CS61A Lecture 3
Higher Order Functions

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June 20, 2012

COMPUTER SCIENCE IN THE NEWS

The (Literal) Evolution of Music

- Scientists from Imperial College London created a computer program powered by Darwinian natural selection.

  Theory that cultural changes in language, art, and music evolve like living things do, since consumers *choose* what is "popular".

- Program would produce loops of random sounds and analyze opinions of musical consumers. The top loops were then "mated".
- "DarwinTunes" (darwintunes.org) has evolved through at least 2513 generations.

Source: http://www3.imperial.ac.uk/newsandeventspggrp/imperialcollege/newssummary/news_19-6-2012-9-59-1

Review

```
def foo(n):
    k = 2
    while k < n:
        if n % k == 0:
            return False
        k = k + 1
    return True

foo(7)? foo(9)? foo(1)?
```

TRIVIA

`foo` is a *metasyntactic variable*, or a commonly used nonsense name to denote an arbitary or temporary thing.

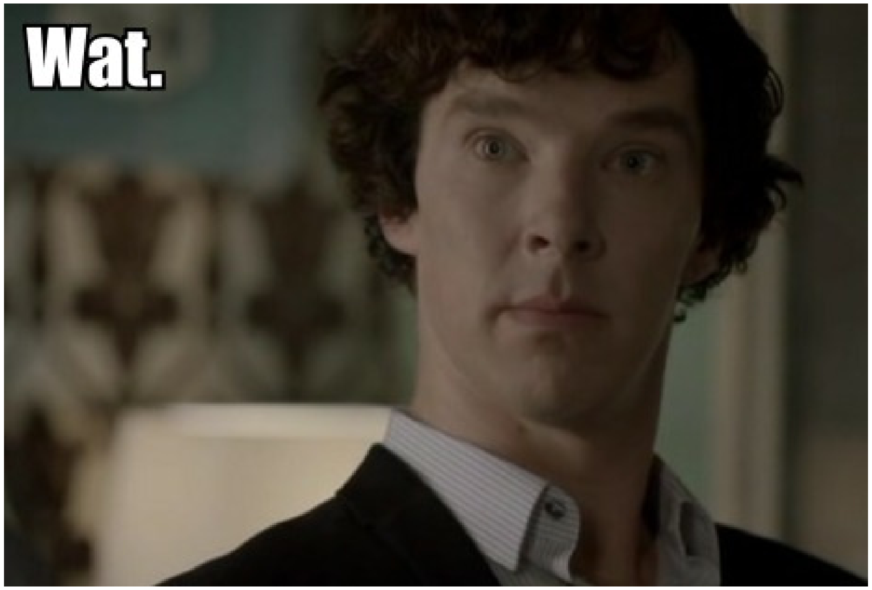
Other examples you may see are `bar`, `garply`, and `baz`. Sometimes, `om` and `nom`. Or even `herp` and `derp`.

This deserves a better name though. Any suggestions?

TODAY

Domain and range of functions

... and feeding functions to functions



Domain and Range

Domain is the *set of values* that the function is *defined for*.

```
square(n):
    return n*n
```

Range (or image) is the *set of values* that the function *returns*.

Domain of `square`: All real *numbers*.

Range of `square`: All non-negative real *numbers*.

DOMAIN AND RANGE

Remember the domain and range of the functions that you are writing and using!

Many bugs arise because programmers forget what a function can and cannot work with.

The domain and range are *especially important* when working with higher order functions.

Problem: Finding Squares

```
>>> square_of_0 = 0*0
>>> square_of_1 = 1*1
>>> square_of_2 = 2*2
>>> square_of_3 = 3*3
...
>>> square_of_65536 = 65536*65536
>>> square_of_65537 = 65537*65537
...
```

PROBLEM: FINDING SQUARES

```
>>> square_of_0 = 0*0
>>> square_of_1 = 1*1
>>> square_of_2 = 2*2
>>> square_of
>>> square_of_65536 = 65536*65536
>>> square_of_65537 = 65537*65537
...
```

There has to be a better way!

PROBLEM: FINDING SQUARES

```
>>> square_of_0 = 0*0
>>> square_of_1 = 1*1
>>> square_of_2 = 2*2
>>> square_of_3 = 3*3
...
>>> square_of_65536 = 65536*65536
>>> square_of_65537 = 65537*65537
...
```

Can we generalize?

FINDING SQUARES: GENERALIZATION

THE SQUARE OF A NUMBER

```
def square(n):
    return n * n
```

IS THE NUMBER MULTIPLIED BY ITSELF

Problem: Sums of Series

```
def sum_of_n_squares(n):
    '''Returns 1² + 2² + 3² + ... + n².'''
    sum, k = 0, 1
    while k <= n:
        sum, k = sum + square(k), k + 1
    return sum

def sum_of_n_cubes(n):
    '''Returns 1³ + 2³ + 3³ + ... + n³.'''
    sum, k = 0, 1
    while k <= n:
        sum, k = sum + cube(k), k + 1
    return sum
```

Problem: Sums of Series

```
from math import sin, sqrt
def sum_of_n_sines(n):
    ‘‘‘Returns sin(1) + sin(2) + sin(3) + ... + sin(n).’’’
    sum, k = 0, 1
    while k <= n:
        sum, k = sum + sin(k), k + 1
    return sum

def sum_of_n_sqrts(n):
    ‘‘‘Returns sqrt(1) + sqrt(2) + ... + sqrt(n).’’’
    sum, k = 0, 1
    while k <= n:
        sum, k = sum + sqrt(k), k + 1
    return sum
```

and so on…

PROBLEM: SUMS OF SERIES

```
from math import sin, sqrt
def sum_of_n_sines(n):
    '''Returns sin(1) + sin(2) + sin(3) + ... + sin(n).'''
    sum, k = 0, 1
    while k <= n:
        sum, k = sum + sin(k), k
    return sum

de        _sqrts(n):
    '''Returns sqrt(1) + sqrt(2) + ... + sqrt(n).'''
    sum, k = 0, 1
    while k <= n:
        sum, k = sum + sqrt(k), k + 1
    return sum
```

There has to be a better way!

and so on...

Problem: Sums of Series

```
def sum_of_n_squares(n):
    sum, k = 0, 1
    while k <= n:
        sum = sum + square(k)
        k = k + 1
    return sum

def sum_of_n_cubes(n):
    sum, k = 0, 1
    while k <= n:
        sum = sum + cube(k)
        k = k + 1
    return sum

def sum_of_n_sines(n):
    sum, k = 0, 1
    while k <= n:
        sum = sum + sin(k)
        k = k + 1
    return sum

def sum_of_n_sqrts(n):
    sum, k = 0, 1
    while k <= n:
        sum = sum + sqrt(k)
        k = k + 1
    return sum
```

Can we generalize?

Announcements

- HW2 is released, due **Tuesday, June 26**.
- Discussions earlier held in 320 Soda will now be held in **310 Soda** for the next six weeks. We will move back to 320 Soda for the last two weeks.
- Groups for studying and midterms will be assembled on **Thursday, June 21** in discussion section.

EVALUATION OF PRIMITIVE EXPRESSIONS

```
>>> 3
3
```

evaluates to

```
>>> x = 5
```

Statement

```
>>> x
5
```

evaluates to

```
>>> 'CS61A'
'CS61A'
```

evaluates to

FUNCTION NAMES

Address where the function is *stored* in the computer. It is not fixed.

```
>>> square
<function square at 0x0000000002279648>
```

evaluates to

Python's representation of

```
square(n):
    return n*n
```

square is the name of the function, or "machine", that squares its input

FUNCTION NAMES

```
>>> square
<function square at 0x0000000002279648>
>>> cube
<function cube at 0x00000000022796C8>
>>> max
<built-in function max>
```

Function names are *primitive expressions* that *evaluate to* the corresponding "machines"

USING PRIMITIVE EXPRESSIONS

We have seen primitive expressions used as arguments to functions:

```
>>> square(3)
9
>>> x = 4
>>> square(x)
16
```

Can we then use function names as arguments?

http://boredommagnified.files.wordpress.com/2011/07/challenge-accepted.jpg

PROBLEM: SUMS OF SERIES

```
def sum_of_n_squares(n):
    sum, k = 0, 1
    while k <= n:
        sum = sum + square(k)
        k = k + 1
    return sum

def sum_of_n_cubes(n):
    sum, k = 0, 1
    while k <= n:
        sum = sum + cube(k)
        k = k + 1
    return sum

def sum_of_n_sines(n):
    sum, k = 0, 1
    while k <= n:
        sum = sum + sin(k)
        k = k + 1
    return sum

def sum_of_n_sqrts(n):
    sum, k = 0, 1
    while k <= n:
        sum = sum + sqrt(k)
        k = k + 1
    return sum
```

Functions as Arguments

```
def summation(n, term):
    sum, k = 0, 1
    while k <= n:
        sum = sum + term(k)
        k = k + 1
    return sum
```

term(1) + term(2) + ... + term(n)

```
def sum_of_n_squares(n):
    return summation(n, square)
```

Functions as Arguments

```
def summation(n, square):
    sum, k = 0, 1
    while k <= n:
        sum = sum + square(k)
        k = k + 1
    return sum

def sum_of_n_squares(n):
    return summation(n, square)
```

Functions as Arguments

3 → `square(n):` `return n*n` → 9

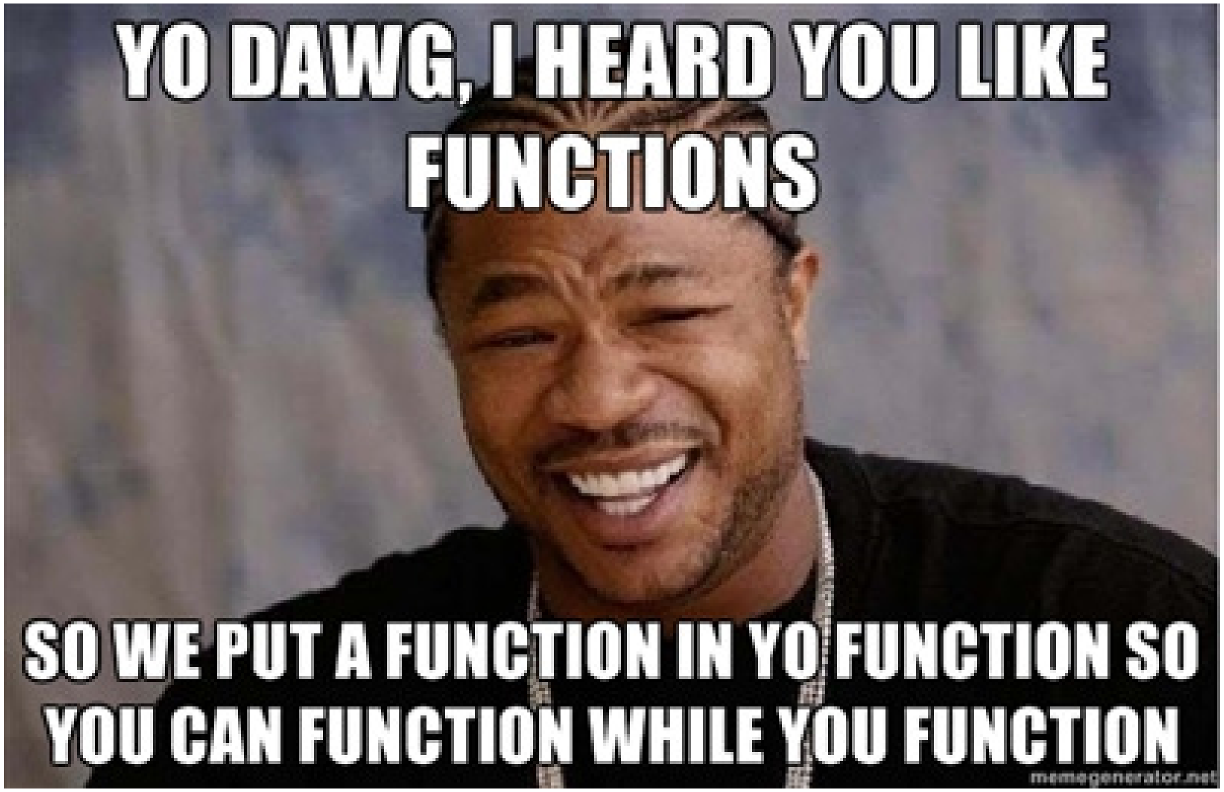
4, `square(n):` `return n*n` → `summation(n, term):`

```
sum, k = 0, 1
while k <= n:
    sum = sum + term(k)
    k = k + 1
return sum
```

→ 27

`summation` takes a function as an argument, and is thus a *higher order function.*

HIGHER ORDER FUNCTIONS



http://cdn.memegenerator.net/instances/400x/13712469.jpg

Higher Order Functions: Practice

```
def summation(n, term):
    sum, k = 0, 1
    while k <= n:
        sum = sum + term(k)
        k = k + 1
    return sum

def sum_of_n_cubes(n):
    return summation(n, _______________)

def sum_of_n_sines(n):
    return summation(n, _______________)

def sum_of_n_positive_ints(n):
    return summation(n, _______________)
```

Higher Order Functions: Practice

```
def summation(n, term):
    sum, k = 0, 1
    while k <= n:
        sum = sum + term(k)
        k = k + 1
    return sum

def sum_of_n_cubes(n):
    return summation(n, lambda x: x ** 3)

def sum_of_n_sines(n):
    return summation(n, sin)

def sum_of_n_positive_ints(n):
    return summation(n, lambda x: x)
```

HIGHER ORDER FUNCTIONS

Functions that can take other functions as arguments are considered *higher order functions*.

The *domains* of these functions now include *other functions*.

Functions can be treated as ***data***!

Higher Order Functions: Aside

Why "higher order"?

First-order functions only take non-function values, like numbers and strings, as arguments.

Second-order functions can take first-order functions as arguments.

Third-order functions can take second-order functions as arguments.

… and so on.

HOFs Elsewhere

$$\int_a^b f(x)dx$$

Integral $\int$ is a **function** that takes three arguments: lower limit *a*, upper limit *b*, and a **function** *f*

HOFs Elsewhere

Recipe
(Function)
+
Ingredients

Food

YOU (Kind of.)
are a function (Kind of.)

HOFs Elsewhere

Basketball

Playing

NOUN
(Data)

VERB
(Function)

Interests: Playing Basketball

NOUN (“Gerund”)
(Function as data)

FUNCTIONS AS ARGUMENTS

```
def summation2(n, term, next):
    sum, k = 0, 1
    while k <= n:
        sum = sum + term(k)
        k = next(k)
    return sum
```

FUNCTIONS AS ARGUMENTS: PRACTICE

```
def summation2(n, term, next):
    sum, k = 0, 1
    while k <= n:
        sum = sum + term(k)
        k = next(k)
    return sum

def summation(n, term):
    return summation2(n, term,
                              ______________)
```

FUNCTIONS AS ARGUMENTS: PRACTICE

```
def summation2(n, term, next):
    sum, k = 0, 1
    while k <= n:
        sum = sum + term(k)
        k = next(k)
    return sum

def summation(n, term):
    return summation2(n, term,
                      lambda x: x + 1)
```

HIGHER ORDER FUNCTIONS: PRACTICE

```
def summation(n, term):
    sum, k = 0, 1
    while k <= n:
        sum = sum + term(k)
        k = k + 1
    return sum

def sum_of_n_even(n):
    '''Returns the sum of the first n even numbers.'''
    return summation(n, ________________)

def sum_of_n_odd(n):
    '''Returns the sum of the first n odd numbers.'''
    return summation(n, ________________)

def sum_of_n_starting_from_m(m, n):
    '''Returns the sum of the first n numbers starting from m.'''
    return summation(n, ________________)
```

HIGHER ORDER FUNCTIONS: PRACTICE

```
def summation(n, term):
    sum, k = 0, 1
    while k <= n:
        sum = sum + term(k)
        k = k + 1
    return sum

def sum_of_n_even(n):
    ‘‘‘Returns the sum of the first n even numbers.’’’
    return summation(n, lambda x: 2 * x)

def sum_of_n_odd(n):
    ‘‘‘Returns the sum of the first n odd numbers.’’’
    return summation(n, lambda x: 2 * x + 1)

def sum_of_n_starting_from_m(m, n):
    ‘‘‘Returns the sum of the first n numbers starting from m.’’’
    return summation(n, lambda x: x + m)
```

Higher Order Functions: Practice

The value of the *derivative* of a function f at a point x can be approximated as

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}$$

if Δx is really small.

Write the function `derivative` that takes as arguments a function `f`, a point `x`, and a really small value `delta_x`, and returns the approximate value of the derivative at `x`.

```
def deriv(f, x, delta_x):
    return _______________
```

Higher Order Functions: Practice

The value of the *derivative* of a function f at a point x can be approximated as

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}$$

if Δx is really small.

Write the function `derivative` that takes as arguments a function `f`, a point `x`, and a really small value `delta_x`, and returns the approximate value of the derivative at `x`.

```
def deriv(f, x, delta_x):
    return (f(x + delta_x) - f(x)) / delta_x
```

CONCLUSION

- It is important to remember the domain and range of functions!
- Functions can take functions as arguments.
- ***Preview***: Functions can also *return* functions.