1 Learning Goals

- Understand the general idea behind recursion
- Understand how to structure recursive functions
- Understand the general structure of counting problems and how to solve them
- Understand how to approach exam-level problems for various topics
2 Recursion Overview

2.1 What are three things you find in every recursive function?

2.2 When you write a Recursive function, you seem to call it before it has been fully defined. Why doesn’t this break the Python interpreter?

2.3 Below is a Python function that computes the $n$th Fibonacci number. Identify the three things it contains as a recursive function (from 1.1).

```python
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-1) + fib(n-2)
```

2.4 With the definition of the Fibonacci function above, draw out a diagram of the recursive calls made when `fib(4)` is called.
2.5 What does the following function \texttt{cascade2} do? What is its domain and range?

\begin{verbatim}
def cascade2(n):
    print(n)
    if n >= 10:
        cascade2(n//10)
        print(n)
\end{verbatim}

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
Implement multiadder, which takes a positive integer \( n \) and returns an order \( n \) numeric function that sums an argument sequence of length \( n \).

```python
def multiadder(n):
    """Return a function that takes N arguments, one at a time, and adds them."
    >>> f = multiadder(3)
    >>> f(5)(6)(7) # 5 + 6 + 7
    18
    >>> multiadder(1)(5)
    5
    >>> multiadder(2)(5)(6) # 5 + 6
    11
    >>> multiadder(4)(5)(6)(7)(8) # 5 + 6 + 7 + 8
    26
    """

    assert n > 0

    if _____________________________________________________________________:
        return __________________________________________________________________

    else:
        return __________________________________________________________________
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
Complete the expression below by writing one integer in each blank so that the whole expression evaluates to 2016. Assume multiadder is implemented correctly.

```python
def compose1(f, g):
    """Return the composition function which given x, computes f(g(x))."
    return lambda x: f(g(x))

>>> add_one = lambda x: x + 1  # adds one to x
>>> square = lambda x: x**2
>>> a1 = compose1(square, add_one)  # (x + 1)^2
>>> a1(4)
25
>>> mul_three = lambda x: x * 3  # multiplies 3 to x
>>> a2 = compose1(mul_three, a1)  # ((x + 1)^2) * 3
>>> a2(4)
75
>>> a2(5)
108
"""

compose1(multiadder(_______)(1000), multiadder(_______)(10)(_______))(1)(2)(3)
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.
4 Reverse Environment Diagram Practice

4.1 Fill in the lines below so that the execution of the program would lead to the environment diagram below. You may not use any numbers in any blanks.

```python
def how(f, x):
    return ________________

def bizarre(___):
    return 2 * _______

scary = ________(_______, 3)
```
4.2 Fill in the lines below so that the execution of the program would lead to the environment diagram below. You may not use any numbers in any blanks.

```python
def what(_______):
    def ______(x):
        return ______
    return ________

def who(n):
    def ________(k):
        return 2 * k + n
    return _________

y = 3
_____.___._______(4)
```