CS61A Lecture 24
Infinite Sequences

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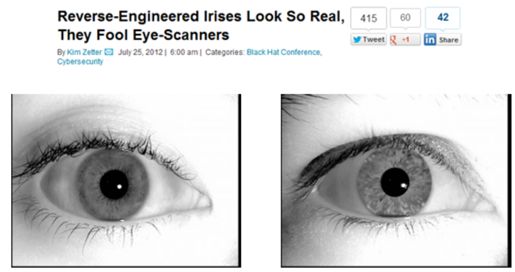
Computer Science in the News



http://www.wired.com/threatlevel/2012/07/reverse-engineering-iris-scans/all/

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Agenda

Weeks 1 to 3: Functional programming
Weeks 4 to 5: Object-oriented programming

Next Two Weeks:
- Streams and Lazy Evaluation (*Today*)
- Logic Programming
- Client/Server Programming
- Parallel Programming

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Lazy Evaluation

```
def is_better(a, b, a_better, b_better):
    if a > b:
        return a_better
    return b_better

is_better(foo, bar, fib(1), fib(1000))
```

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Lazy Evaluation

```
is_better(foo, bar, fib(1), fib(1000))
```

Remember that the operand and operators are evaluated first, before the body of the function is executed.

Since the operators are evaluated *before* the body is executed, `fib(1000)` is calculated, even though its value may not be needed.

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Lazy Evaluation

In ***lazy*** (or ***deferred***) ***evaluation***, expressions are only evaluated *when they are needed*.

Lazy evaluation is native to many languages, such as Haskell.

By contrast, Python is an *eager* language, evaluating expressions immediately.

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Lazy Evaluation

We can modify `is_better` to evaluate "lazily" by passing in functions. These functions will then provide the necessary values when – and if – they are called.

```
def is_better(a, b, a_better_fn, b_better_fn):
    if a > b:
        return a_better_fn()
    return b_better_fn()

is_better(foo, bar,
          lambda: fib(1), lambda: fib(1000))
```

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Infinite Sequences: Examples

A ***sequence*** is an ordered collection of data values.

There are many kinds of sequences, and all share certain properties.

What about infinite collections of data, or infinite sequences?

Length: A sequence has a *finite length*.

Element selection: A sequence has an element for any non-negative integer less than its length.

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Infinite Sequences: Examples

- Mathematical sequences
  Prime numbers, Fibonacci sequence, ...
- Internet and cell phone traffic
- Real-time data
  Instrument measurements, stock prices, weather, social media data, ...

Problem: How do we represent an infinite sequence in a finite-memory computer?

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Announcements

- Homework 12 due **Tuesday, July 31**.
- Project 4 due **Tuesday, August 7**.
  – Partnered project, in two parts.
  – Twelve questions, so *please start early*!
  – Two extra credit questions.

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Announcements

- Project 4 contest due **Friday, August 3**.
  – Generate recursive art using Scheme.
  – Prizes awarded in two categories:
    - **Featherweight**: At most 128 words of Scheme.
    - **Heavyweight**: At most 1024 words of Scheme.
  – One question on homework 14 will ask you to vote for your favorite drawing.
  – Three extra credit points.
  – *Prize*: *Logicomix*

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Announcements: Midterm 2

- Scores available on `glookup`.
  – Average: 39.9, standard deviation: 6.9.
  – Will be handed back in lab today.
- Solutions are available online.
  – Regrade requests due **Tuesday, August 7**.
- Post-midterm de-stress potluck this week.
  – Food and games.
  – Come and leave when you want.

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Announcements: Final

- Final is **Thursday, August 9**.
  - *Where*? 1 Pimentel.
  - *When*? 6PM to 9PM.
  - *How much*? *All* of the material in the course, from June 18 to August 8, will be tested.
- Closed book and closed electronic devices.
- One 8.5" x 11" 'cheat sheet' allowed.
- No group portion.
- We will get back to you this week if you have conflicts and have told us. If you haven't told us yet, please *let us know*.

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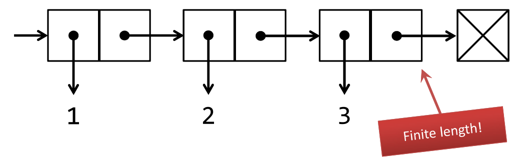
Review: Recursive Lists

`<1, 2, 3>`



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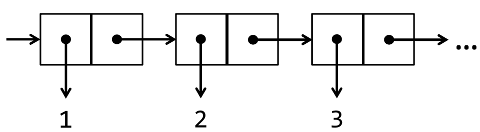
Infinite Recursive Lists

`<1, 2, 3, ...>`



Observe that we depict only as many values of an infinite sequence *as we will need*.

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Infinite Recursive Lists

Idea: We only construct as much of an infinite sequence *as we will need*.

We also remember *how to construct* the rest of the sequence, in case we need to find more values of the infinite sequence.

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Streams

Streams are *lazily computed* recursive lists that represent (potentially infinite) sequences.

Like a recursive list, a stream is a *pair*:
the first element is *the first element* of the stream,
the second element stores how to compute the rest of the stream when needed, and will compute it when asked.

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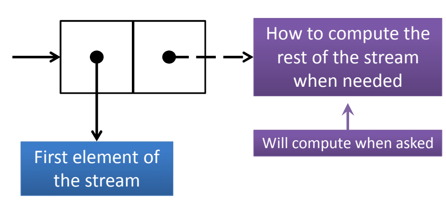
Streams



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Streams

```
class Stream(object):
    def __init__(self, first,
                       compute_rest,
                       empty=False):
        self.first = first
        self._compute_rest = compute_rest
        self.empty = empty
        self._rest = None
        self._computed = False
```

First element of the stream

How to compute the rest of the stream when needed

Is this stream empty?

Has the rest of this stream already been computed?

Streams

```
class Stream:
    ...
    @property
    def rest(self):
        assert not self.empty, \
        'Empty streams have no rest.'
        if not self._computed:
            self._rest = self._compute_rest()
            self._computed = True
        return self._rest

Stream.the_empty_stream = Stream(None, None, True)
```

If the rest of the stream has not been computed yet...

... compute it and remember the result.

Streams: Example

To construct the (finite) stream
`1, 2, 3`
we make a `Stream` object
whose first element is `1`, and
whose second element computes the smaller (finite) stream
`2, 3`
when called.

```
Stream(1,
       lambda: Stream(2,
                      lambda: Stream(3,
                                     lambda:
                                     Stream.the_empty_stream)))
```


Streams: Example

Notice that the definition

```
Stream(1,
       lambda: Stream(2,
                      lambda: Stream(3,
                                     lambda: Stream.the_empty_stream)))
```

is similar to the definition of the recursive list `<1, 2, 3>`:

```
make_rlist(1,
           make_rlist(2,
                      make_rlist(3,
                                 Rlist.the_empty_rlist)))
```

This is no accident: streams are recursively defined.
The rest of a `Stream` is also a `Stream`.

Streams: Example

```
>>> s = Stream(1,
               lambda:
               Stream(2, lambda:
                          Stream(3,
                                 lambda:
                                 Stream.the_empty_stream)))
>>> s.first
1
>>> s.rest.first
2
>>> s.rest.rest.first
3
>>> s.rest.rest.rest
<empty stream>
```

The interaction with a stream is very similar to that with a recursive list.

Infinite Streams: Example

We want to make an infinite stream of only 1s.

`1, 1, 1, 1, 1, ...`

First element is 1.

The rest is *also* an infinite stream of only 1s.

Infinite Streams: Example

```
def make_one_stream():
    def compute_rest():
        return make_one_stream()
    return Stream(1, compute_rest)
```

First element is 1.

Second element is a function that, when called, will return a stream of 1s.

Infinite Streams: Example

make_one_stream calls make_one_stream recursively, but there is no base case! This is because we want an *infinite* stream.

```
def make_one_stream():
    def compute_rest():
        return make_one_stream()
    return Stream(1, compute_rest)
```


Infinite Streams: Example

```
>>> ones = make_one_stream()
>>> ones.first
1
>>> ones.rest.first
1
>>> ones.rest.rest.first
1
>>> ones.rest.rest.rest.rest.rest.first
1
```


Infinite Streams: Practice

Write the function make_stream_of that generalizes the stream of 1s to produce a stream of any given number.

```
>>> fives = make_stream_of(5)
>>> fives.first
5
>>> fives.rest.rest.rest.rest.first
5
```


Infinite Streams: Practice

Write the function make_stream_of that generalizes the stream of 1s to produce a stream of any given number.

```
def make_stream_of(n):
    def compute_rest():
        return _________________
    return _______________________
```


Infinite Streams: Practice

Write the function make_stream_of that generalizes the stream of 1s to produce a stream of any given number.

```
def make_stream_of(n):
    def compute_rest():
        return make_stream_of(n)
    return Stream(n, compute_rest)
```

Infinite Streams: Practice

Write the function `integers_starting_from` that returns an infinite stream of integers starting from a given number.

```
>>> positive_ints = \
    integers_starting_from(0)
>>> positive_ints.first
0
>>> positive_ints.rest.rest.rest.first
3
```


Infinite Streams: Practice

Write the function `integers_starting_from` that returns an infinite stream of integers starting from a given number.

```
def integers_starting_from(n):
    def compute_rest():
        return ___________________________
    return _______________________
```


Infinite Streams: Practice

Write the function `integers_starting_from` that returns an infinite stream of integers starting from a given number.

```
def integers_starting_from(n):
    def compute_rest():
        return integers_starting_from(n+1)
    return Stream(n, compute_rest)
```


Infinite Streams

Write a function add_one that adds one to every element of a stream of integers.

```
>>> s = make_stream_of(3)
>>> t = add_one(s)
>>> t.first
4
```


Infinite Streams

Write a function add_one that adds one to every element of a stream of integers.

```
def add_one(s):
    if s.empty:
        return Stream.the_empty_stream
    def compute_rest():
        return add_one(s.rest)
    return Stream(s.first + 1,
                  compute_rest)
```

If the stream is empty, return the empty stream.

The rest of the new stream…

… is one added to the rest of the original stream.

The first of the new stream is one more than the first of the original stream.

Infinite Streams

Write a function add_one that adds one to every element of a stream of integers.

```
def add_one(s):
    if s.empty:
        return Stream.the_empty_stream
    def compute_rest():
        return add_one(s.rest)
    return Stream(s.first + 1,
                  compute_rest)
```

We will never reach this base case if the stream is infinite. Why is that okay?

INFINITE STREAMS: PRACTICE

Write a function `map_stream` that applies a given function to each element of a stream, and returns the new stream.

```
>>> s = integers_starting_from(1)
>>> t = map_stream(lambda x: x*x, s)
>>> t.rest.first
4
```

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INFINITE STREAMS: PRACTICE

Write a function `map_stream` that applies a given function to each element of a stream, and returns the new stream.

```
def map_stream(fn, s):
    if s.empty:
        return ______________________
    def compute_rest():
        return _____________________
    return Stream(___________, compute_rest)
```

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INFINITE STREAMS: PRACTICE

Write a function `map_stream` that applies a given function to each element of a stream, and returns the new stream.

```
def map_stream(fn, s):
    if s.empty:
        return Stream.the_empty_stream
    def compute_rest():
        return map_stream(fn, s.rest)
    return Stream(fn(s.first), compute_rest)
```

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INFINITE STREAMS: PRACTICE

Write a function `filter_stream`, which takes in a predicate function and a stream, and returns a new stream of values that satisfy the predicate function.

```
>>> s = integers_starting_from(2)
>>> t = filter_stream(lambda x: x%2 == 0, s)
>>> t.rest.first
4
```

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INFINITE STREAMS: PRACTICE

```
def filter_stream(pred, s):
    if s.empty:
        return _______________________
    def compute_rest():
        return ___________________________
    if _____________:
        return Stream(_______, compute_rest)
    return ___________________________
```

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INFINITE STREAMS: PRACTICE

```
def filter_stream(pred, s):
    if s.empty:
        return Stream.the_empty_stream
    def compute_rest():
        return filter_stream(pred, s.rest)
    if pred(s.first):
        return Stream(s.first, compute_rest)
    return filter_stream(pred, s.rest)
```

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Infinite Streams: Example

Write a function `add_streams` that takes two *infinite* streams of numbers, `s1` and `s2`, and produces a new stream of the sum of the numbers at the same positions in the two streams.

```
>>> s = integers_starting_from(2)
>>> t = integers_starting_from(5)
>>> sum_stream = add_streams(s, t)
>>> sum_stream.rest.first
9
```


Infinite Streams: Example

Write a function `add_streams` that takes two *infinite* streams of numbers, `s1` and `s2`, and produces a new stream of the sum of the numbers at the same positions in the two streams.

```
def add_streams(s1, s2):
    def compute_rest():
        return add_streams(s1.rest,
                           s2.rest)
    return Stream(s1.first + s2.first,
                  compute_rest)
```


Integer Stream: Alternate Construction

Now that we have defined `add_streams`, we can construct the stream of non-negative integers, `nonnegative_ints`, in another way:

```
    1, 2, 3, 4, 5, ...
= 1, 1, 1, 1, 1, ...
+    1, 2, 3, 4, ...
```

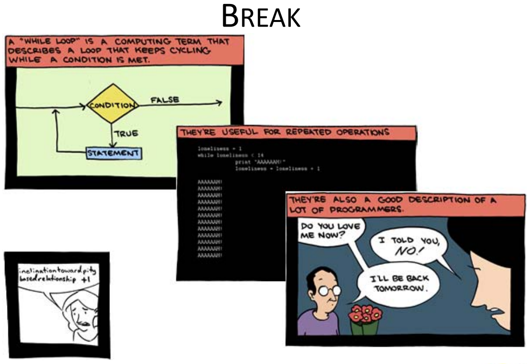
We can define `nonnegative_ints` in terms of itself!

Integer Stream: Alternate Construction

```
nonnegative_ints = \
Stream(1,
       lambda:
       add_streams(ones,
                   nonnegative_ints))
```

Protip: As with recursion, when coding with streams, it helps if you "trust" that the stream is available for you already. How would you then use this information?

Break



http://www.smbc-comics.com/index.php?db=comics&id=2656

Iterators

Python natively supports iterators.

Iterators are objects that give out one item at a time and save the next item until they are asked: this evaluation is *lazy*.

Iterators

Python has the following *interface* for iterators:

- The `__iter__` method should return an iterator object.
- The `__next__` method should:
  - return a value, *or*
  - raise a `StopIteration` when the end of the sequence is reached, and on all subsequent calls.

Iterators: Example

We want to write an iterator that returns the characters of a word.

```
>>> jom_chars = Characters('jom')
>>> jom_chars.__next__()
'j'
>>> jom_chars.__next__()
'o'
>>> next(jom_chars)
'm'
>>> jom_chars.__next__()
Traceback (most recent call last):
    ...
StopIteration
```

The built-in function `next` calls the `__next__` method of its argument.

Iterators: Example

```
class Characters:
    def __init__(self, word):
        self.word = word
        self.curr_pos = 0

    def __iter__(self):
        return self
```

Remember the position of the character to return.

Iterators: Example

```
class Characters:
    ...
    def __next__(self):
        if self.curr_pos >= len(self.word):
            raise StopIteration

        result = self.word[self.curr_pos]
        self.curr_pos += 1
        return result
```

If we are done with the word, we raise a `StopIteration`.

Otherwise, return the current character, and "move forward".

Iterators

Why does Python provide this interface?
Among other things, so that we can iterate using `for`-loops.

```
>>> for char in Characters('jom'):
...     print(char)
...
j
o
m
```


Iterators

What does a `for`-loop do "under the hood"?

```
for x in iterable:
    function(x)
```

is equivalent to

```
iterator = iterable.__iter__()
try:
    while True:
        element = iterator.__next__()
        function(element)
except StopIteration as e:
    pass
```

Create an iterator.

Try to get an element from the iterator.

Apply the function on the element.

If we could not get an element, we *catch* the `StopIteration` exception and do not apply the function.

Other Lazy Objects

We have seen a few other lazy objects before, or objects that only produce new values when they are asked.

The objects returned by the built-in `map` and `filter` functions are lazy; the `range` constructor is also lazy.

Other Lazy Objects

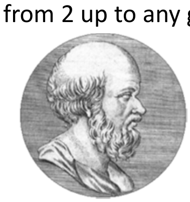
```
>>> my_squares = map(square,
                     (4, 5, 6, 7))
>>> my_squares
<map object at 0x...>
>>> my_squares.__next__()
16
>>> next(my_squares)
25
```


Conclusion

- In lazy evaluation, expressions are not evaluated until they are needed.
- Streams allow us to represent infinite sequences.
- Streams are pairs whose first element is the first element of the stream, and whose second element stores how the rest of the stream can be calculated.
  - This way, only as much of the stream is created as is needed.
- Python has built-in support for iterators, or objects that compute new values only when asked.
- ***Preview***: Generators, logic programming.

Extras: The Sieve of Eratosthenes

Eratosthenes (Ἐρατοσθένους) of Cyrene was a Greek mathematician (276 BC – 195 BC) who discovered an algorithm for generating all prime numbers from 2 up to any given limit.



http://upload.wikimedia.org/wikipedia/commons/thumb/a/a2/Portrait_of_Eratosthenes.png/220px-Portrait_of_Eratosthenes.png

Extras: The Sieve of Eratosthenes

1. First, remove all the numbers that are multiples of 2, except for 2 itself.
2. Then, remove all the numbers that are multiples of the next non-removed number, except for that number itself.
3. Repeat step 2 until no more numbers can be removed.

Demo:
http://en.wikipedia.org/wiki/File:Sieve_of_Eratosthenes_animation.gif

Extras: The Sieve of Eratosthenes

```
def primes(positive_ints):
    # positive_ints is a stream that starts at least at 2.
    def not_div_by_first(num):
        return num % positive_ints.first != 0
    def sieve():
        return primes(filter_stream(not_div_by_first,
                                    positive_ints.rest))
    return Stream(positive_ints.first, sieve)
```