61A LECTURE 14 – MULTIPLE REPRESENTATIONS

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Generic Functions

An abstraction might have more than one representation.

- Python has many sequence types: tuples, ranges, lists, etc.

An abstract data type might have multiple implementations.

- Some representations are better suited to some problems

A function might want to operate on multiple data types.

Message passing enables us to accomplish all of the above, as we will see today and next time

String Representations

An object value should **behave** like the kind of data it is meant to represent;

For instance, by **producing a string** representation of itself.

Strings are important: they represent *language* and *programs*.

In Python, all objects produce two string representations:

- The "str" is legible to **humans**.
- The "repr" is legible to the **Python interpreter**.

"str" and "repr" strings are often the same! Think: numbers.

When the "str" and "repr" **strings are the same**, that's evidence that a programming language is legible by humans!

Message Passing Enables Polymorphism

Polymorphic function: A function that can be applied to many (*poly*) different forms (*morph*) of data

`str` and `repr` are both polymorphic; they apply to anything.

`repr` invokes a zero-argument method `__repr__` on its argument.

```
>>> today.__repr__()
'datetime.date(2013, 7, 16)'
```

`str` invokes a zero-argument method `__str__` on its argument. (But `str` is a class, not a function!)

```
>>> today.__str__()
'2013-07-16'
```

Aside: duck typing

- "If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck."
- In Python terms: it doesn't matter what the official type of something is if it understands the correct messages
- The `repr` function knows nothing about the types of objects that it's getting…
- …except that they can quack (they have a `__repr__` method).

Inheritance and Polymorphism

Inheritance also enables polymorphism, since subclasses provide at least as much behavior as their base classes

Example of function that works on all accounts:

```
def welfare(account):
    """Deposit $100 into an account if it has less
    than $100."""
    if account.balance < 100:
        return account.deposit(100)

>>> alice_account = CheckingAccount(0)
>>> welfare(alice_account)
100
>>> bob_account = SavingsAccount(0)
>>> welfare(bob_account)
98
```

Interfaces

Message passing allows **different data types** to respond to the **same message**.

A shared message that elicits similar behavior from different object classes is a powerful method of abstraction.

An *interface* is a **set of shared messages**, along with a specification of **what they mean**.

Classes that implement `__repr__` and `__str__` methods *that return Python- and human-readable* strings thereby **implement an interface** for producing Python string representations.

Classes that implement `__len__` and `__getitem__` are sequences.

Special Methods

Python operators and generic functions make use of methods with names like "`__name__`"

These are *special* or *magic methods*

Examples:

`len`	`__len__`
`+, +=`	`__add__, __iadd__`
`[], []=`	`__getitem__, __setitem__`
`.`	`__getattr__, __setattr__`

Example: Rational Numbers

```
class Rational(object):
    def __init__(self, numer, denom):
        g = gcd(numer, denom)
        self.numerator = numer // g
        self.denominator = denom // g
    def __repr__(self):
        return 'Rational({0}, {1})'.format(self.numerator,
                                           self.denominator)
    def __str__(self):
        return '{0}/{1}'.format(self.numerator,
                                self.denominator)
    def __add__(self, num):
        denom = self.denominator * num.denominator
        numer1 = self.numerator * num.denominator
        numer2 = self.denominator * num.numerator
        return Rational(numer1 + numer2, denom)
    def __eq__(self, num):
        return (self.numerator == num.numerator and
                self.denominator == num.denominator)
```

Property Methods

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numerator = 4
>>> f.float_value
0.8
>>> f.denominator -= 3
>>> f.float_value
2.0
```

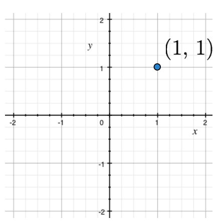
```
@property
def float_value(self):
    return (self.numerator //
            self.denominator)
```

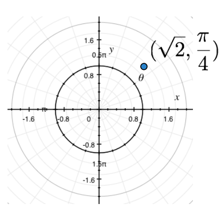
The `@property` decorator on a method designates that it will be called whenever it is *looked up* on an instance.

It allows zero-argument methods to be called without an explicit call expression.

Multiple Representations of Abstract Data

Rectangular and polar representations for complex numbers





Most operations don't care about the representation.

Some mathematical operations are easier on one than the other.

Arithmetic Abstraction Barriers

Complex numbers as whole data values

`add_complex` `mul_complex`

Complex numbers as two-dimensional vectors

`real` `imag` `magnitude` `angle`

Rectangular representation | *Polar representation*

An Interface for Complex Numbers

All complex numbers should have real and imag components.

All complex numbers should have a magnitude and angle.

Using this interface, we can implement complex arithmetic:

```
def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
                     z1.imag + z2.imag)

def mul_complex(z1, z2):
    return ComplexMA(z1.magnitude * z2.magnitude,
                     z1.angle + z2.angle)
```

The Rectangular Representation

```
class ComplexRI(object):

    def __init__(self, real, imag):
        self.real = real
        self.imag = imag

    @property
    def magnitude(self):
        return (self.real ** 2 + self.imag ** 2) ** 0.5

    @property
    def angle(self):
        return atan2(self.imag, self.real)

    def __repr__(self):
        return 'ComplexRI({0}, {1})'.format(self.real,
                                            self.imag)
```

Property decorator: "Call this function on attribute look-up"

math.atan2(y,x): Angle between x-axis and the point (x,y)

The Polar Representation

```
class ComplexMA(object):

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

    @property
    def real(self):
        return self.magnitude * cos(self.angle)

    @property
    def imag(self):
        return self.magnitude * sin(self.angle)

    def __repr__(self):
        return 'ComplexMA({0}, {1})'.format(self.magnitude,
                                            self.angle)
```

Using Complex Numbers

Either type of complex number can be passed as either argument to **add_complex** or **mul_complex**:

```
def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
                     z1.imag + z2.imag)

def mul_complex(z1, z2):
    return ComplexMA(z1.magnitude * z2.magnitude,
                     z1.angle + z2.angle)
```

```
>>> from math import pi
>>> add_complex(ComplexRI(1, 2), ComplexMA(2, pi/2))
ComplexRI(1.0000000000000002, 4.0)
>>> mul_complex(ComplexRI(0, 1), ComplexRI(0, 1))
ComplexMA(1.0, 3.141592653589793)
```

We can also define **__add__** and **__mul__** in both classes.

The Independence of Data Types

Data abstraction and class definitions keep types separate

Some operations need to cross type boundaries

How do we add a complex number and a rational number together?

`add_rational  mul_rational`

Rational numbers as numerators & denominators

`add_complex  mul_complex`

Complex numbers as two-dimensional vectors

There are many different techniques for doing this!

Type Dispatching

Define a different function for each possible combination of types for which an operation (e.g., addition) is valid

```
def iscomplex(z):
    return type(z) in (ComplexRI, ComplexMA)
def isrational(z):
    return type(z) is Rational
def add_complex_and_rational(z, r):
    return ComplexRI(z.real + r.numerator / r.denominator,
                     z.imag)
def add_by_type_dispatching(z1, z2):
    """Add z1 and z2, which may be complex or rational."""
    if iscomplex(z1) and iscomplex(z2):
        return add_complex(z1, z2)
    elif iscomplex(z1) and isrational(z2):
        return add_complex_and_rational(z1, z2)
    elif isrational(z1) and iscomplex(z2):
        return add_complex_and_rational(z2, z1)
    else:
        add_rational(z1, z2)
```

Converted to a real number (float)

Tag-Based Type Dispatching

Idea: Use dictionaries to dispatch on type (like we did for message passing)

```
def type_tag(x):
    return type_tags[type(x)]

type_tags = {ComplexRI:  'com',
             ComplexMA:  'com',
             Rational:   'rat'}

def add(z1, z2):
    types = (type_tag(z1), type_tag(z2))
    return add_implementations[types](z1, z2)

add_implementations = {}
add_implementations[('com', 'com')] = add_complex
add_implementations[('rat', 'rat')] = add_rational
add_implementations[('com', 'rat')] = add_complex_and_rational
add_implementations[('rat', 'com')] = add_rational_and_complex
```

Declares that **ComplexRI** and **ComplexMA** should be treated uniformly

`lambda r, z: add_complex_and_rational(z, r)`

Type Dispatching Analysis

Minimal violation of abstraction barriers: we define cross-type functions as necessary, but use abstract data types

Extensible: Any new numeric type can "install" itself into the existing system by adding new entries to various dictionaries

```
def add(z1, z2):
    types = (type_tag(z1), type_tag(z2))
    return add_implementations[types](z1, z2)
```

Question: How many cross-type implementations are required to support *m* types and *n* operations?

integer, rational, real, complex

$m \cdot (m-1) \cdot n$

add, subtract, multiply, divide

$4 \cdot (4-1) \cdot 4 = 48$

Type Dispatching Analysis

Minimal violation of abstraction barriers: we define cross-type functions as necessary, but use abstract data types

Extensible: Any new numeric type can "install" itself into the existing system by adding new entries to various dictionaries

Arg 1	Arg 2	Add	Multiply
Complex	Complex		
Rational	Rational		
Complex	Rational		
Rational	Complex		

Type Dispatching

Message Passing

Data-Directed Programming

There's nothing addition-specific about **add**

Idea: One dispatch function for (operator, types) pairs

```
def apply(operator_name, x, y):
    tags = (type_tag(x), type_tag(y))
    key = (operator_name, tags)
    return apply_implementations[key](x, y)

apply_implementations = {
    ('add', ('com', 'com')): add_complex,
    ('add', ('rat', 'rat')): add_rational,
    ('add', ('com', 'rat')): add_complex_and_rational,
    ('add', ('rat', 'com')): add_rational_and_complex,
    ('mul', ('com', 'com')): mul_complex,
    ('mul', ('rat', 'rat')): mul_rational,
    ('mul', ('com', 'rat')): mul_complex_and_rational,
    ('mul', ('rat', 'com')): mul_rational_and_complex
    }
```

Coercion

Idea: Some types can be converted into other types

Takes advantage of structure in the type system

```
def rational_to_complex(x):
    return ComplexRI(x.numerator / x.denominator, 0)

coercions = {('rat', 'com'): rational_to_complex}
```

Question: Can any numeric type be coerced into any other?

Question: Have we been repeating ourselves with data-directed programming?

Applying Operators with Coercion

1. Attempt to coerce arguments into values of the same type

2. Apply type-specific (not cross-type) operations

```
def coerce_apply(operator_name, x, y):
    tx, ty = type_tag(x), type_tag(y)
    if tx != ty:
        if (tx, ty) in coercions:
            tx, x = ty, coercions[(tx, ty)](x)
        elif (ty, tx) in coercions:
            ty, y = tx, coercions[(ty, tx)](y)
        else:
            return 'No coercion possible.'
    assert tx == ty
    key = (operator_name, tx)
    return coerce_implementations[key](x, y)
```

Coercion Analysis

Minimal violation of abstraction barriers: we define cross-type coercion as necessary, but use abstract data types

Requires that all types can be coerced into a common type

More sharing: All operators use the same coercion scheme

Arg 1	Arg 2	Add	Multiply
Complex	Complex		
Rational	Rational		
Complex	Rational		
Rational	Complex		

From	To	Coerce
Complex	Rational	
Rational	Complex	

Type	Add	Multiply
Complex		
Rational		