

Trees

## Announcements

## Tree-Structured Data

def tree(label, branches=[]): return [labet] + list(branches)
def label( t ):
return t [0]
def branches(t):
return t [1:]
def is_leaf( t$)$ :
return not branches ( $t$ )
class Tree:
def __init_(self, label, branches=[]): self. label = label self.branches = list(branches)
def is_leaf(self): return not self.branches

A tree can contains other trees
[5, [6, 7], 8, [[9], 10]]
$(+5(-67) 8(*(-9) 10))$
(s)
(NP (JJ Short) (NNS cuts))
(VP (VBP make)
(NP (JJ long) (NNS delays)))
(. .))

<ul>
<li>Midterm <b>1</b></li>
<li>Midterm <b>2</b></li>
</ul>
Tree processing often involves recursive calls on subtrees

Tree Processing

## Solving Tree Problems

Implement bigs, which takes a Tree instance $t$ containing integer labels. It returns the number of nodes in $t$ whose labels are larger than all labels of their ancestor nodes. (Assume the root label is always larger than all of its ancestors, since it has none.
def bigs(t):
"""Return the number of nodes in $t$ that are larger than all their ancestors.
>>> $a=\operatorname{Tree}(1,[\operatorname{Tree}(4,[\operatorname{Tree}(4), \operatorname{Tree}(5)]), \operatorname{Tree}(3,[\operatorname{Tree}(0,[\operatorname{Tree}(2)])])])$
$\rightarrow \gg$ bigs(a)

if t.is_leaf ():
Somehow track a
list of ancestors
else: $\qquad$ ( [ for b in trbranches])

## Somehow increment

 Somehow incrementthe total count
l > max(ancestors):
Somehow track the largest ancestor
if node. label > max_ancestors:

## Solving Tree Problems

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def bigs(t):
"""Return the number of nodes in $t$ that are larger than all their ancestors.
>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])]
$\ggg$ bigs(a)
$4>$ bigs(a) """ $\quad \begin{gathered}\text { Somehow track the } \\ \text { largest ancestor }\end{gathered}$
$\underset{A \text { node }}{\boldsymbol{d}} \underset{\mathrm{X}}{ } \mathrm{k}_{\text {m }}$
A node $\bar{T}$
if $\qquad$
return $1+\operatorname{sum}([f(b, a . l a b e l)$ for $b$ in a.branches $])$ else: Somehow increment the total count

$$
\text { return } \operatorname{sum}([f(b, x) \text { for } b \text { in a.branches]) }
$$

$$
f\left(\begin{array}{cc}
4 \\
\boldsymbol{f}, 4) & 5(\boldsymbol{\mathcal { A }}, 4)
\end{array}\right.
$$

return $f(t, t . l a b e l-\mathbf{1})<$ Root label is always larger than its ancestors

[^0]
## Solving Tree Problems

Implement bigs, which takes a Tree instance $t$ containing integer labels. It returns the number of nodes in $t$ whose labels are larger than any labels of their ancestor nodes.
def bigs(t):
"Return the number of nodes in $t$ that are larger than all their ancestors.""
$\mathrm{n}=$ [0]
def $f(\mathbf{a}, \mathbf{x}):<\begin{gathered}\text { Somehow track the } \\ \text { largest ancestor }\end{gathered}$
if a label $>\mathrm{x}$ node. label > max_ancestors $\mathrm{n}[0]+=1$ Somehow increment the total count
for $\mathbf{b}$ in a.branches
$f(\underline{b}, \max (a . l a b e l, x) \quad)$
$f(t, t . l a b e l-1) \sqrt{\text { Root label }}$ is always larger than its ancestors
eturn n[0]

## How to Design Programs

From Problem Analysis to Data Definitions
Identify the information that must be represented and how it is represented in the chosen programming language. Formulate data definitions and illustrate them with examples.

Signature, Purpose Statement, Header
State what kind of data the desired function consumes and produces. Formulate a concise answer to the question what the function computes. Define a stub that lives up to the signature.

## Functional Example

Work through examples that illustrate the function's purpose
Function Template
Translate the data definitions into an outline of the function
Function Definition
Fill in the gaps in the function template. Exploit the purpose statement and the examples
Articulate the examples as tests and ensure that the function passes all. Doing so discovers mistakes. Tests also supplement examples in that they help others read and
understand the definition when the need arises-and it will arise for any serious program.

## Designing a Function

Implement smalls, which takes a Tree instance $t$ containing integer labels. It returns the non-leaf nodes in $t$ whose labels are smaller than any labels of their descendant nodes.
def smalls(t): Signature: Tree -> List of Trees
"""Return the non-leaf nodes in $t$ that are smaller than all their descendants.
 [0, 2]
"""
"result $=$ [] Signature: Tree $\rightarrow$ number def process $(t)$ : if $t$.is_leaf(): return $t$. label
else:

process( t )
return result
"Find smallest label in $t$ \& maybe add $t$ to result"
turn min(...)

## Designing a Function

Implement smalls, which takes a Tree instance $t$ containing integer labels. It returns the non-leaf nodes in t whose labels are smaller than any labels of their descendant nodes.
def smalls(t): Signature: Tree -> List of Trees
""nReturn a list of the non-leaf nodes in $t$ that are smaller than all descendants. $\ggg>\operatorname{a}=\operatorname{Tree}(1,[\operatorname{Tree}(2,[\operatorname{Tree}(4), \operatorname{Tree}(5)]), \operatorname{Tree}(3,[\operatorname{Tree}(0,[\operatorname{Tree}(6)])])])$
$\ggg$ sorted([t.label for $t$ in smalis(a)])
10, 2
result $=[]$
result $=[]$
def process $(t)$
Signature: Tree -> number
def process( $t$ ):
"Find smallest label in $t$ \& maybe add $t$ to result"
return
t.label
else:
smallest label smallest $=\underline{\min (\text { [process(b) }}$ for b in t .branches )
smallest label if $\begin{gathered}\text { t. label < smallest } \\ \text { in a branch of } t\end{gathered}$
return min(smallest, t.label)
process(t)
return result

Interpreter Analysis
What expressions are passed to scheme_eval when evaluating the following expressions?
(define x (o)
(define (fy) (forn)


[^0]:    Some initial value for the largest ancestor so far...

