

Example: Prime Factorization

Each positive integer n has a set of prime factors: primes whose product is n

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9 = 3 * 3
10 = 2 * 5
11 = 11
12 = 2 * 2 * 3
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One approach: Find the smallest prime factor of n, then divide by it

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858

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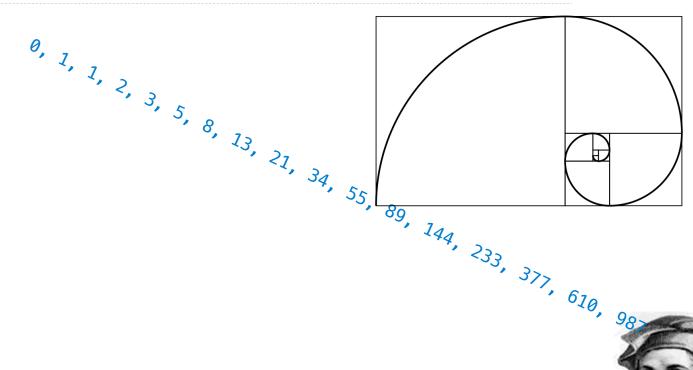
(Demo)

Example: Iteration

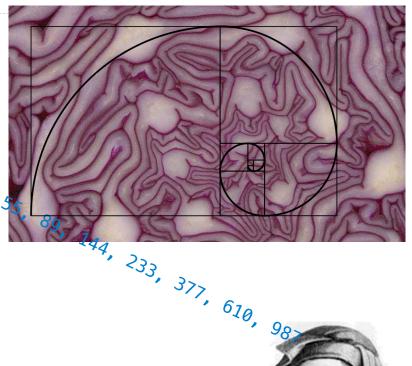


0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 98

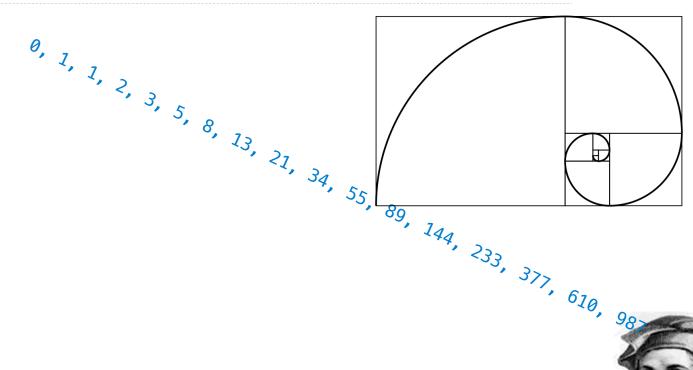




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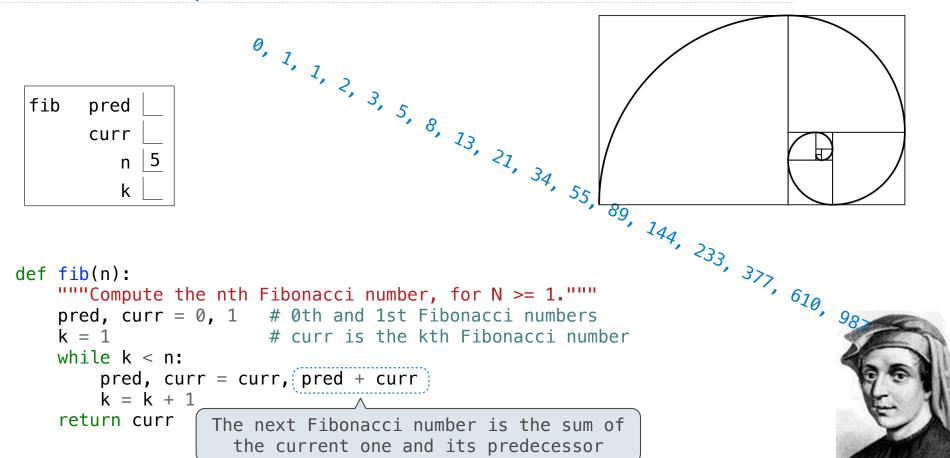


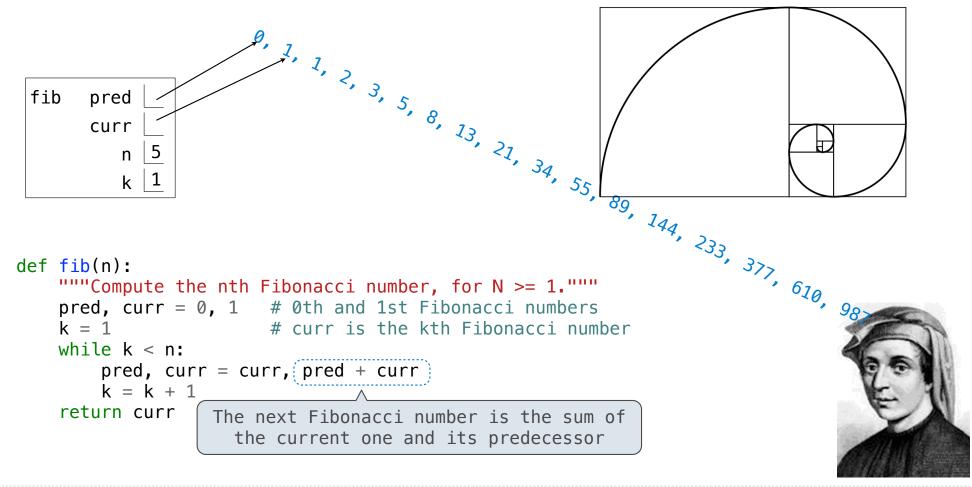


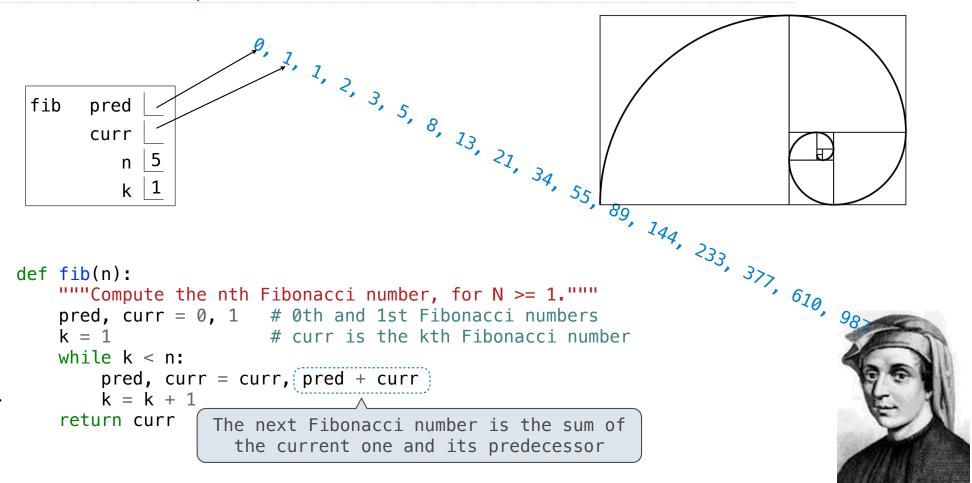


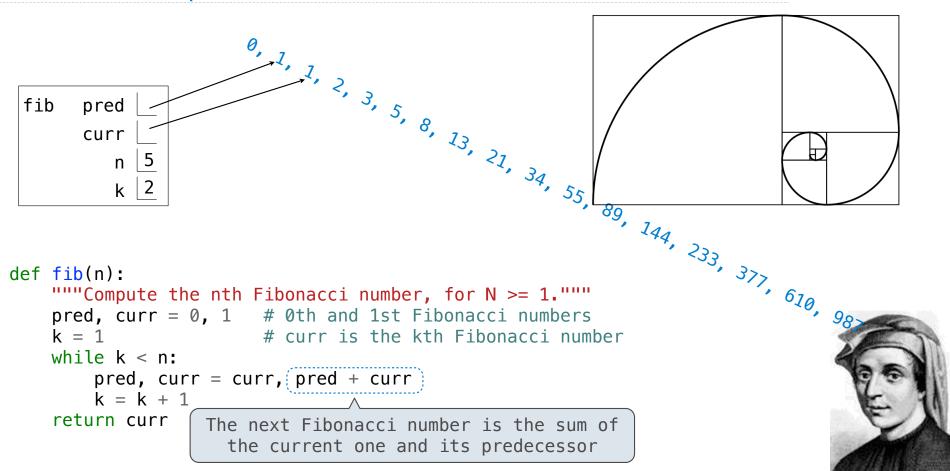
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def fib(n):
    """Compute the nth Fibonacci number, for N >= 1."""
    pred, curr = 0, 1  # 0th and 1st Fibonacci numbers
    k = 1
```

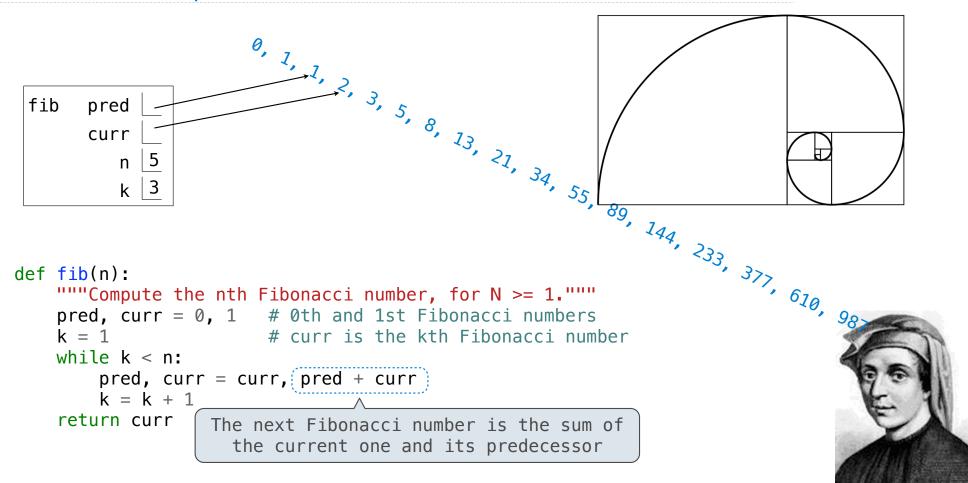
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def fib(n):
    """Compute the nth Fibonacci number, for N >= 1."""
    pred, curr = 0, 1  # 0th and 1st Fibonacci numbers
    k = 1
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
                  The next Fibonacci number is the sum of
                    the current one and its predecessor
```

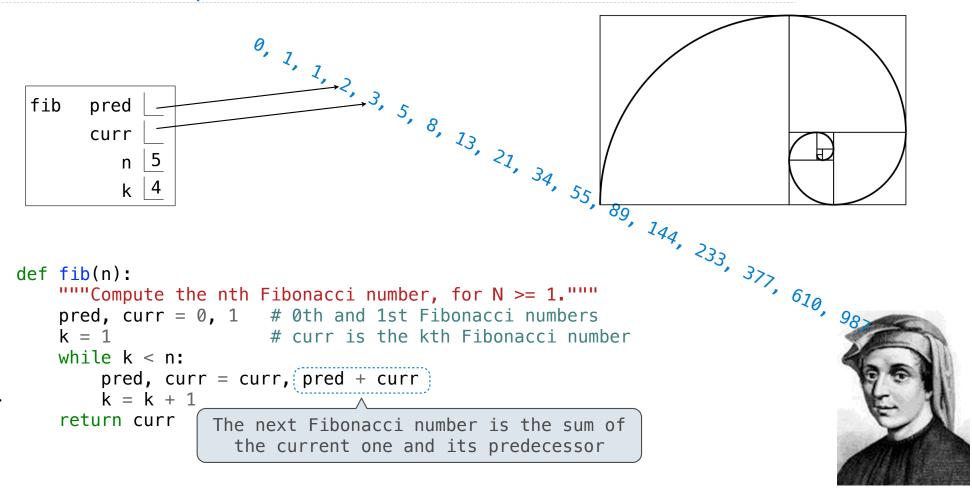


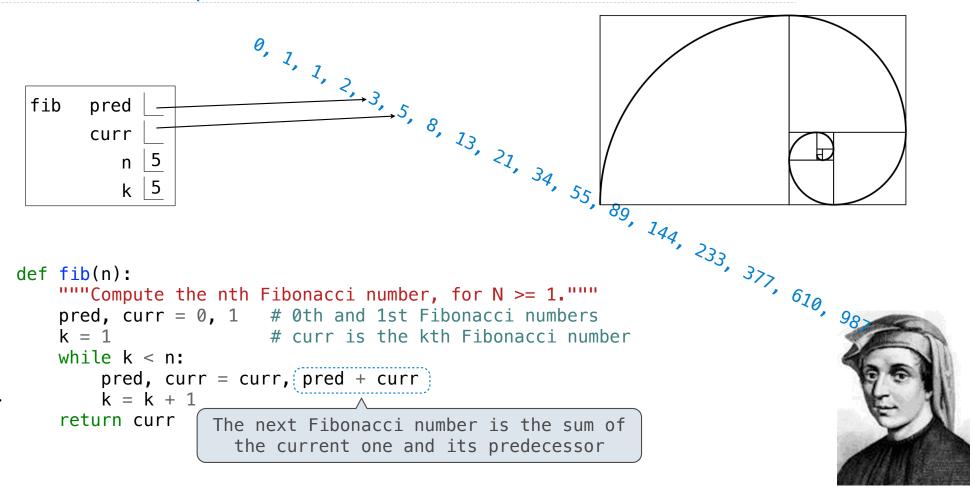




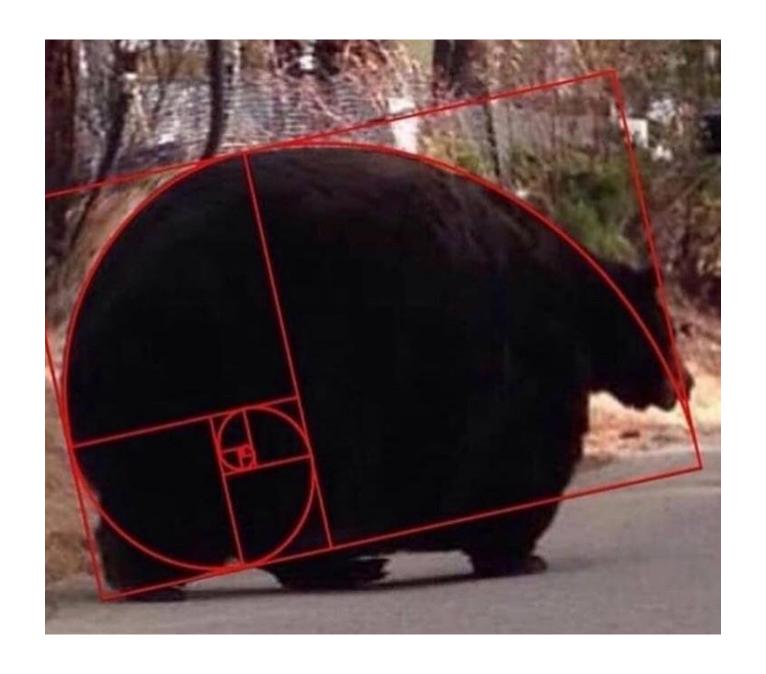


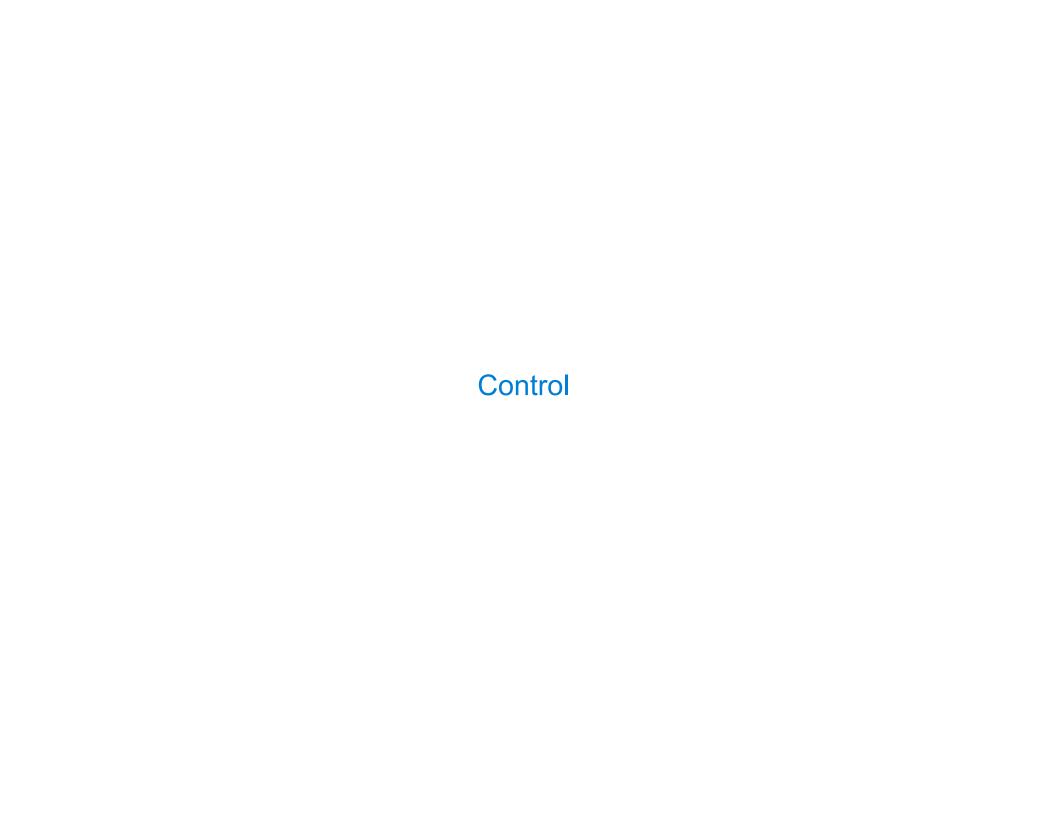






Go Bears!







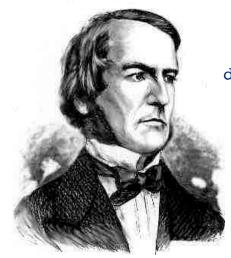
George Boole

```
def absolute_value(x):
    """Return the absolute value of x."""
    if x < 0:
        return -x
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        return 0
    else:
        return x</pre>
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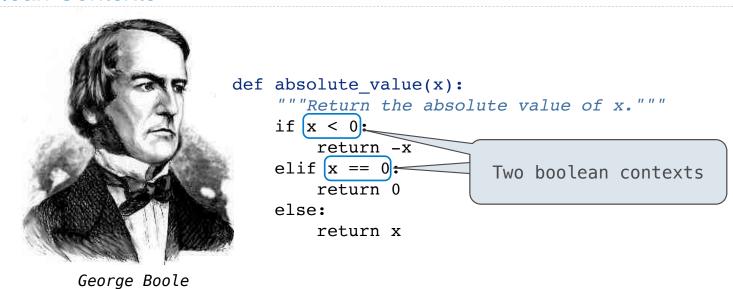
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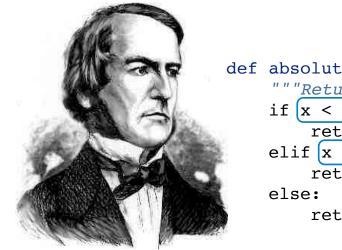


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Boolean Contexts



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(Demo)

Let's try to write a function that does the same thing as an if statement.

	lf	Statements	and	Call	Exp	oression	าร
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if ____:

else:

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Execution Rule for Conditional Statements:

If Statements and Call Expression	l Expressions	Call E	and	tements	t Stat	ΙŤ
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else:

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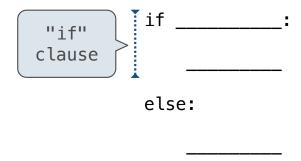
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_		
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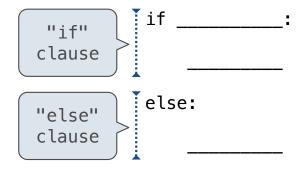
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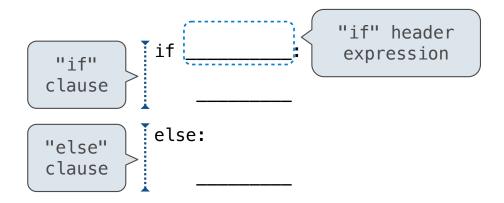
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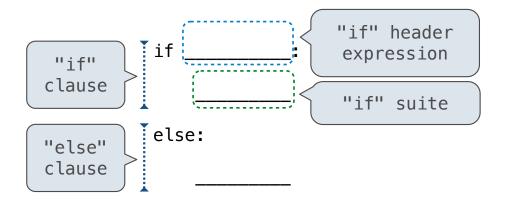
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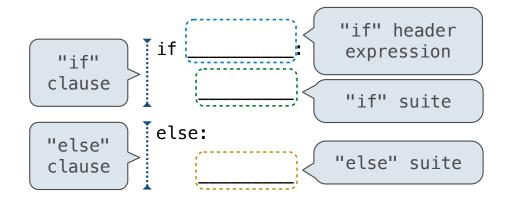
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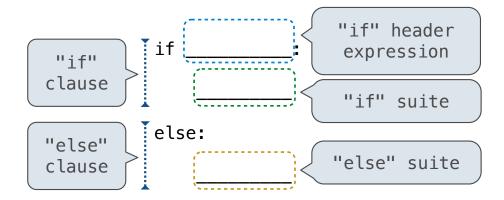
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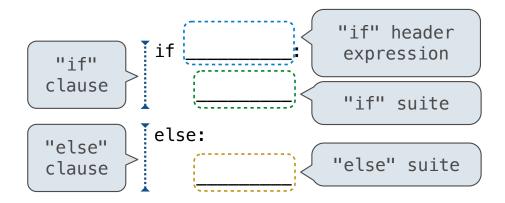
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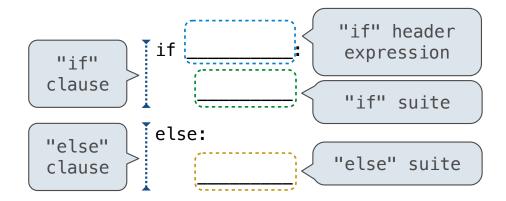


if_(____, ____)

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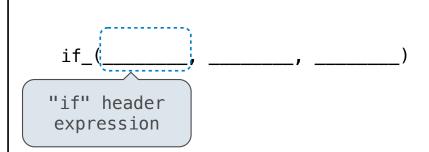
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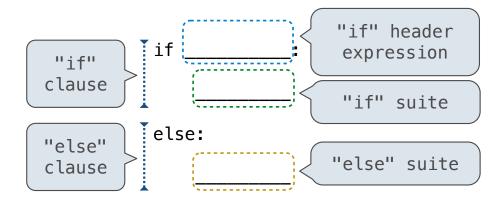


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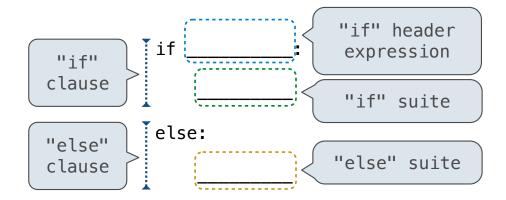
"if" header expression "if" suite

if

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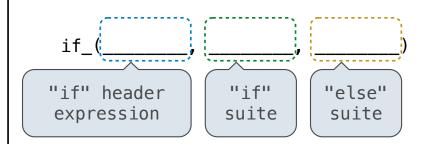
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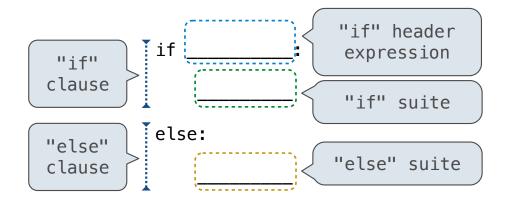


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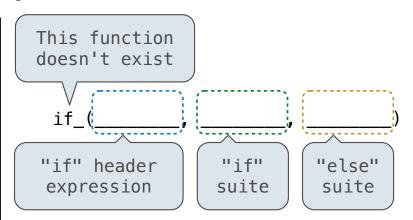


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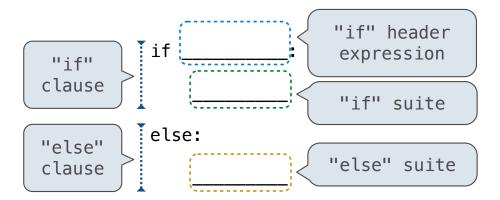


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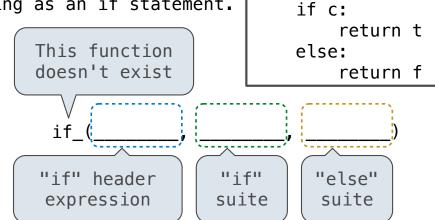
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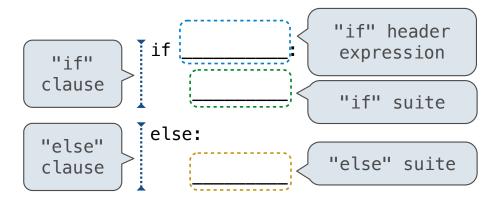
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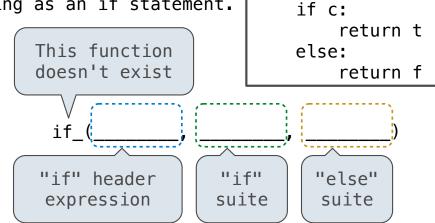
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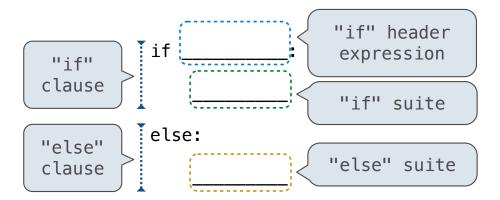
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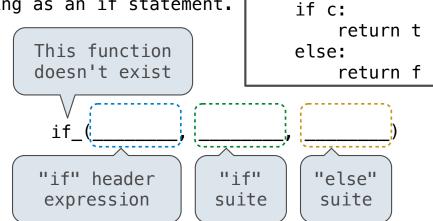
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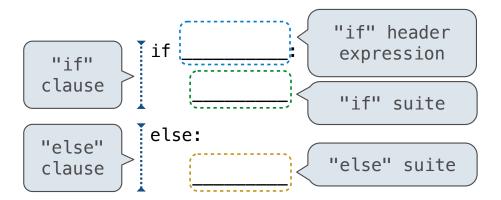


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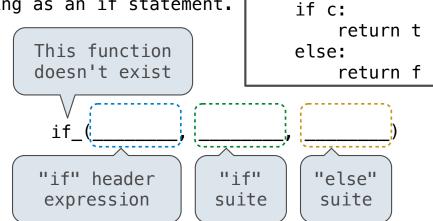
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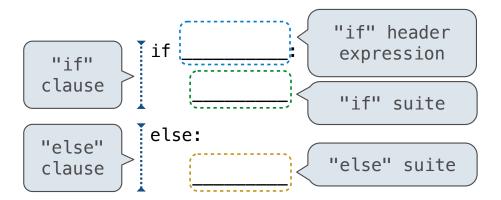


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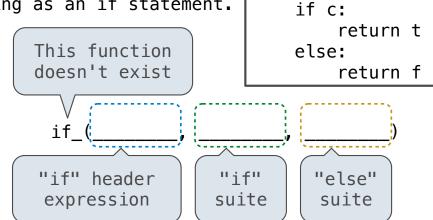


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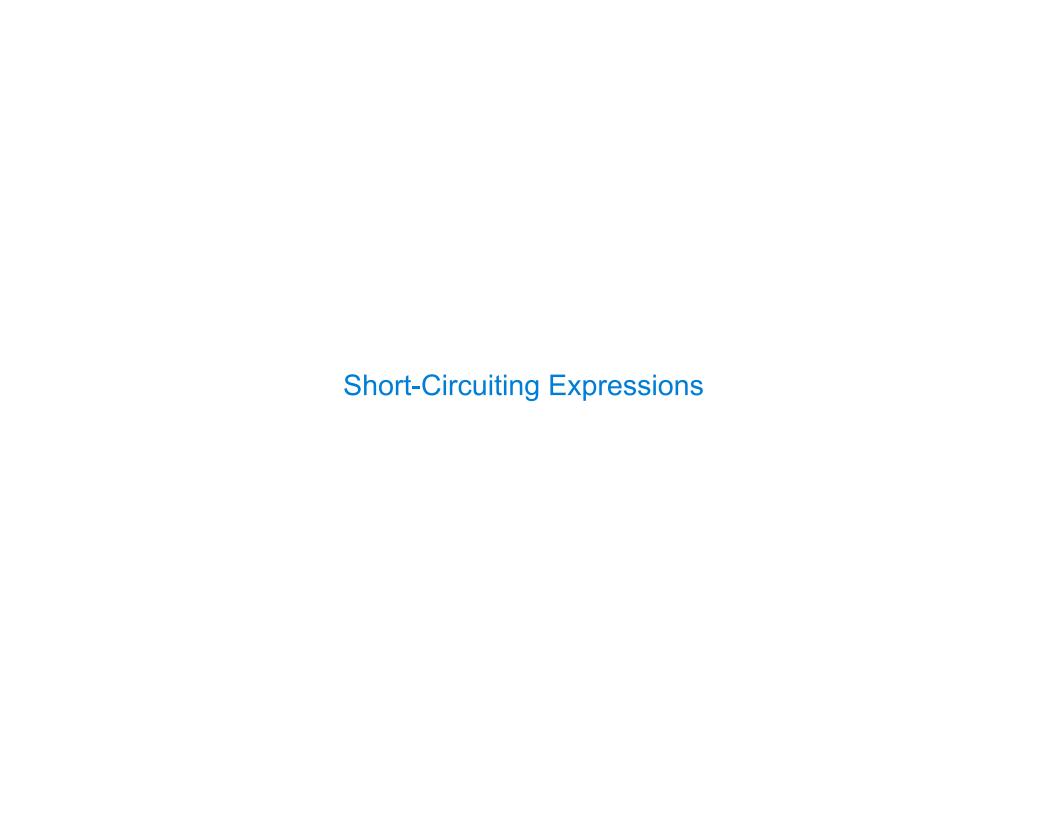
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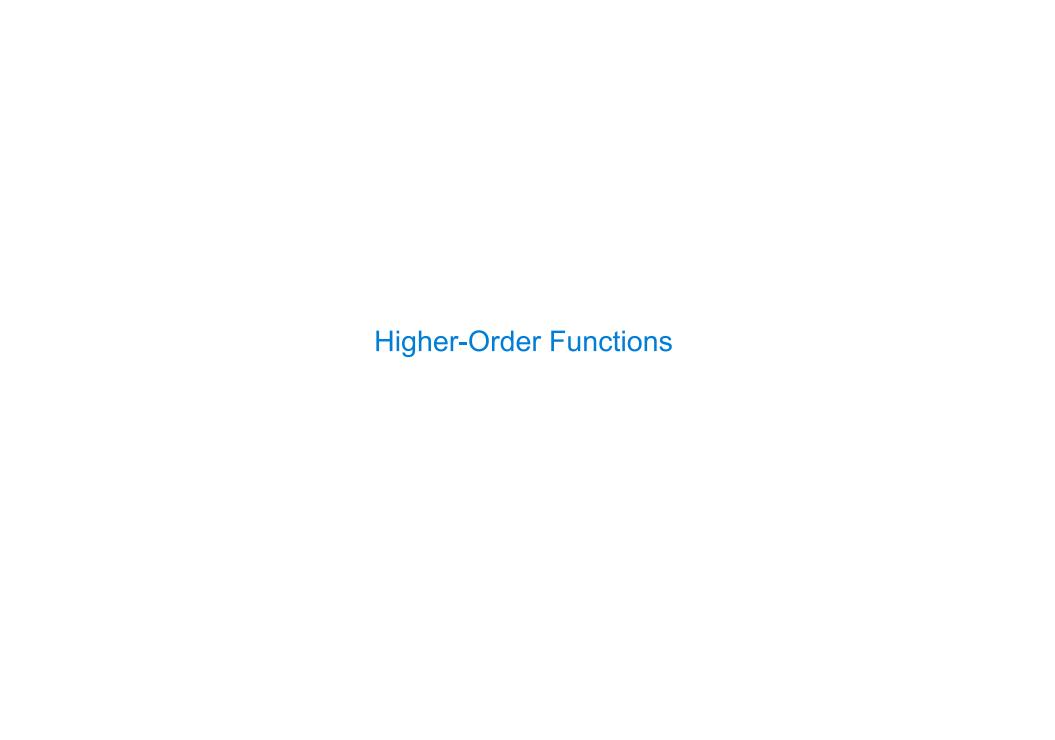
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(Demo)



Generalizing Over Co	mputational Processes	5	

	Generalizing	Over	Com	outation	nal	Processes
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$$\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$$

$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} = 3.04$$

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(Demo)

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def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

>>> summation(5, cube)
    225
    """

    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total</pre>
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>>> summation(5, cube)
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"""

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while k <= n:
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return total</pre>
```

```
Function of a single argument
def cube(k):
                                (not called "term")
    return pow(k, 3)
                           A formal parameter that will
def summation(n, term)
                              be bound to a function
     """Sum the first n terms of a sequence.
    >>> summation(5, cube)
    225
     11 11 11
    total, k = 0, 1
    while k <= n:
         total, k = total + term(k), k + 1
    return total
                            The function bound to term
                                gets called here
```

```
Function of a single argument
def cube(k):
                                 (not called "term")
     return pow(k, 3)
                            A formal parameter that will
def summation(n, term)
                               be bound to a function
     """Sum the first n terms of a sequence.
     >>> summation(5, cube)
     225
                           The cube function is passed
     11 11 11
                              as an argument value
     total, k = 0, 1
     while k <= n:</pre>
          total, k = total + term(k), k + 1
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          total, k = total + term(k), k + 1
     return total
                             The function bound to term
  0 + 1 + 8 + 27 + 64 + 125
                                 gets called here
```