61A LECTURE 15 – MEMOIZATION, RECURSIVE DATA, SETS

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Now in a wider screen format!

Who am I? What am I doing here?

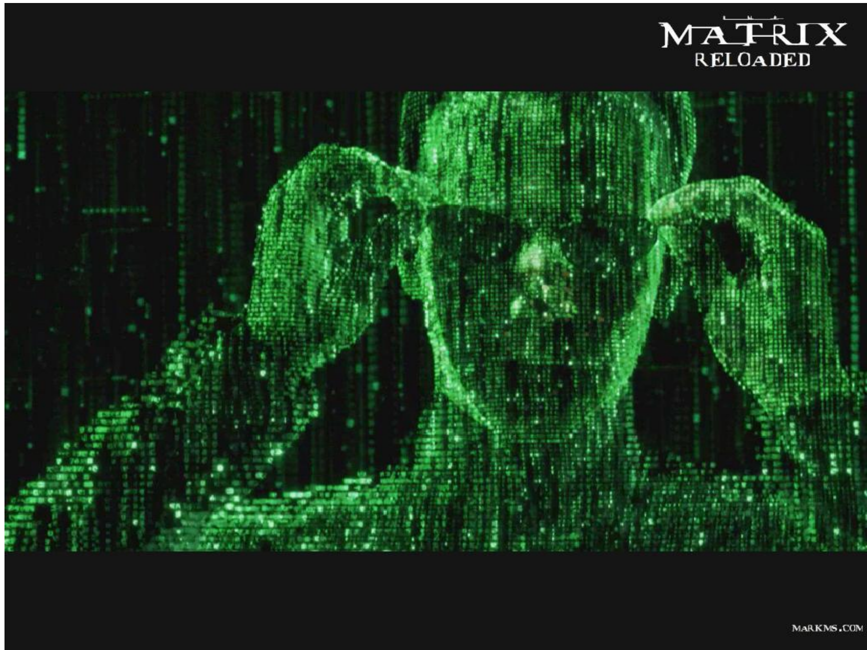
- First two weeks of class (Chapter 1):
  - FUNCTIONS
  - Computational processes, role of functions
- Past 2 weeks of class (Chapter 2):
  - DATA
  - Real-world phenomena are complex – Mining Twitter data!
- Today and the next 2 weeks (Chapter 3):
  - **PROGRAMS and their INTERPRETATION**

Next 2 weeks

- A Python program is just a collection of text
- This text only has meaning through *interpretation*
- Programming languages like Python are useful because we can define an *interpreter,* a program that carries out Python's evaluation and execution procedures

An interpreter, which determines the meaning of expressions, is really just another program.

We are not only *users* of languages designed by others, we are *designers* of languages.



Interpreters...

- Writing our own interpreters will be exciting! We will cover this next week.
- First, though, we need to learn a few background tools and techniques
- Today's lecture focuses on several ideas that will help us later create our interpreters

Announcements

- Do homework!
- Potluck next Friday, July 26 6-8pm, in the Woz Lounge (same place as last time)
  - Don’t make any other plans for Friday!
  - Bring some food, enjoy other people’s food!
  - Come mingle with fellow students and the teaching staff! No project or midterm due that week!

Speeding up computation

```
def fib_iter(n):
    if n == 0:
        return 0
    fib_n, fib_n_1 = 1, 0
    k = 1
    while k < n:
        fib_n, fib_n_1 = fib_n_1 + fib_n, fib_n
        k += 1
    return fib_n
```

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    return fib(n - 1) + fib(n - 2)
```

Speeding up computation

- fib_iter seems to be much faster when the input is large!
- Why?
  - The recursive function calls fib many times (about 2 fib recursive calls are generated for each call to fib). When you have a LOT of function calls, then computation will take much longer (think: a new frame has to be created for each call)
- How can we speed up the recursive version?

Memoization

Tree recursive functions often compute the same thing many times

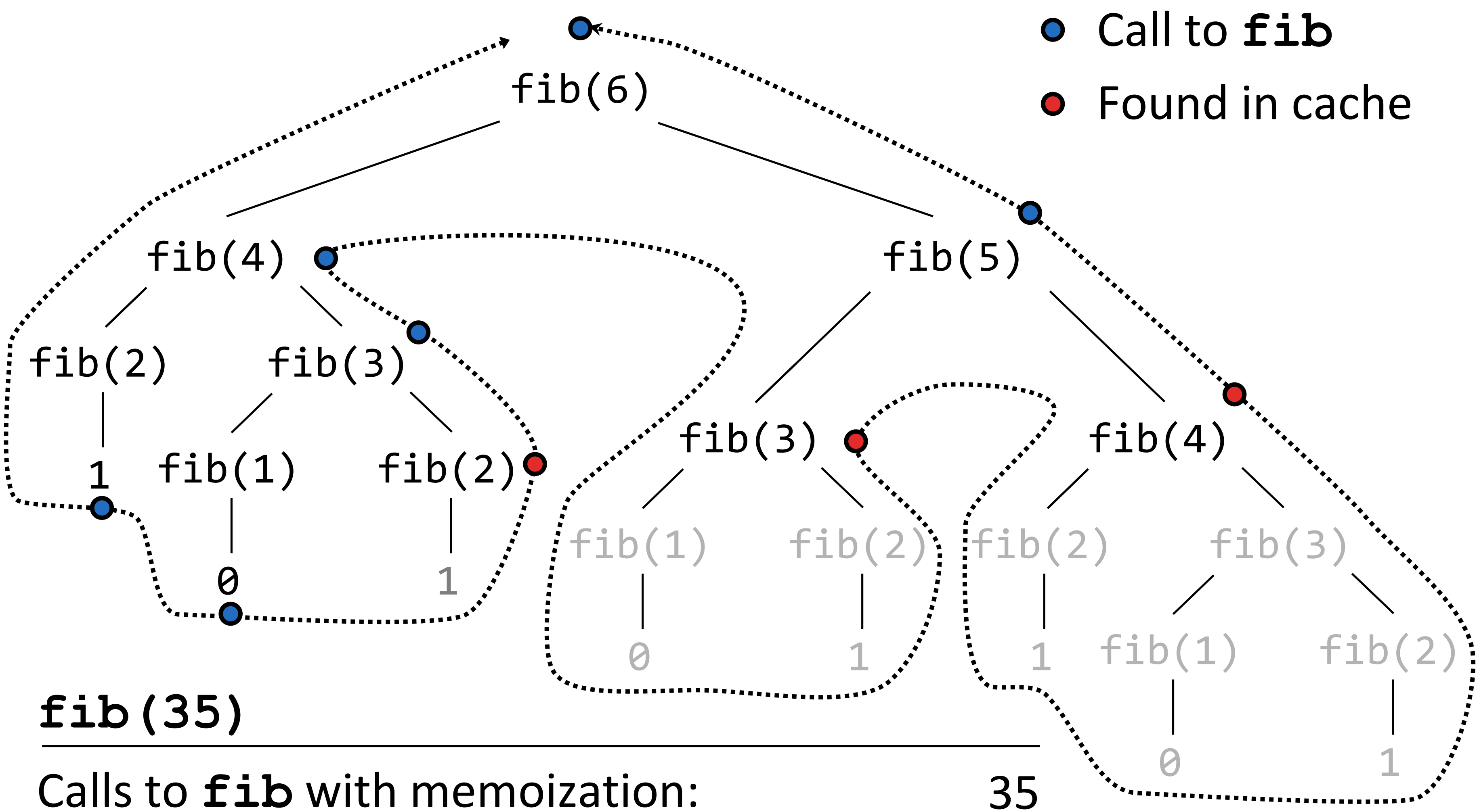
Idea: Remember the results that have been computed before

```
def memo(f):
    cache = {}
    def memoized(n):
        if n not in cache:
            cache[n] = f(n)
        return cache[n]
    return memoized
```

Keys are arguments that map to return values

Same behavior as **f**, if **f** is a pure function

Memoized Tree Recursion



fib(35)

Calls to **fib** with memoization:	35
Calls to **fib** without memoization:	18,454,929

When does memo speed computation up?

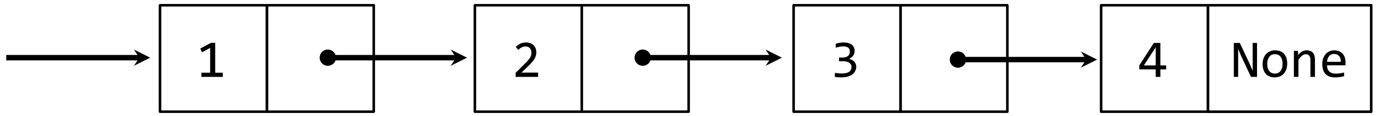
- `memo_factorial(5)` – not sped up the first time we call it
- What if we called `memo_factorial(5)` again?
- Memoization speeds computation up when the function is called more than once, perhaps through recursion; otherwise, no effect other than minor assignments to the memo dictionary
- The memoized version of fib computes more efficiently
- We will discuss a more precise definition for “computes more efficiency” tomorrow

Closure Property of Data

A tuple can contain another tuple as an element.

Pairs are sufficient to represent sequences.

Recursive list representation of the sequence 1, 2, 3, 4:



Recursive lists are recursive: the rest of the list is a list.

Nested pairs (old): `(1, (2, (3, (4, None))))`

Rlist class (new): `Rlist(1, Rlist(2, Rlist(3, Rlist(4))))`

Recursive List Class

Methods can be recursive as well!

```
class Rlist(object):
    class EmptyList(object):
        def __len__(self):
            return 0
    empty = EmptyList()
    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest
    def __len__(self):
        return 1 + len(self.rest)
    def __getitem__(self, i):
        if i == 0:
            return self.first
        return self.rest[i - 1]
```

There's the base case!

Yes, this call is recursive

Recursive Operations on Rlists

Recursive list processing almost always involves a recursive call on the rest of the list.

```
>>> s = Rlist(1, Rlist(2, Rlist(3)))

>>> s.rest
Rlist(2, Rlist(3))

>>> extend_rlist(s.rest, s)
Rlist(2, Rlist(3, Rlist(1, Rlist(2, Rlist(3)))))
```

```
def extend_rlist(s1, s2):
    if s1 is Rlist.empty:
        return s2
    return Rlist(s1.first, extend_rlist(s1.rest, s2))
```

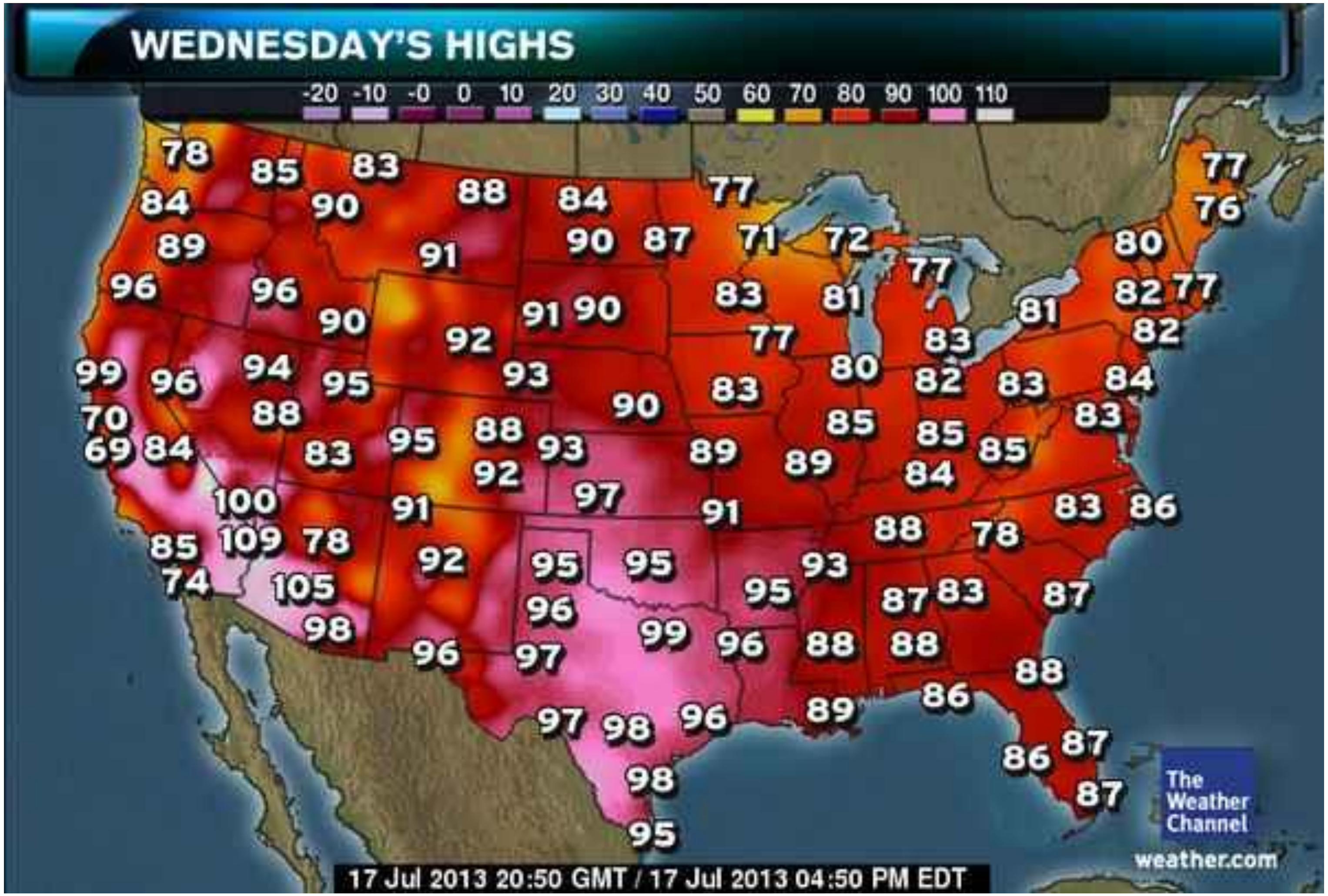
Map and Filter on Rlists

We want operations on a whole list, not an element at a time.

```
def map_rlist(s, fn):
    if s is Rlist.empty:
        return s
    return Rlist(fn(s.first), map_rlist(s.rest, fn))

def filter_rlist(s, fn):
    if s is Rlist.empty:
        return s
    rest = filter_rlist(s.rest, fn)
    if fn(s.first):
        return Rlist(s.first, rest)
    return rest
```

Break!



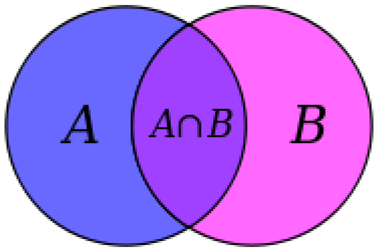
WEDNESDAY'S HIGHS
-20 -10 -0 0 10 20 30 40 50 60 70 80 90 100 110
17 Jul 2013 20:50 GMT / 17 Jul 2013 04:50 PM EDT
The Weather Channel
weather.com

Sets

A built-in Python container type

- Set literals are enclosed in braces
- Duplicate elements are removed on construction
- Sets are unordered, just like dictionary entries

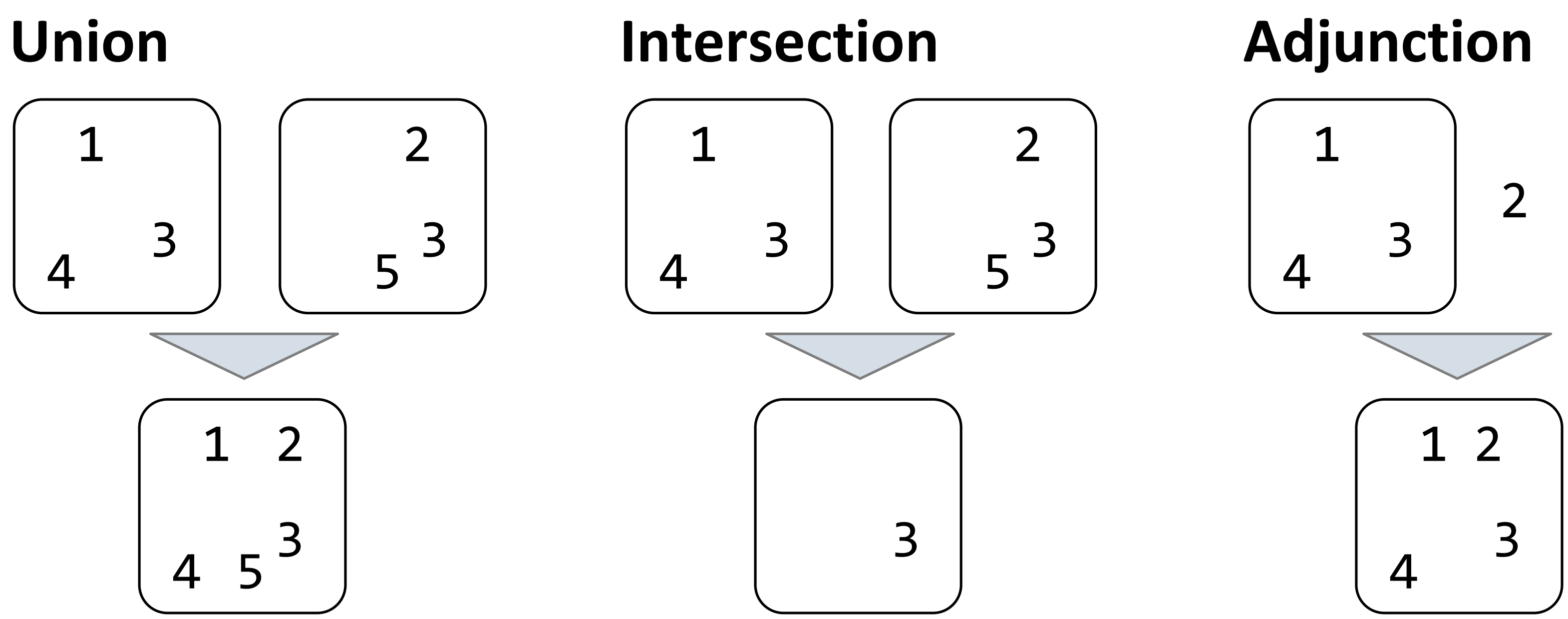

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
>>> 3 in s
True
>>> len(s)
4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```



Implementing Sets

What we should be able to do with a set:

- Membership testing: Is a value an element of a set?
- Union: Return a set with all elements in *set1* **or** *set2*
- Intersection: Return a set with any elements in *set1* **and** *set2*
- Adjunction: Return a set with all elements in *s* and a value *v*



Sets as Unordered Sequences

Proposal 1: A set is represented by a recursive list that contains no duplicate items

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
        return False
    elif s.first == v:
        return True
    return set_contains(s.rest, v)
```

Sets as Unordered Sequences

```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    return Rlist(v, s)


def intersect_set(set1, set2):
    f = lambda v: set_contains(set2, v)
    return filter_rlist(set1, f)


def union_set(set1, set2):
    f = lambda v: not set_contains(set2, v)
    set1_not_set2 = filter_rlist(set1, f)
    return extend_rlist(set1_not_set2, set2)
```

We will talk about how “efficient” these operations are next class!

Sets as Ordered Sequences

Proposal 2: A set is represented by a recursive list with unique elements ordered from least to greatest

```
def set_contains2(s, v):
    if empty(s) or s.first > v:
        return False
    elif s.first == v:
        return True
    return set_contains(s.rest, v)
```

Set Intersection Using Ordered Sequences

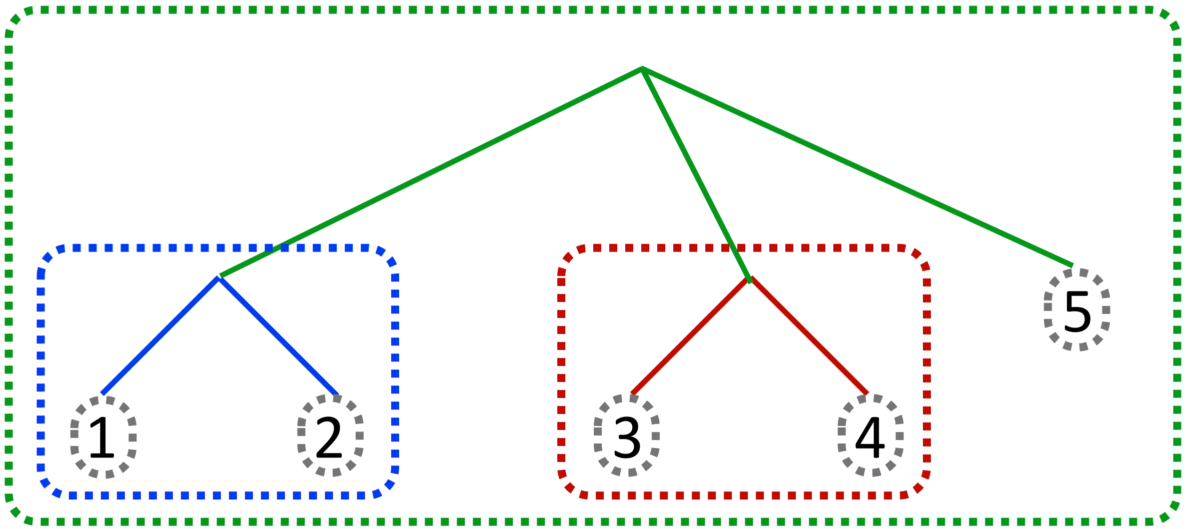
This algorithm assumes that elements are in order.

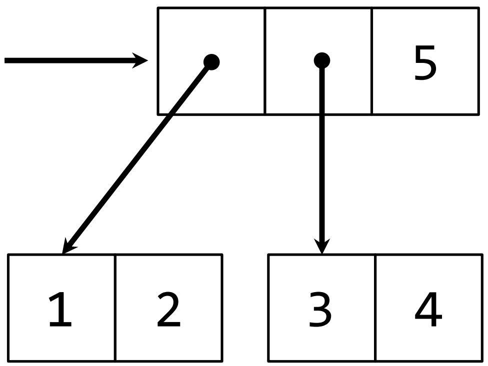
```
def intersect_set2(set1, set2):
    if empty(set1) or empty(set2):
        return Rlist.empty
    e1, e2 = set1.first, set2.first
    if e1 == e2:
        rest = intersect_set2(set1.rest, set2.rest)
        return Rlist(e1, rest)
    elif e1 < e2:
        return intersect_set2(set1.rest, set2)
    elif e2 < e1:
        return intersect_set2(set1, set2.rest)
```

Tree Structured Data

Nested Sequences are Hierarchical Structures.

`((1, 2), (3, 4), 5)`





In every tree, a vast forest

Recursive Tree Processing

Tree operations typically make recursive calls on branches

```
def count_leaves(tree):
    if type(tree) != tuple:
        return 1
    return sum(map(count_leaves, tree))


def map_tree(tree, fn):
    if type(tree) != tuple:
        return fn(tree)
    return tuple(map_tree(branch, fn)
                 for branch in tree)
```

Trees with Internal Node Values

Trees can have values at internal nodes as well as their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right

def fib_tree(n):
    if n == 0:
        return Tree(0)
    if n == 1:
        return Tree(1)
    left = fib_tree(n - 2)
    right = fib_tree(n - 1)
    return Tree(left.entry + right.entry, left, right)
```

Trees with Internal Node Values

Trees can have values at internal nodes as well as their leaves.