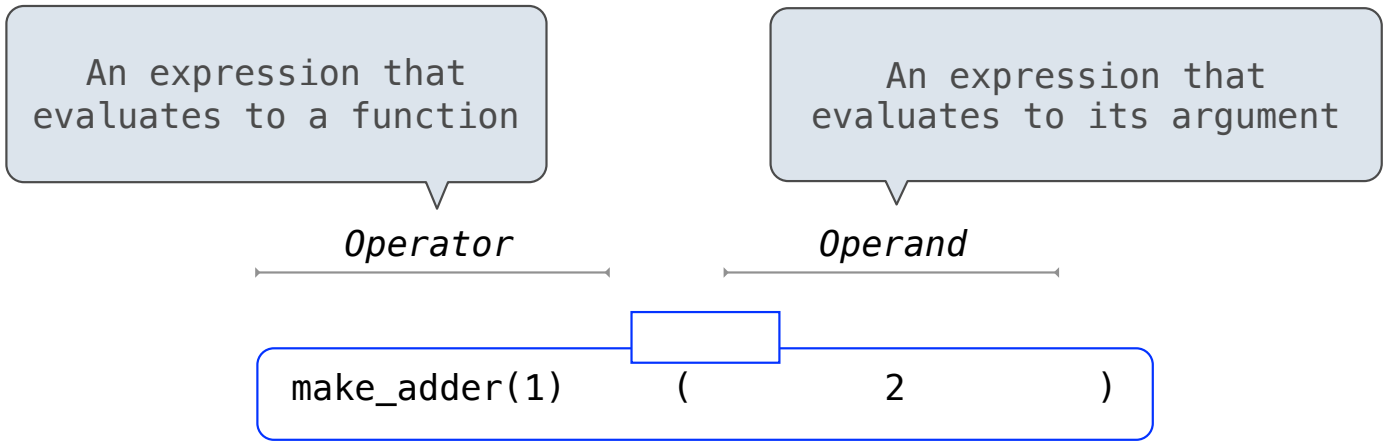

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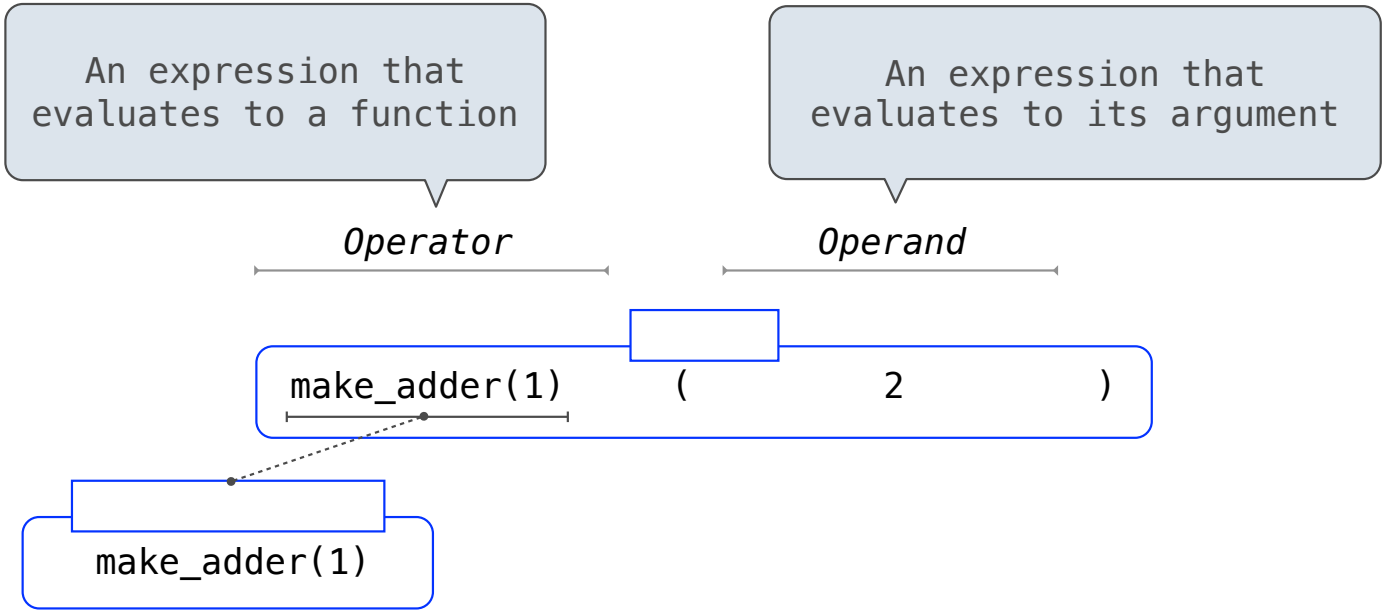

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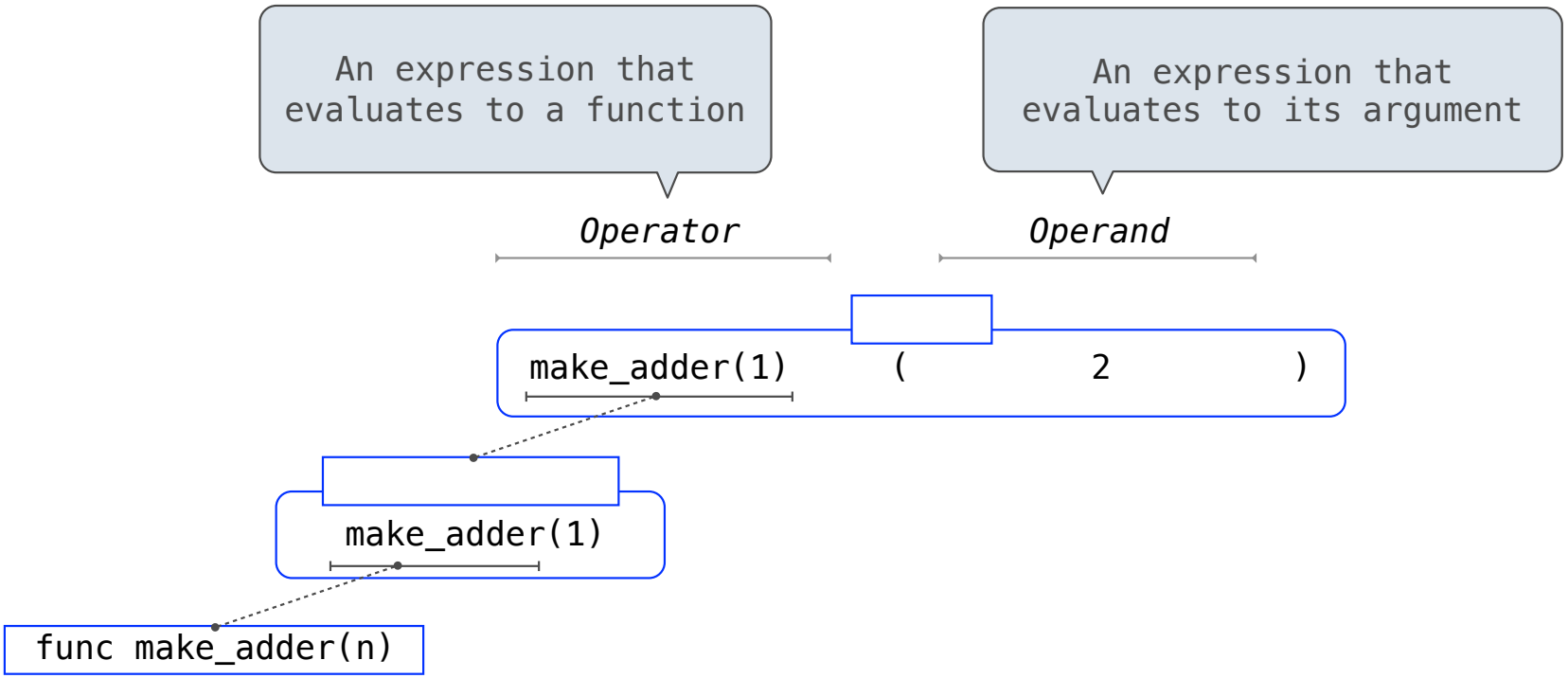

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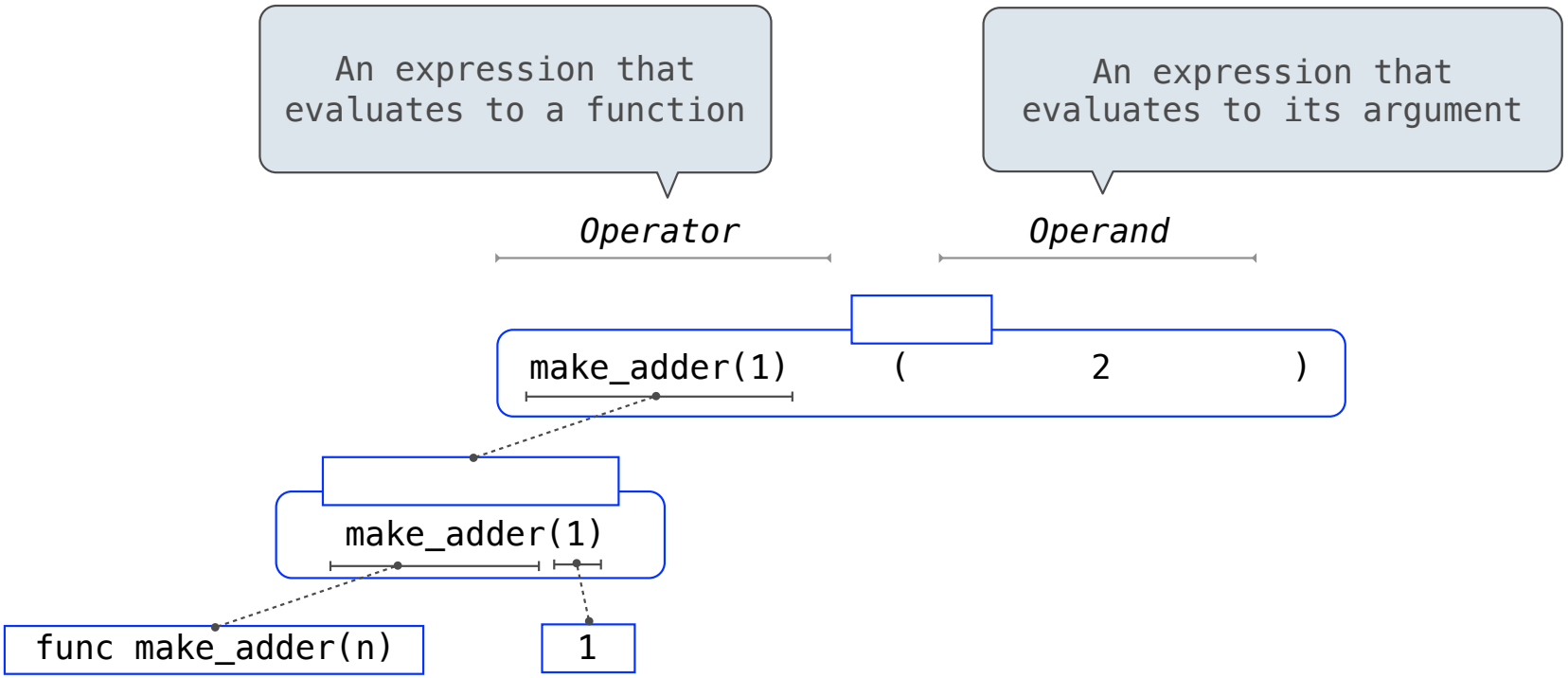

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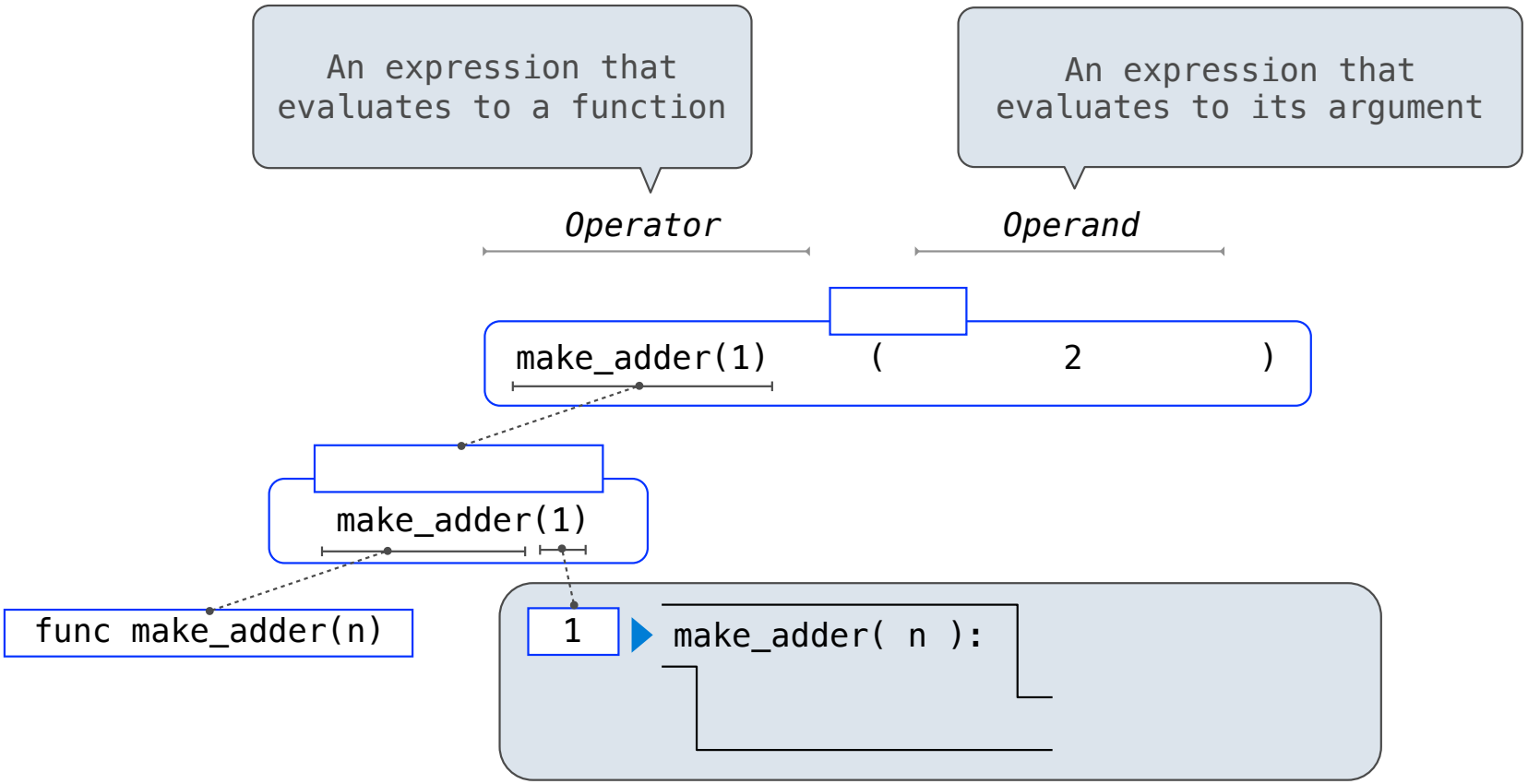

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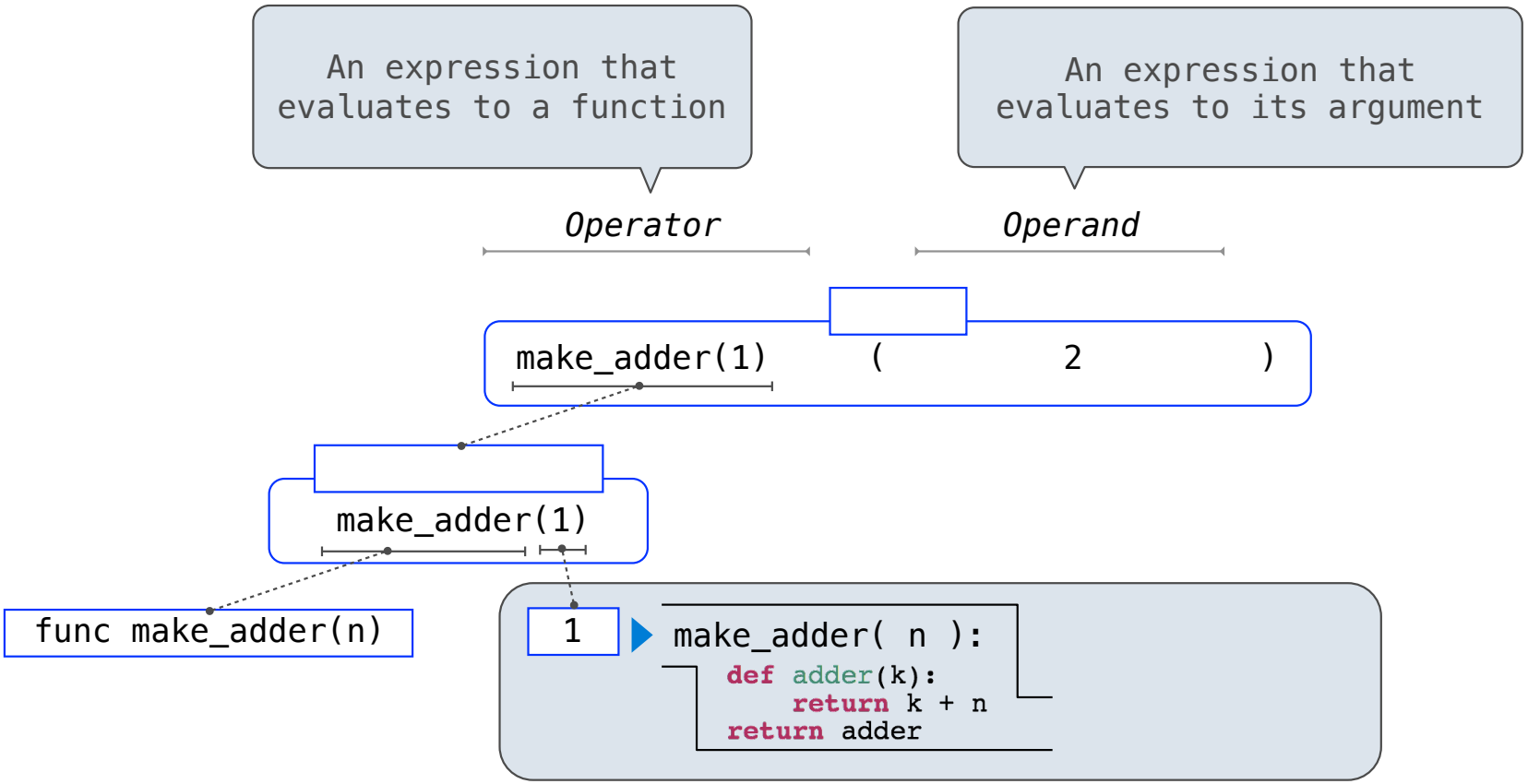

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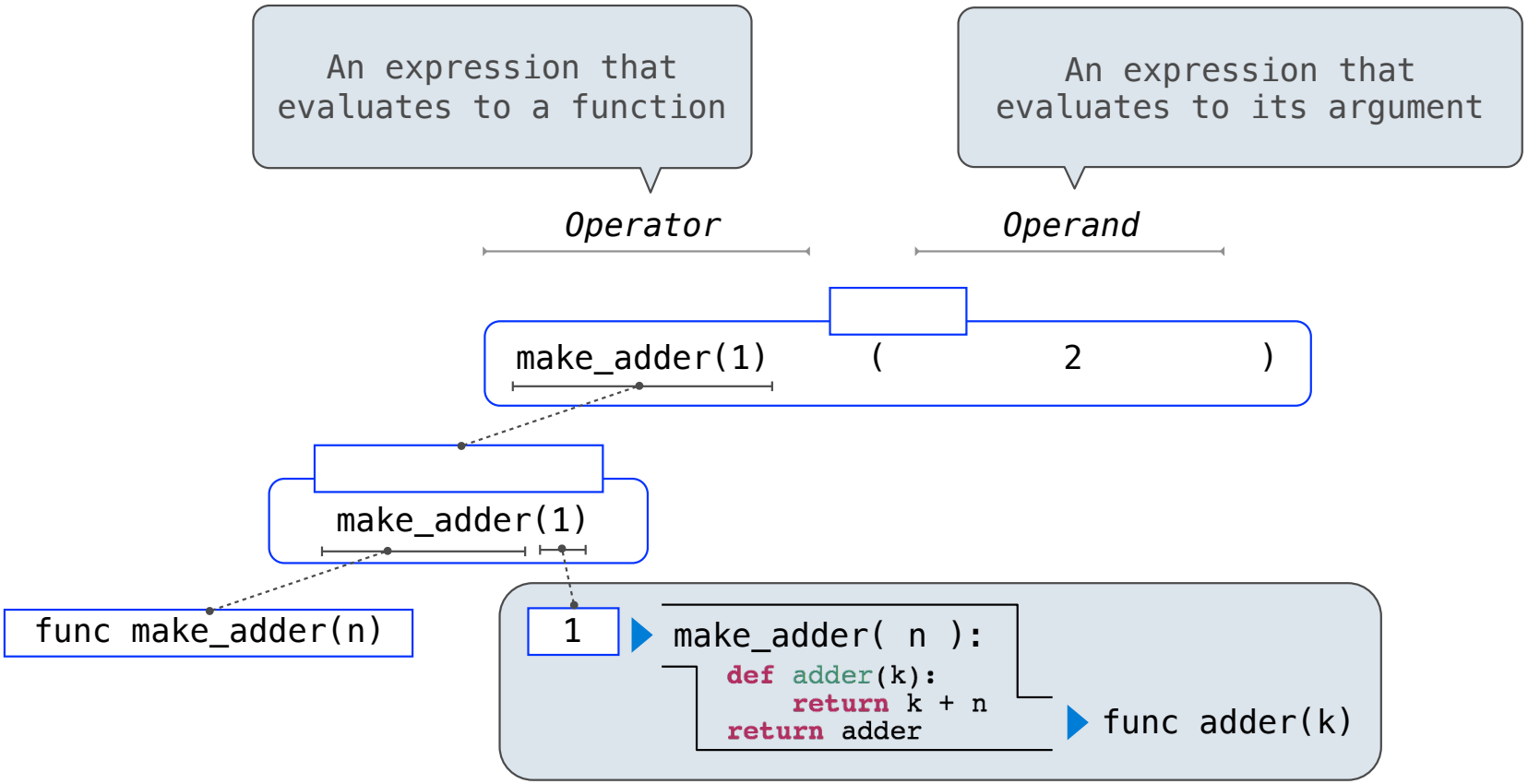

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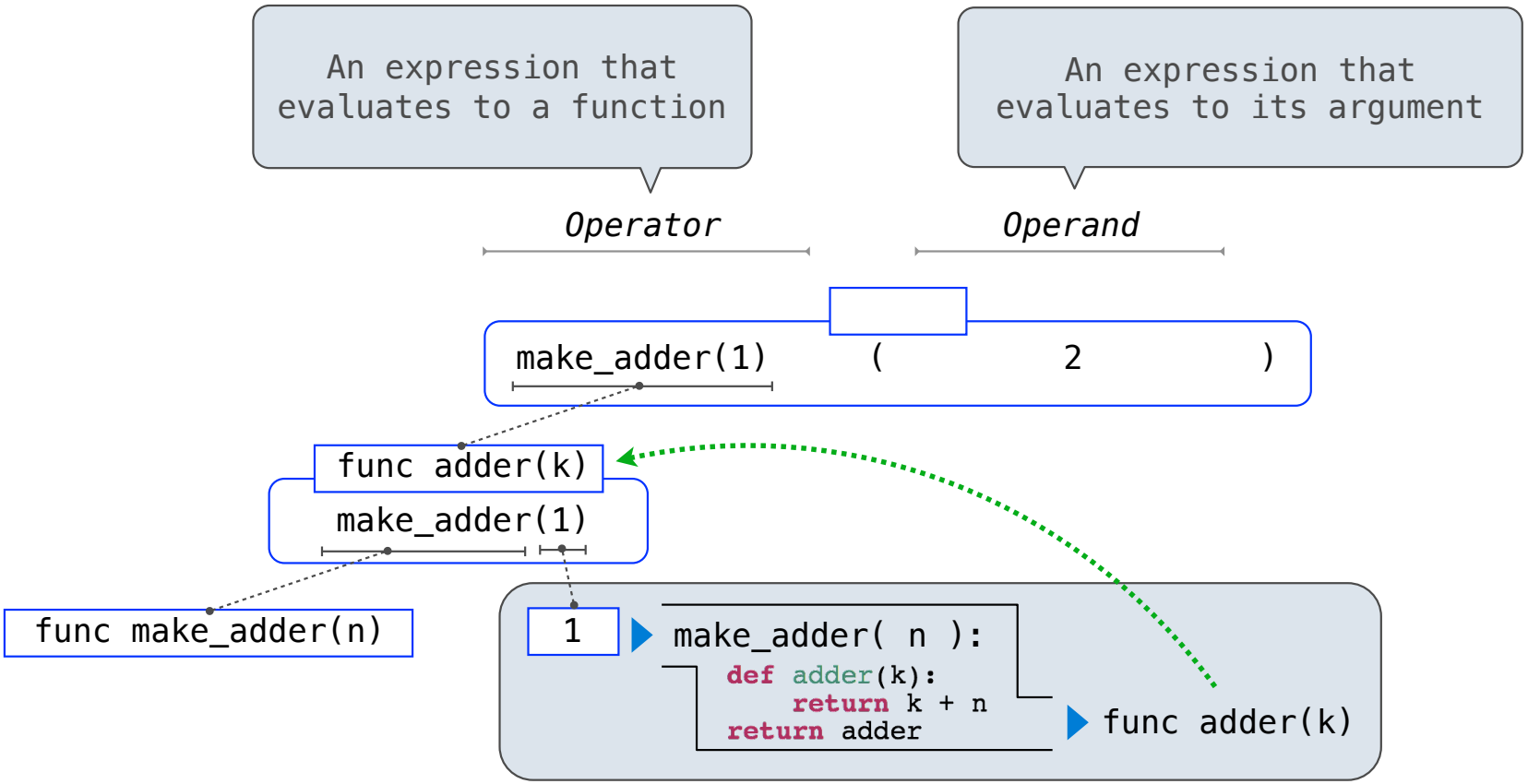

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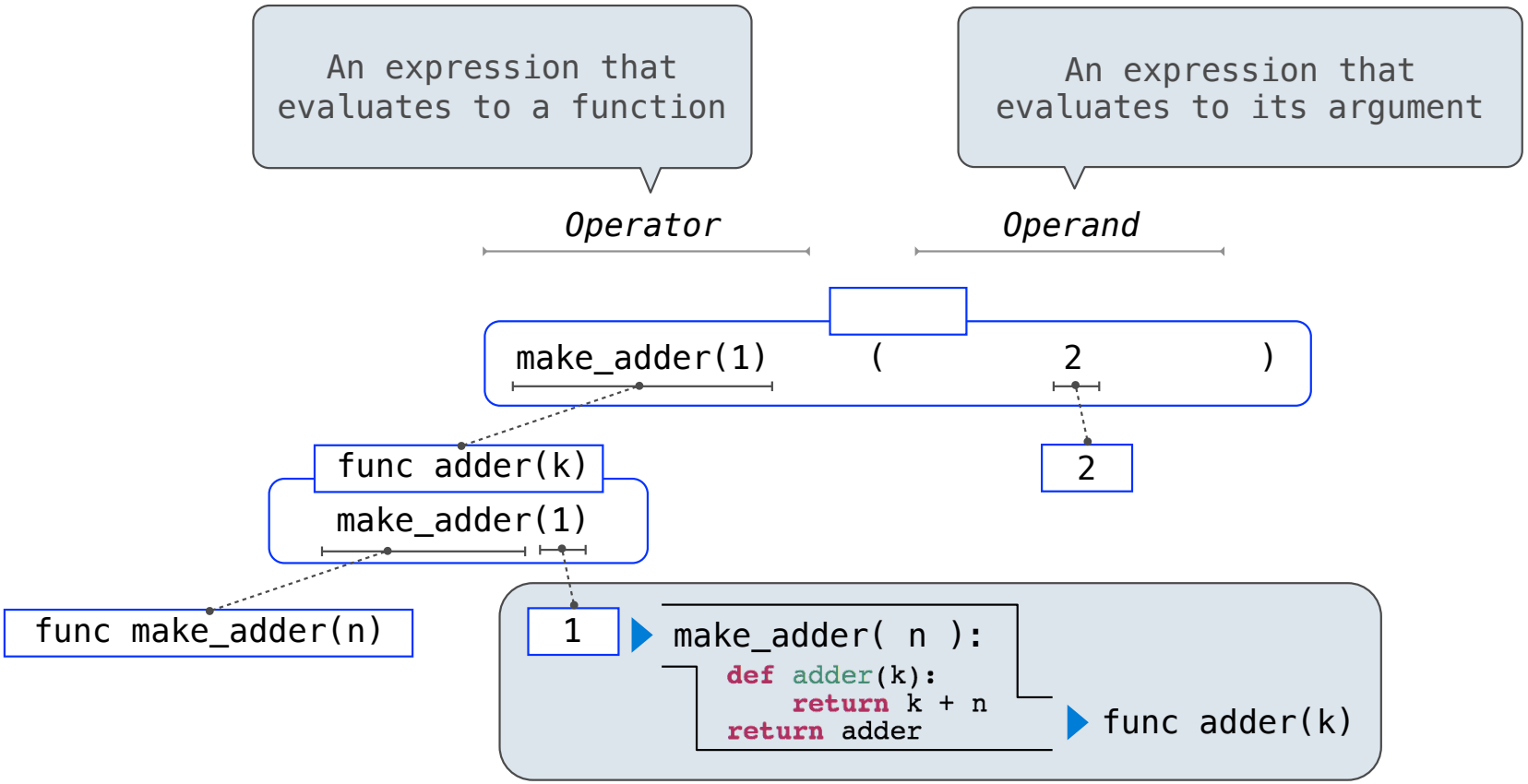

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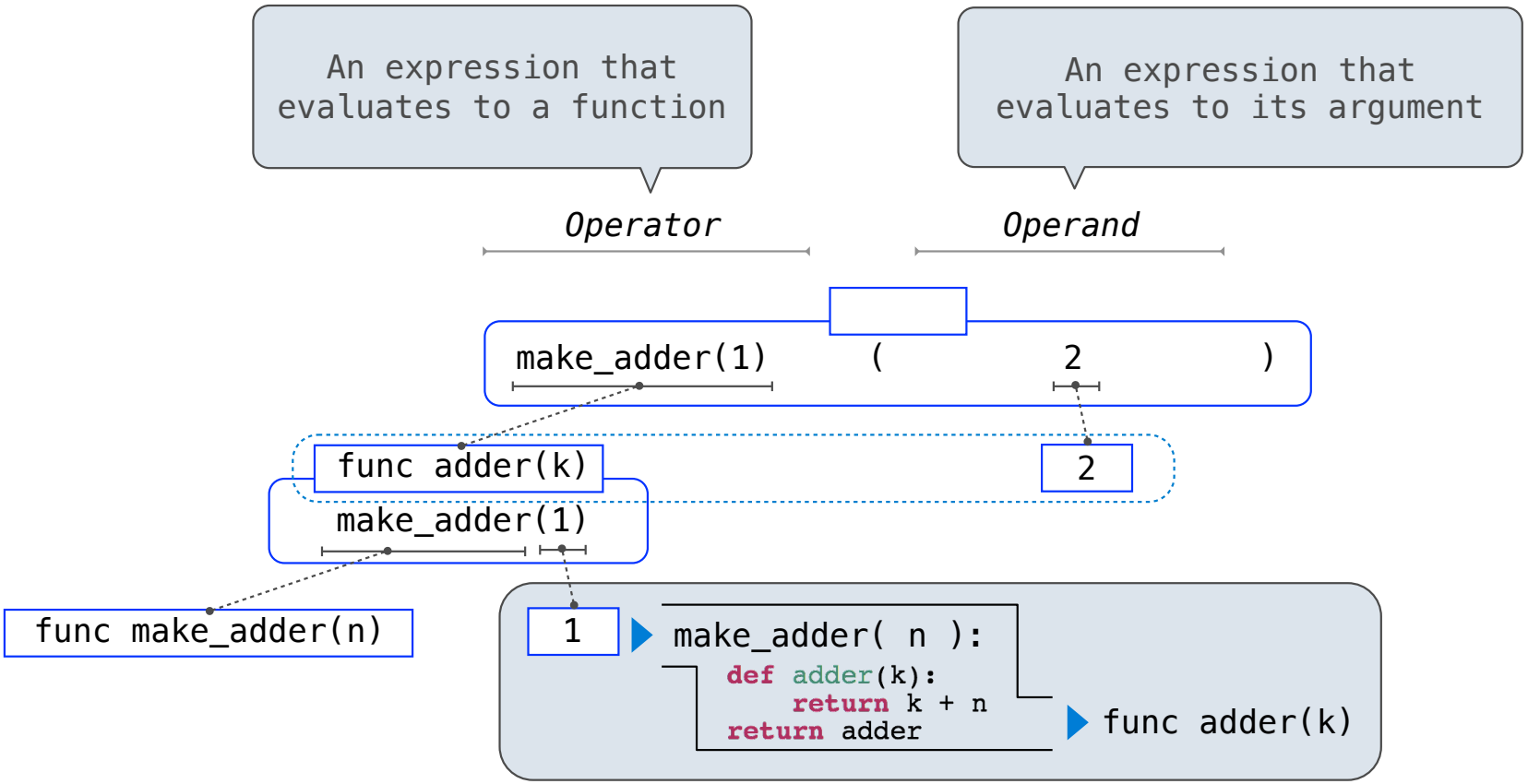

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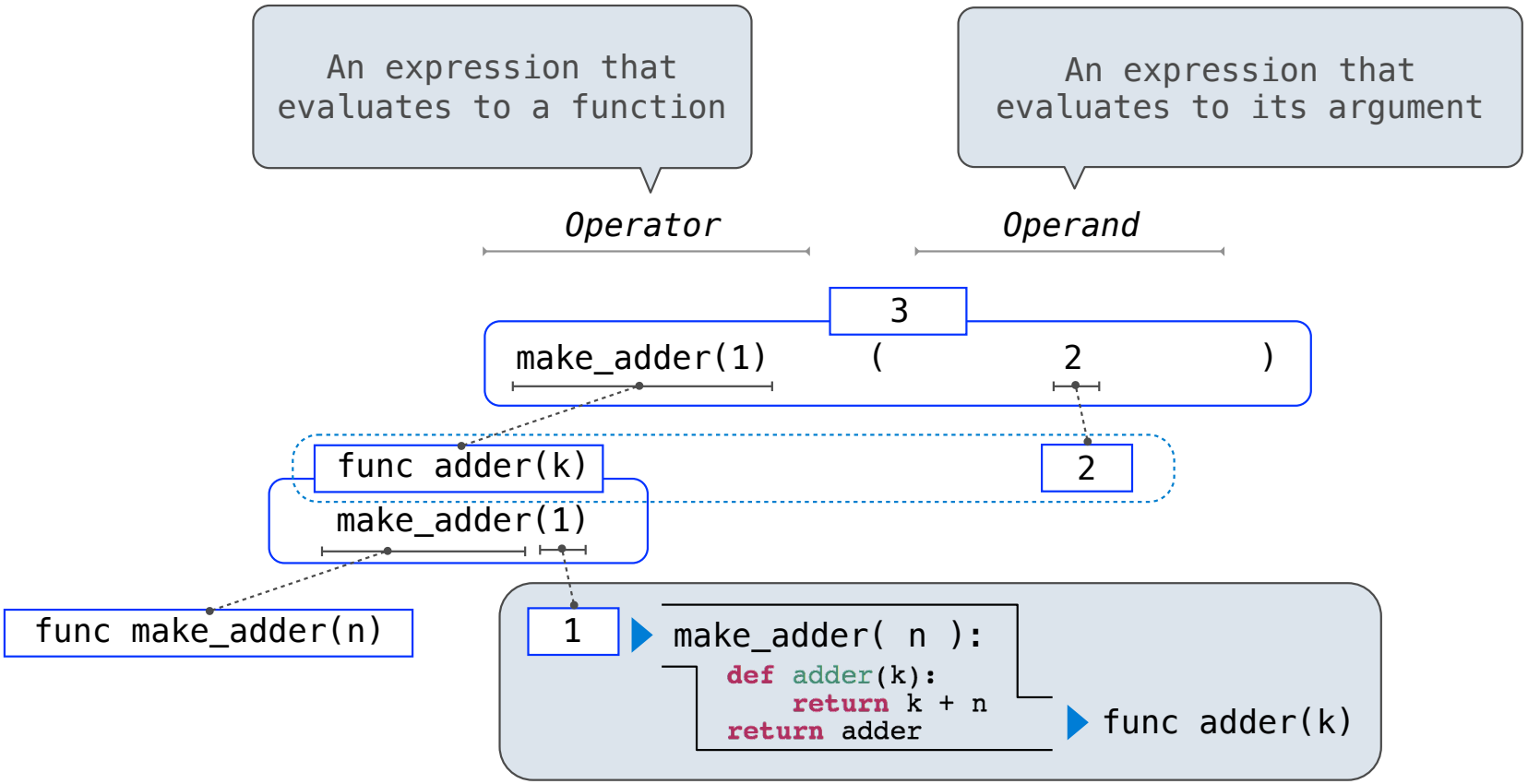

Lambda Expressions

(Demo)

Lambda Expressions

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>>> x = 10
```

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```
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A function

with formal parameter x

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Important: No "return" keyword!

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Must be a single expression

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Lambda expressions are not common in Python, but important in general

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Lambda expressions are not common in Python, but important in general

Lambda expressions in Python cannot contain statements at all!

Lambda Expressions Versus Def Statements

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VS

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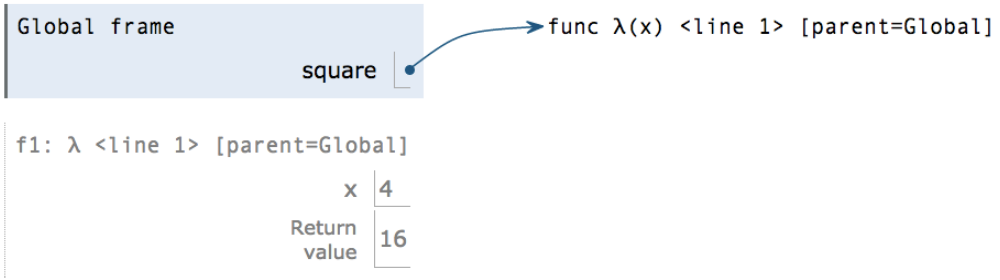

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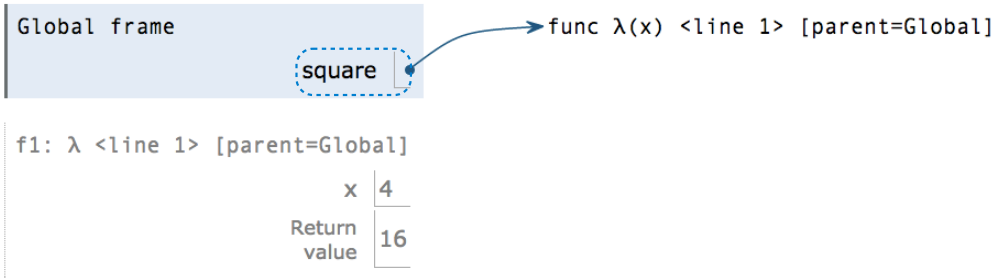

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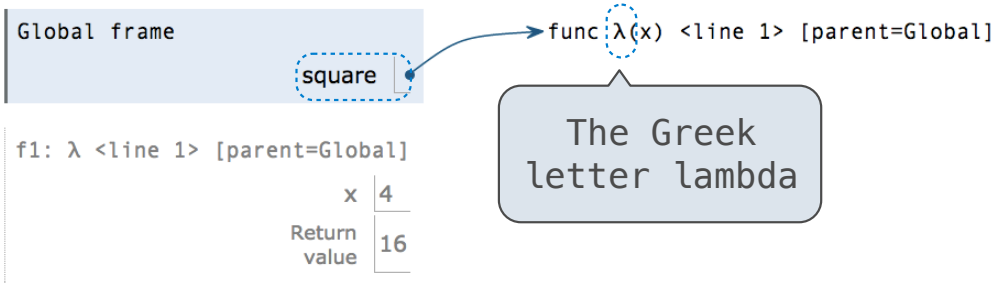

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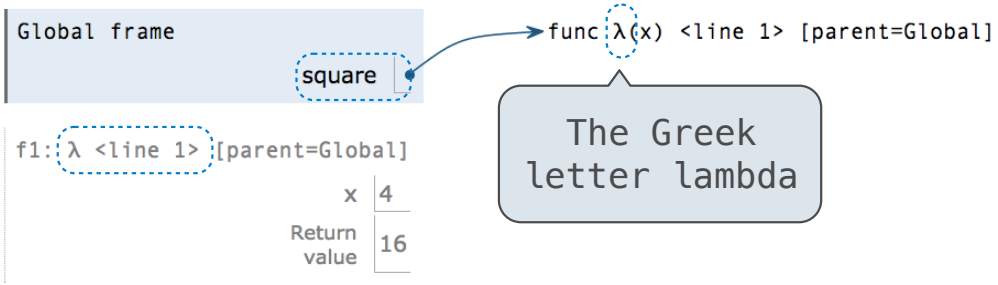

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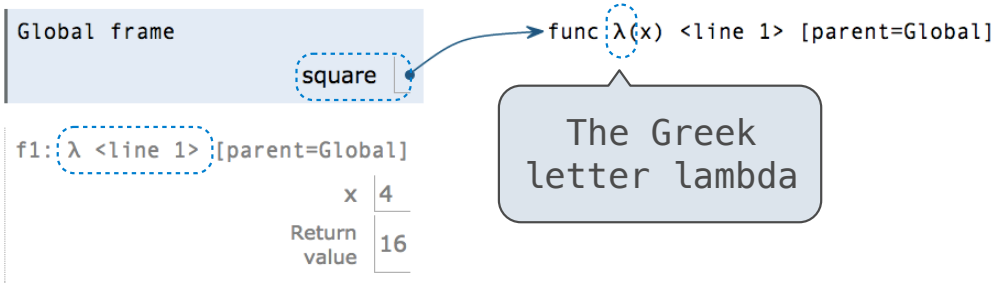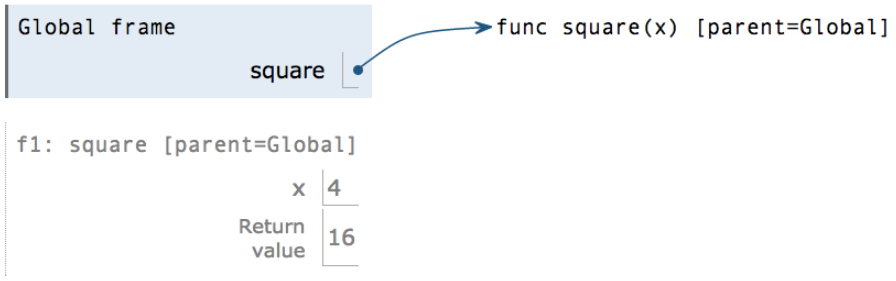
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