Lecture #4: Higher-Order Functions

Announcements

- Pair-programming demo (Pamela Fox & Patricia Ouyang).
- Homework 1 due Thursday.
- Project 1 (Hog) release today.
- Nine new tutorials added:
  - 2 on Wed. @4PM
  - 1 on Thu. @7AM
  - 1 on Thu. @8AM
  - 3 on Thu. @11AM
  - 2 on Thu. @12PM
- "Lost" sections starting Friday at 12–2PM and 4–6PM. See Piazza @239.
- Ask questions on the Piazza thread for today's lecture (@245).

Comments on Functions in General: Terminology

- The set of possible argument values of a function is known as its *domain*.
- The set of values that the function can return (all values that result from inputting some value from its domain) is called its *range*.
- The *codomain* of a function is a set of values that includes the range, and possibly other values.
- Thus, we might say that the square function has the real numbers as its domain, and the non-negative numbers as its range. We can choose to describe its codomain as the real numbers or as just the non-negative real numbers.

Documenting Functions

- Ideally, a *documentation comment* for a function provides enough information so that a programmer can use the function properly and understand what it does *without* having to read its body.
- It should make clear what inputs are valid or under what conditions the function may be called. This is the *precondition*.
- Likewise, it should make clear what the resulting output or effect of the function will be for correct inputs. This is the *postcondition*.
- Together, these are the *behavior* or *semantics* (meaning) of the function.

Two Design Principles

- Functions should do one well-defined thing (a complicated documentation comment might suggest your function does too much).
- *DRY* (Don't Repeat Yourself).
  - Multiple segments of code that look really similar to each other cry out for *refactoring*...
  - That is, for replacing the segments with simple calls to a single general function that states their shared structure just once, with parameters used to specialize to the various cases.

Functions As Templates

- If we think of a function body as a template for a computation, parameters are “blanks” in that template.
- For example:

```
def sum_squares(N):
    """Returns the sum of x**2 for all integers x with 1 <= x <= N."""
    k = 1
    sum = 0
    while k <= N:
        sum += k**2
        k += 1
    return sum
```

  is a template for an infinite set of computations that add squares of numbers up to 0, 1, 2, 3, . . . , in place of the N.

- But the `sum_squares` function is specialized to the summing k^2.
- A function for summing k^3, $\sin k$, or $1/k$ would have the same structure, differing only in what comes after `sum +=`.
- How do we practice DRY here?

Functions on Functions

- Function parameters allow us to have templates with slots for *computations*:

```
def summation(N, term):
    k = 1
    sum = 0
    while k <= N:
        sum += term(k)
        k += 1
    return sum
```

- Generalizes **sum_squares**. We can write `sum_squares(5)` as:

```
def square(x):
    return x*x
summation(5, square)
```

- or (if we don't really need a "square" function elsewhere), we can create the function argument anonymously on the fly:

```
summation(5, lambda x: x*x)
```

Quick Review of Lambda

- In Python, **lambda** is just an abbreviation.
- Writing lambda *PARAMS: EXPRESSION* is the same as writing NEWNAME, where NEWNAME is a name that appears nowhere else in the program and is defined by

  ```
  def NEWNAME(PARAMS):
      return EXPRESSION
  ```

  evaluated in the same environment in which the original **lambda** was.
- There is no return: the body must be a single expression.
- Now we can write any number of summations succinctly:

  ```
  summation(10, lambda x: x**3)       # Sum of cubes
  summation(10, lambda x: 1 / x)      # Harmonic series
  summation(10, lambda k: x**(k-1) / factorial(k-1))
                                      # Approximate e**x
  ```

Functions that Produce Functions

- Functions are *first-class values,* meaning that we can assign them to variables, pass them to functions, and return them from functions.
- Example: let's generalize the class of functions that—like

  ```
  def h(x):  return sin(x) + cos(x)
  ```

  —add the results of applying two functions to the same argument:

  ```
  >>> def add_func(f, g):
  ...     """Return function that returns F(x)+G(x) for argument x."""
  ...     def adder(x):             #
  ...         return f(x) + g(x)  # or return lambda x: f(x) + g(x)
  ...     return adder            #

  >>> from math import sin, cos, pi
  >>> h = add_func(sin, cos)
  >>> sin(pi/4) + cos(pi/4)
  1.414213562373095
  >>> h(pi / 4)
  1.414213562373095
  ```

Generalize!

- Let's make a general function-combining function (that goes beyond addition):

```
>>> def combine_funcs(op):
...     """combine_funcs(OP)(f, g)(x) = OP(f(x), g(x))."""
...     def combined(f, g):
...         def val(x):
...             return op(f(x), g(x))
...         return val
...     return combined
```

- Now `add_func` itself can be constructed by a call to `combine_funcs`:

```
>>> from operator import add
>>> add_func = ??
>>> from math import sin, cos, pi
>>> h = add_func(sin, cos)
>>> h(pi / 4)
1.414213562373095
```

- What do the environments look like here? Think about it and try it out.

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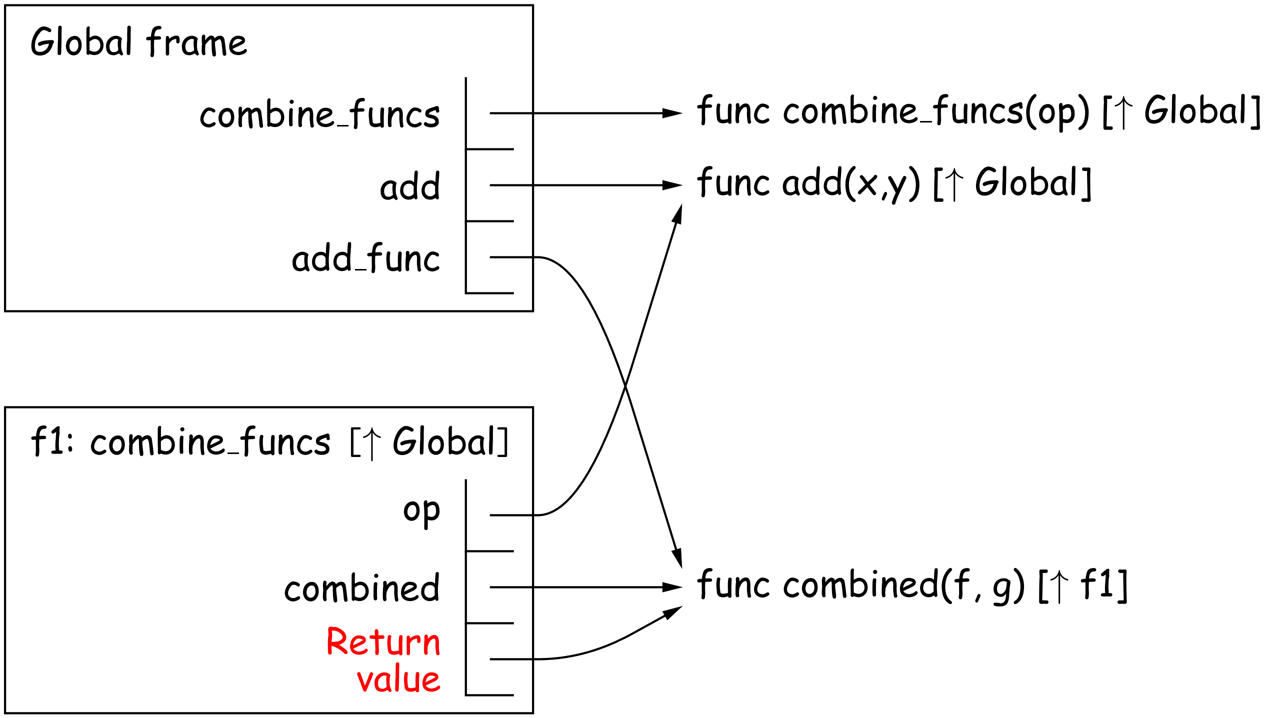
- Now `add_func` itself can be constructed by a call to `combine_funcs`:

```
>>> from operator import add
>>> add_func = combine_funcs(lambda x, y: x + y)
>>> from math import sin, cos, pi
>>> h = add_func(sin, cos)
>>> h(pi / 4)
1.414213562373095
```

- What do the environments look like here? Think about it and try it out.

The Environment Picture (I)

```
def combine_funcs(op):
  def combined(f, g):
    def val(x):
      return op(f(x), g(x))
    return val
  return combined
add_func = combine_funcs(add)
```



Global frame

combine_funcs

add

add_func

func combine_funcs(op) [↑ Global]

func add(x,y) [↑ Global]

f1: combine_funcs [↑ Global]

op

combined

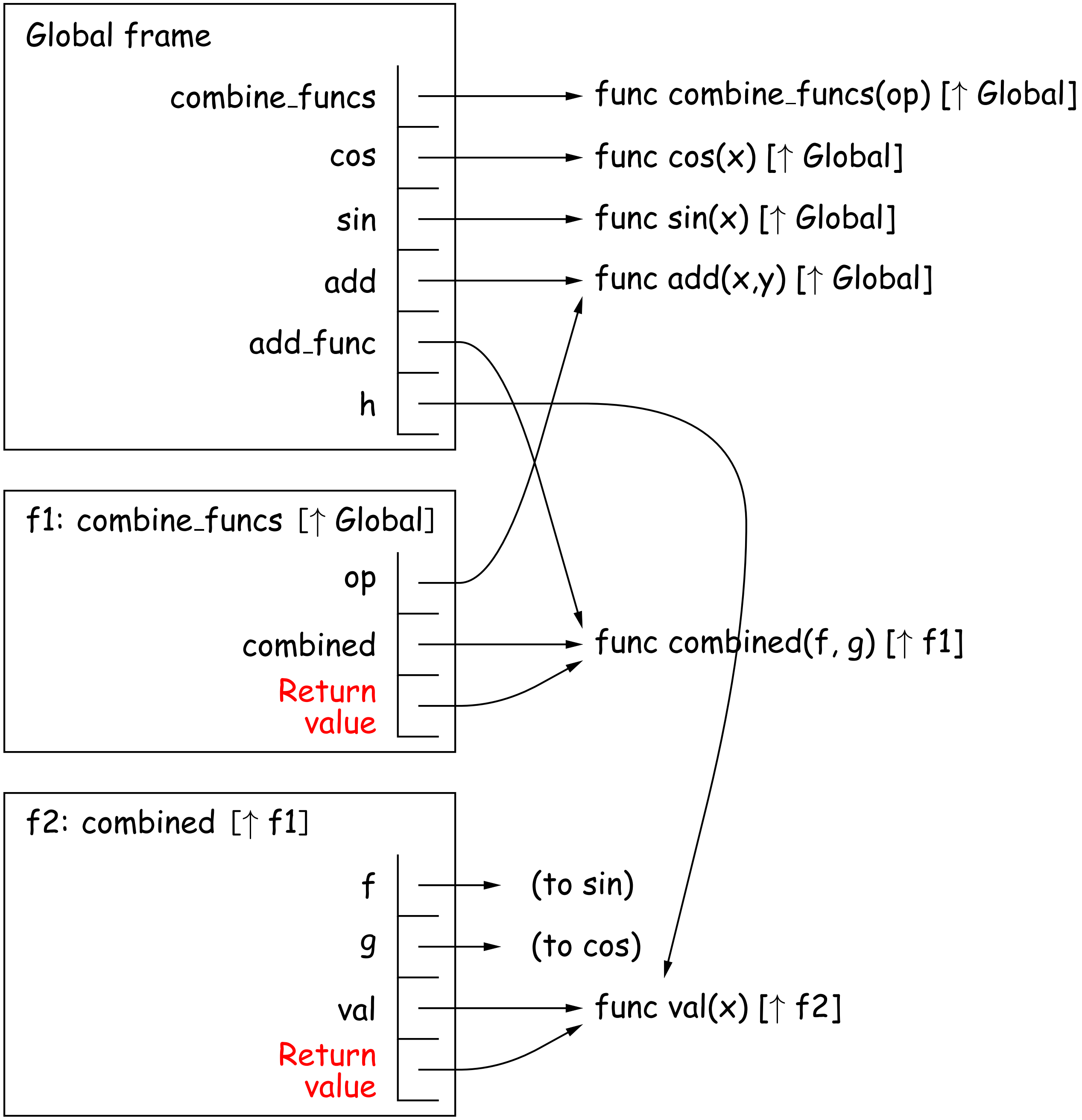
Return value

func combined(f, g) [↑ f1]

Legend: ↑ is short for "parent=".

The Environment Picture (II)

```
def combine_funcs(op):
  def combined(f, g):
    def val(x):
      return op(f(x), g(x))
    return val
  return combined
add_func = combine_funcs(add)
h = add_func(sin, cos)
```



Global frame

combine_funcs

cos

sin

add

add_func

h

func combine_funcs(op) [↑ Global]

func cos(x) [↑ Global]

func sin(x) [↑ Global]

func add(x,y) [↑ Global]

f1: combine_funcs [↑ Global]

op

combined

Return value

func combined(f, g) [↑ f1]

f2: combined [↑ f1]

f

g

val

Return value

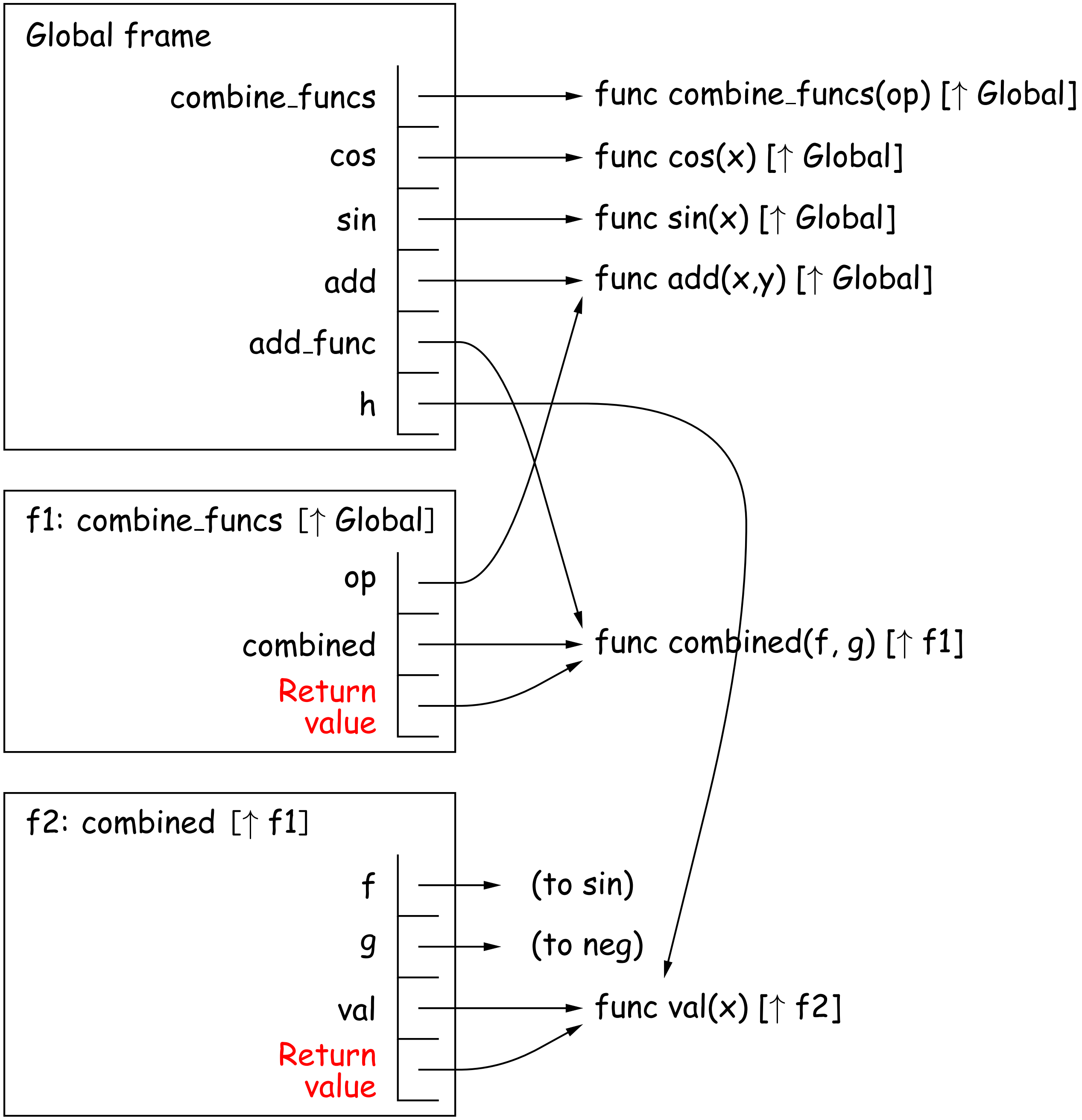
(to sin)

(to cos)

func val(x) [↑ f2]

The Environment Picture (III)

```
def combine_funcs(op):
  def combined(f, g):
    def val(x):
      return op(f(x), g(x))
    return val
  return combined
add_func = combine_funcs(add)
h = add_func(sin, cos)
h(-5)
```



Global frame

combine_funcs → func combine_funcs(op) [↑ Global]

cos → func cos(x) [↑ Global]

sin → func sin(x) [↑ Global]

add → func add(x,y) [↑ Global]

add_func

h

f1: combine_funcs [↑ Global]

op

combined → func combined(f, g) [↑ f1]

Return value

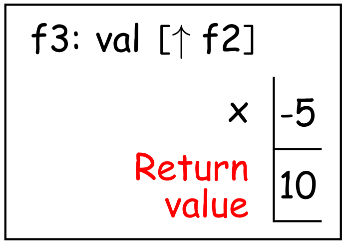
f2: combined [↑ f1]

f → (to sin)

g → (to neg)

val → func val(x) [↑ f2]

Return value



f3: val [↑ f2]

x -5

Return value 10

+ local frames for calls to

- add (value of op),
- sin (value of f), and
- cos (value of g)

Challenge: Conditional Function?

- Write a Python function, `if_func`, such that, for example

```
if_func(1/x, x > 0, 0)
```

always returns the same value as the conditional expression

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1/x if x > 0 else 0
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- But all is not lost, because we can define instead

  ```
  def if_func(then_expr, condition, else_expr):
      return then_expr() if condition else else_expr()
  ```

  and call

  ```
  if_func(lambda: 1/x, x > 0, lambda: 0)
  ```

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- Why don't we need a thunk for the condition?

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  ```

- (The jargon term for those parameterless lambdas is *thunks*.)
- Why don't we need a thunk for the condition?
  Answer: Because the condition parameter must always be evaluated first anyway.