

Rational implementation using functions:

```
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
        elif name == 'd':
            return d
    return select

def numer(x):
    return x('n')

def denom(x):
    return x('d')
```

This function represents a rational number

Constructor is a higher-order function

Selector calls x

Lists:

```
>>> digits = [1, 8, 2, 8]
>>> len(digits)
4
>>> digits[3]
8
>>> [2, 7] + digits * 2
[2, 7, 1, 8, 2, 8, 1, 8, 2, 8]
>>> pairs = [[10, 20], [30, 40]]
>>> pairs[1]
[30, 40]
>>> pairs[1][0]
30
```

digits

pairs

Executing a for statement:

```
for <name> in <expression>:
    <suite>
```

- Evaluate the header `<expression>`, which must yield an iterable value (a list, tuple, iterator, etc.)
- For each element in that sequence, in order:
 - Bind `<name>` to that element in the current frame
 - Execute the `<suite>`

Unpacking in a for statement:

```
>>> pairs=[[1, 2], [2, 2], [3, 2], [4, 4]]
>>> same_count = 0
for x, y in pairs:
    if x == y:
        same_count = same_count + 1
>>> same_count
2
```

A sequence of fixed-length sequences

A name for each element in a fixed-length sequence

```
..., -3, -2, -1, 0, 1, 2, 3, 4, ...
range(-2, 2)
Length: ending value - starting value
Element selection: starting value + index
>>> list(range(-2, 2))
[-2, -1, 0, 1]
>>> list(range(4))
[0, 1, 2, 3]
```

List constructor

Range with a 0 starting value

Membership:

```
>>> digits = [1, 8, 2, 8]
>>> 2 in digits
True
>>> 1828 not in digits
True
```

Slicing:

```
>>> digits[0:2]
[1, 8]
>>> digits[1:]
[8, 2, 8]
```

Slicing creates a new object

Identity:

```
<exp0> is <exp1>
evaluates to True if both <exp0> and <exp1> evaluate to the same object
```

Equality:

```
<exp0> == <exp1>
evaluates to True if both <exp0> and <exp1> evaluate to equal values
Identical objects are always equal values
```

```
iter(iterable):
    Return an iterator over the elements of an iterable value
next(iterator):
    Return the next element
>>> s = [3, 4, 5]
>>> t = iter(s)
>>> next(t)
3
>>> next(t)
4
>>> d = {'one': 1, 'two': 2, 'three': 3}
>>> k = iter(d)
>>> next(k)
'one'
>>> next(k)
'two'
```

A generator function is a function that yields values instead of returning them.

```
>>> def plus_minus(x):
...     yield x
...     yield -x
>>> t = plus_minus(3)
>>> next(t)
3
>>> next(t)
-3
>>> def a_then_b(a, b):
...     yield from a
...     yield from b
>>> list(a_then_b([3, 4], [5, 6]))
[3, 4, 5, 6]
```

List comprehensions:

```
[<map exp> for <name> in <iter exp> if <filter exp>]
Short version: [<map exp> for <name> in <iter exp>]
```

A combined expression that evaluates to a list using this evaluation procedure:

- Add a new frame with the current frame as its parent
- Create an empty result list that is the value of the expression
- For each element in the iterable value of `<iter exp>`:
 - Bind `<name>` to that element in the new frame from step 1
 - If `<filter exp>` evaluates to a true value, then add the value of `<map exp>` to the result list

```
>>> repr(12e12)
'12000000000000.0'
>>> print(repr(12e12))
12000000000000.0
>>> today = datetime.date(2019, 10, 13)
>>> print(today)
2019-10-13
str and repr are both polymorphic; they apply to any object
repr invokes a zero-argument method __repr__ on its argument
>>> today.__repr__()
'datetime.date(2019, 10, 13)'
>>> today.__str__()
'2019-10-13'
```

Type dispatching: Look up a cross-type implementation of an operation based on the types of its arguments

Type coercion: Look up a function for converting one type to another, then apply a type-specific implementation.

Functions that aggregate iterable arguments

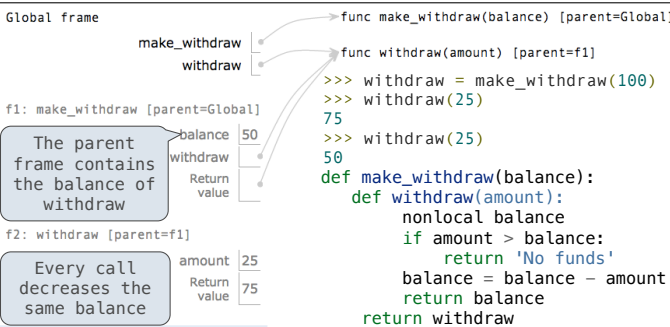
- `sum(iterable[, start])` -> value *sum of all values*
- `max(iterable[, key=func])` -> value *largest value*
- `max(a, b, c, ...[, key=func])` -> value
- `min(iterable[, key=func])` -> value *smallest value*
- `min(a, b, c, ...[, key=func])` -> value
- `all(iterable)` -> bool *whether all are true*
- `any(iterable)` -> bool *whether any is true*

Many built-in Python sequence operations return iterators that compute results lazily

- `map(func, iterable)`: Iterate over `func(x)` for `x` in `iterable`
- `filter(func, iterable)`: Iterate over `x` in `iterable` if `func(x)`
- `zip(first_iter, second_iter)`: Iterate over co-indexed `(x, y)` pairs
- `reversed(sequence)`: Iterate over `x` in a sequence in reverse order
- `list(iterable)`: Create a list containing all `x` in `iterable`
- `tuple(iterable)`: Create a tuple containing all `x` in `iterable`
- `sorted(iterable)`: Create a sorted list containing `x` in `iterable`

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
>>> cascade(123)
123
12
1
12
123
123
```

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
n: 0, 1, 2, 3, 4, 5, 6, 7, 8
fib(n): 0, 1, 1, 2, 3, 5, 8, 13, 21
```



List & dictionary mutation:

```
>>> a = [10]
>>> b = a
>>> a == b
True
>>> a.append(20)
>>> a == b
True
>>> a
[10, 20]
>>> b
[10, 20]
>>> a = [10]
>>> b = [10]
>>> a == b
True
>>> a.append(20)
>>> a
[10]
>>> b
[10, 20]
>>> a == b
False
```

```
>>> nums = {'I': 1.0, 'V': 5, 'X': 10}
>>> nums['X']
10
>>> nums['I'] = 1
>>> nums['L'] = 50
>>> nums
{'X': 10, 'L': 50, 'V': 5, 'I': 1}
>>> sum(nums.values())
66
>>> dict([(3, 9), (4, 16), (5, 25)])
{3: 9, 4: 16, 5: 25}
>>> nums.get('A', 0)
0
>>> nums.get('V', 0)
5
>>> {x: x*x for x in range(3,6)}
{3: 9, 4: 16, 5: 25}
```

```
>>> sum([1, 2])
3
>>> sum([1, 2], 3)
6
>>> sum([])
0
>>> all([False, True])
False
>>> all([])
True
>>> any([False, True])
True
>>> any([])
False
>>> max(1, 2)
2
>>> max([1, 2])
2
>>> max([1, -2], key=abs)
-2
```

You can copy a list by calling the list constructor or slicing the list from the beginning to the end.

```
>>> suits = ['coin', 'string', 'myriad']
>>> suits.pop()
'myriad'
>>> suits.remove('string')
>>> suits.append('cup')
>>> suits.extend(['sword', 'club'])
>>> suits[2] = 'spade'
>>> suits
['coin', 'cup', 'spade', 'club']
>>> suits[0:2] = ['diamond']
>>> suits
['diamond', 'spade', 'club']
>>> suits.insert(0, 'heart')
>>> suits
['heart', 'diamond', 'spade', 'club']
```

Remove and return the last element

Remove a value

Add all values

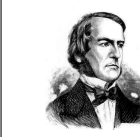
Replace a slice with values

Add an element at an index

False values:

- Zero
- False
- None
- An empty string, list, dict, tuple

```
>>> bool(0)
False
>>> bool(1)
True
>>> bool('')
False
>>> bool('0')
True
>>> bool([])
False
>>> bool([[]])
True
>>> bool({})
False
>>> bool(())
False
>>> bool(lambda x: 0)
True
```



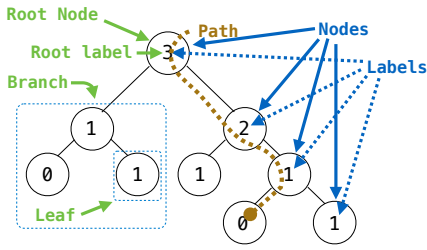
Status	Effect
•No nonlocal statement	Create a new binding from name "x" to number 2 in the first frame of the current environment
•"x" is not bound locally	
•No nonlocal statement	Re-bind name "x" to object 2 in the first frame of the current environment
•"x" is bound locally	
•nonlocal x	Re-bind "x" to 2 in the first non-local frame of the current environment in which "x" is bound
•"x" is bound in a non-local frame	
•nonlocal x	SyntaxError: no binding for nonlocal 'x' found in a non-local frame
•nonlocal x	
•"x" is bound in a non-local frame	SyntaxError: name 'x' is parameter and nonlocal
•"x" also bound locally	

Recursive description:

- A tree has a root label and a list of branches
- Each branch is a tree
- A tree with zero branches is called a leaf

Relative description:

- Each location is a node
- Each node has a label
- One node can be the parent/child of another



```
def tree(label, branches=[]):
```

Verifies the tree definition

```
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)
```

```
def label(tree):
```

Creates a list from a sequence of branches

```
    return tree[0]
```

```
def branches(tree):
```

Verifies that tree is bound to a list

```
    return tree[1:]
```

```
def is_tree(tree):
```

```
    if type(tree) != list or len(tree) < 1:
        return False
```

```
    for branch in branches(tree):
```

```
        if not is_tree(branch):
```

```
            return False
```

```
    return True
```

```
def is_leaf(tree):
```

```
    return not branches(tree)
```

```
def leaves(t):
```

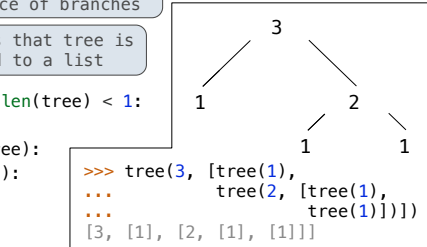
```
    """The leaf values in t.
    >>> leaves(fib_tree(5))
    [1, 0, 1, 0, 1, 1, 0, 1]
    """
```

```
    if is_leaf(t):
```

```
        return [label(t)]
```

```
    else:
```

```
        return sum([leaves(b) for b in branches(t)], [])
```



```
def fib_tree(n):
```

```
    if n == 0 or n == 1:
```

```
        return tree(n)
```

```
    else:
```

```
        left = fib_tree(n-2),
```

```
        right = fib_tree(n-1)
```

```
        fib_n = label(left) + label(right)
```

```
        return tree(fib_n, [left, right])
```

```
class Tree:
```

```
    def __init__(self, label, branches=[]):
```

```
        self.label = label
```

```
        for branch in branches:
```

```
            assert isinstance(branch, Tree)
```

```
        self.branches = list(branches)
```

```
    def is_leaf(self):
```

```
        return not self.branches
```

```
    def leaves(self):
```

```
        """The leaf values in a tree."
        if tree.is_leaf():
```

```
            return [tree.label]
```

```
        else:
```

```
            return sum([leaves(b) for b in tree.branches], [])
```

```
class Link:
```

```
    empty = ()
```

Some zero length sequence

```
    def __init__(self, first, rest=empty):
```

```
        assert rest is Link.empty or isinstance(rest, Link)
```

```
        self.first = first
```

```
        self.rest = rest
```

```
    def __repr__(self):
```

```
        if self.rest:
```

```
            rest = ', ' + repr(self.rest)
```

```
        else:
```

```
            rest = ''
```

```
        return 'Link(' + repr(self.first) + rest + ')'
```

```
    def __str__(self):
```

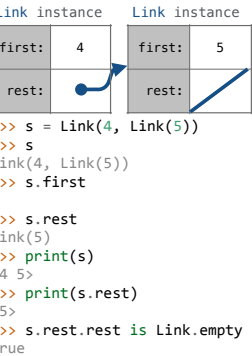
```
        string = '<'
```

```
        while self.rest is not Link.empty:
```

```
            string += str(self.first) + '> , <'
```

```
            self = self.rest
```

```
        return string + str(self.first) + '>'
```



Anatomy of a recursive function:

- The def statement header is like any function
- Conditional statements check for base cases
- Base cases are evaluated without recursive calls
- Recursive cases are evaluated with recursive calls

```
def sum_digits(n):
    """Sum the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = n // 10, n % 10
        return sum_digits(all_but_last) + last
```

Recursive decomposition: finding simpler instances of a problem.

- E.g., count_partitions(6, 4)
- Explore two possibilities:
 - Use at least one 4
 - Don't use any 4
- Solve two simpler problems:
 - count_partitions(2, 4)
 - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Python object system:

Idea: All bank accounts have a balance and an account holder; the Account class should add those attributes to each of its instances

```
>>> a = Account('Jim')
>>> a.holder
'Jim'
>>> a.balance
0
```

An account instance
balance: 0 holder: 'Jim'

When a class is called:

1. A new instance of that class is created:
2. The `__init__` method of the class is called with the new object as its first argument (named `self`), along with any additional arguments provided in the call expression.

```
class Account:
    def __init__(self, account_holder):
        self.balance = 0
        self.holder = account_holder
    def deposit(self, amount):
        self.balance = self.balance + amount
        return self.balance
    def withdraw(self, amount):
        if amount > self.balance:
            return 'Insufficient funds'
        self.balance = self.balance - amount
        return self.balance
```

`__init__` is called a constructor

`self` should always be bound to an instance of the Account class or a subclass of Account

Function call: all arguments within parentheses

Method invocation: One object before the dot and other arguments within parentheses

```
>>> type(Account.deposit)
<class 'function'>
>>> type(a.deposit)
<class 'method'>

>>> Account.deposit(a, 5)
10
>>> a.deposit(2)
12
```

Call expression

Dot expression



The `<expression>` can be any valid Python expression. The `<name>` must be a simple name. Evaluates to the value of the attribute looked up by `<name>` in the object that is the value of the `<expression>`.

- To evaluate a dot expression:
1. Evaluate the `<expression>` to the left of the dot, which yields the object of the dot expression
 2. `<name>` is matched against the instance attributes of that object; if an attribute with that name exists, its value is returned
 3. If not, `<name>` is looked up in the class, which yields a class attribute value
 4. That value is returned unless it is a function, in which case a bound method is returned instead

Assignment statements with a dot expression on their left-hand side affect attributes for the object of that dot expression

- If the object is an instance, then assignment sets an instance attribute
- If the object is a class, then assignment sets a class attribute

```
Account class attributes: interest: 0.02 0.04 0.05 (withdraw, deposit, __init__)

Instance attributes of jim_account: balance: 0 holder: 'Jim' interest: 0.08
Instance attributes of tom_account: balance: 0 holder: 'Tom'

>>> jim_account = Account('Jim')
>>> tom_account = Account('Tom')
>>> tom_account.interest
0.02
>>> jim_account.interest
0.02
>>> Account.interest = 0.04
>>> tom_account.interest
0.04
>>> jim_account.interest
0.04
>>> Account.interest = 0.05
>>> tom_account.interest
0.05
>>> jim_account.interest
0.08
```

```
class CheckingAccount(Account):
    """A bank account that charges for withdrawals."""
    withdraw_fee = 1
    interest = 0.01
    def withdraw(self, amount):
        return Account.withdraw(self, amount + self.withdraw_fee)
        or
        return super().withdraw(amount + self.withdraw_fee)
```

To look up a name in a class:

1. If it names an attribute in the class, return the attribute value.
2. Otherwise, look up the name in the base class, if there is one.

```
>>> ch = CheckingAccount('Tom') # Calls Account.__init__
>>> ch.interest # Found in CheckingAccount
0.01
>>> ch.deposit(20) # Found in Account
20
>>> ch.withdraw(5) # Found in CheckingAccount
14
```