61A LECTURE 17 – ORDERS OF GROWTH, EXCEPTIONS

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July 23, 2013

Announcements

- Regrades for project 1 composition scores, due by next **Monday**
  - See Piazza post for more details
- **Midterm 2** is next Thursday, August 1, at 7pm.
  - If you have a conflict at that time, fill out the conflict form on Piazza ASAP
- Potluck on Friday in the Woz at 6PM. See you there!

Order of Growth

A method for bounding the resources used by a function as the "size" of a problem increases

n: size of the problem

R(n): Measurement of some resource used (time or space)

$$R(n) = \Theta(f(n))$$

means that there are positive constants k_1 and k_2 such that

$$k_1 \cdot f(n) \leq R(n) \leq k_2 \cdot f(n)$$

for sufficiently large values of ***n***.

A graphical explanation

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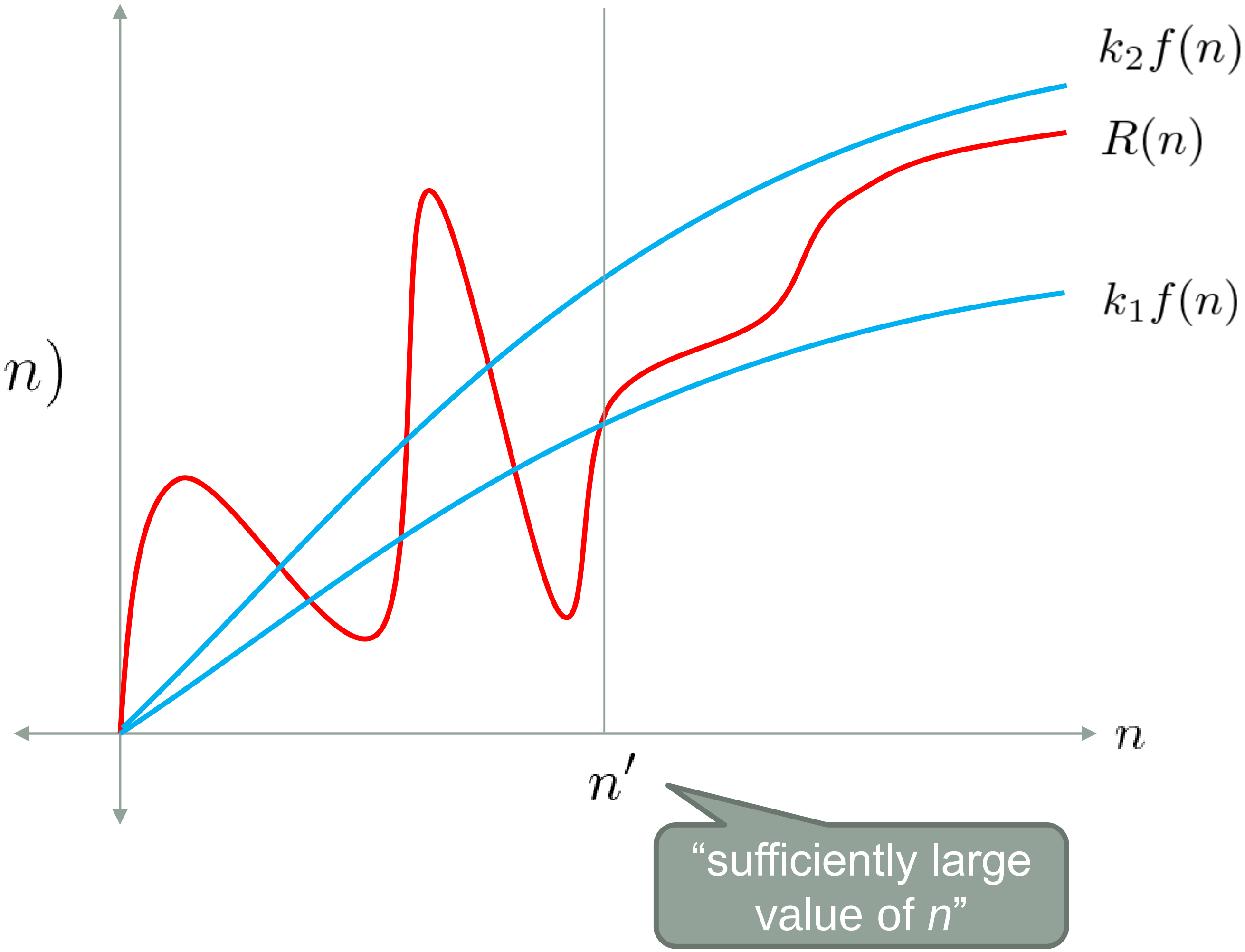
for sufficiently large values of ***n***.



Warm up!

Time

```
def factorial(n):
    if n == 0:
        return 1
    return n * factorial(n - 1)
```

$\Theta(n)$

```
def sunshine(n):
    if n == 0:
        return 0
    happiness = 1
    while happiness < 10000000:
        happiness += 1
    return happiness + sunshine(n - 1)
```

A constant amount of work – doesn't contribute to the order of growth!

$\Theta(n)$

```
def eternity(n):
    i = 0
    while i < n:
        factorial(n)
        i += 1
```

$\Theta(n^2)$

Comparing Orders of Growth (*n* is problem size)

$\Theta(b^n)$ Exponential growth! Recursive fib takes

$\Theta(\phi^n)$ steps, where $\phi = \frac{1+\sqrt{5}}{2} \approx 1.61828$

$\Theta(n^6)$ → Incrementing the problem scales *R(n)* by a factor.

$\Theta(n^2)$ Quadratic growth. E.g., operations on all pairs.

Incrementing *n* increases *R(n)* by the problem size *n*.

$\Theta(n)$ Linear growth. Resources scale with the problem.

$\Theta(\sqrt{n})$ →

$\Theta(\log n)$ Logarithmic growth. These processes scale well.

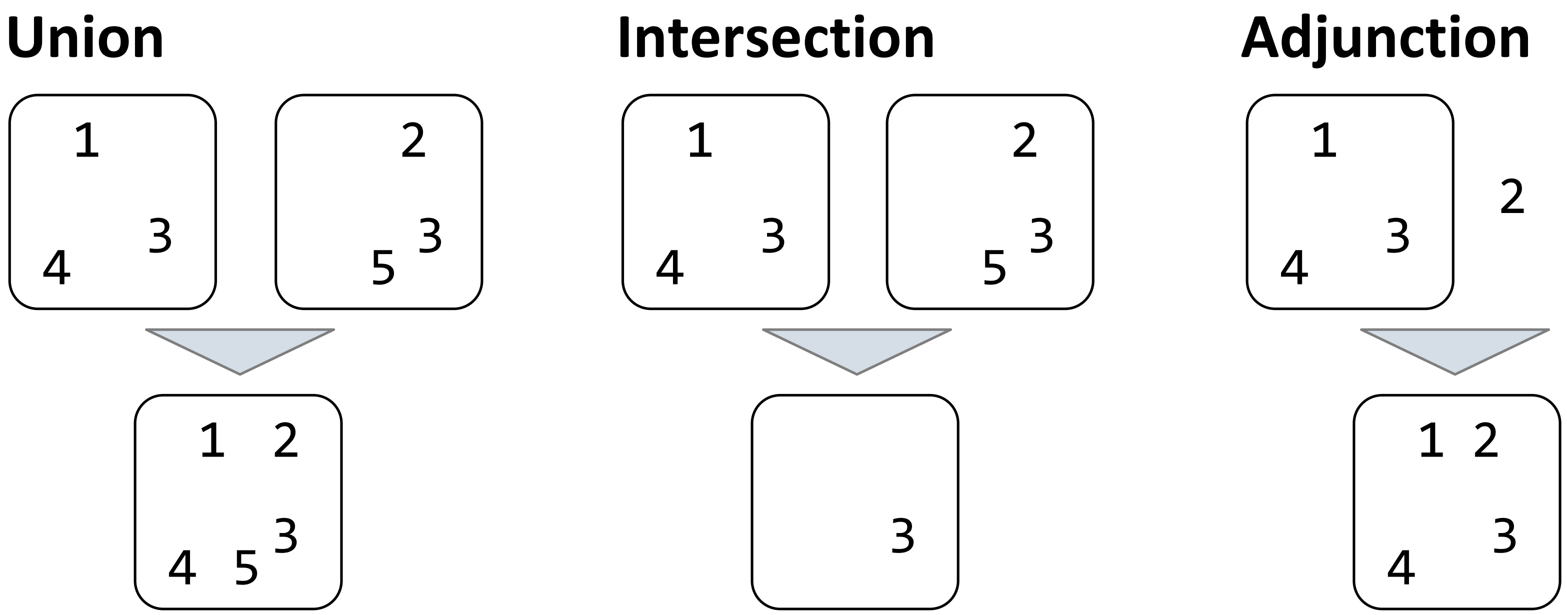
Doubling the problem only increments *R(n)*.

$\Theta(1)$ Constant. The problem size doesn't matter.

Implementing Sets

What we should be able to do with a set:

- Membership testing: Is a value an element of a set?
- Union: Return a set with all elements in *set1* **or** *set2*
- Intersection: Return a set with any elements in *set1* **and** *set2*
- Adjunction: Return a set with all elements in *s* and a value *v*



Implementation considerations

- Many ways to accomplish this
- Not all solutions are made equal!
- Some implementations might be better than other implementations when performing certain operations

Sets as Unordered Sequences

Proposal 1: A set is represented by a recursive list that contains no duplicate items

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
        return False
    elif s.first == v:
        return True
    return set_contains(s.rest, v)
```

$$\Theta(n)$$

The size of the set

Sets as Unordered Sequences

```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    return Rlist(v, s)
```

```
def intersect_set(set1, set2):
    f = lambda v: set_contains(set2, v)
    return filter_rlist(set1, f)
```

```
def union_set(set1, set2):
    f = lambda v: not set_contains(set2, v)
    set1_not_set2 = filter_rlist(set1, f)
    return extend_rlist(set1_not_set2, set2)
```

Time order of growth

$\Theta(n)$

The size of the set

$\Theta(n^2)$

Assume sets are the same size

$\Theta(n^2)$

Sets as Ordered Sequences

Proposal 2: A set is represented by a recursive list with unique elements ordered from least to greatest

```
def set_contains2(s, v):
    if empty(s) or s.first > v:
        return False
    elif s.first == v:
        return True
    return set_contains2(s.rest, v)
```

Order of growth? $\Theta(n)$

Compare

```
def set_contains(s, v):
    if empty(s):
        return False
    elif s.first == v:
        return True
    return set_contains(s.rest, v)

def set_contains2(s, v):
    if empty(s) or s.first > v:
        return False
    elif s.first == v:
        return True
    return set_contains(s.rest, v)
```

Both functions have an order of growth $\Theta(n)$

set_contains2 is slightly more optimized than set_contains, but they are still both linear time operations.

Set Intersection Using Ordered Sequences

This algorithm assumes that elements are in order.

```
def intersect_set2(set1, set2):
    if empty(set1) or empty(set2):
        return Rlist.empty
    e1, e2 = set1.first, set2.first
    if e1 == e2:
        rest = intersect_set2(set1.rest, set2.rest)
        return Rlist(e1, rest)
    elif e1 < e2:
        return intersect_set2(set1.rest, set2)
    elif e2 < e1:
        return intersect_set2(set1, set2.rest)
```

Order of growth? $\Theta(n)$

Compare to the first version of intersect_set.

Trees with Internal Node Values

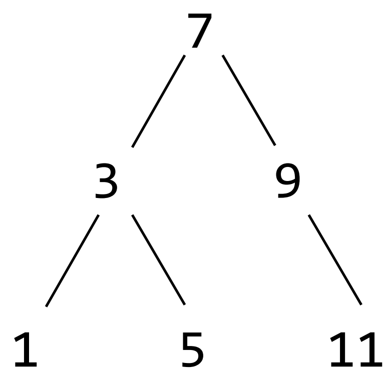
Trees can have values at internal nodes as well as their leaves.

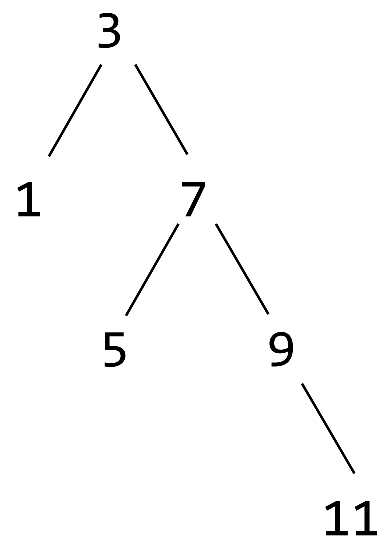
```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right
```

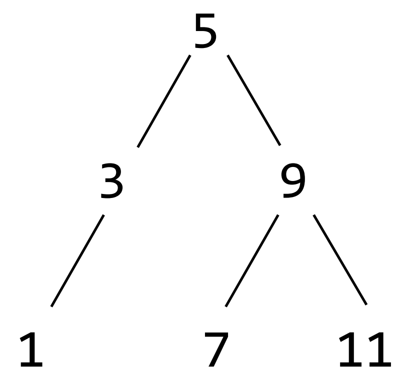
Tree Sets

Proposal 3: A set is represented as a Tree. Each entry is:

- Larger than all entries in its left branch and
- Smaller than all entries in its right branch





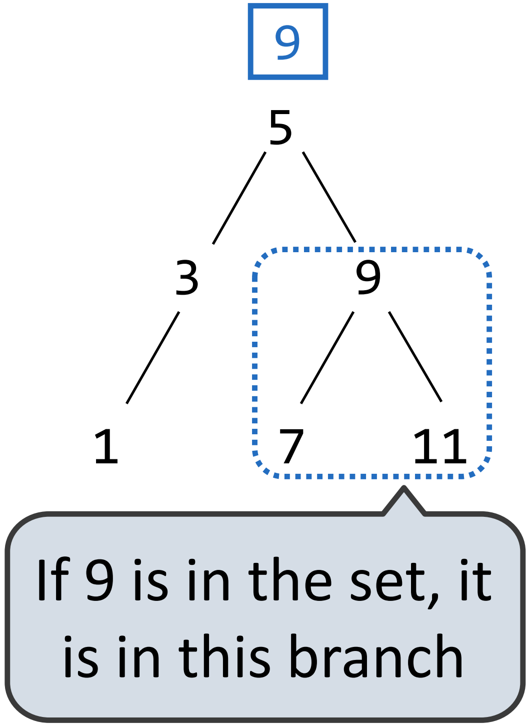


Membership in Tree Sets

Set membership tests traverse the tree

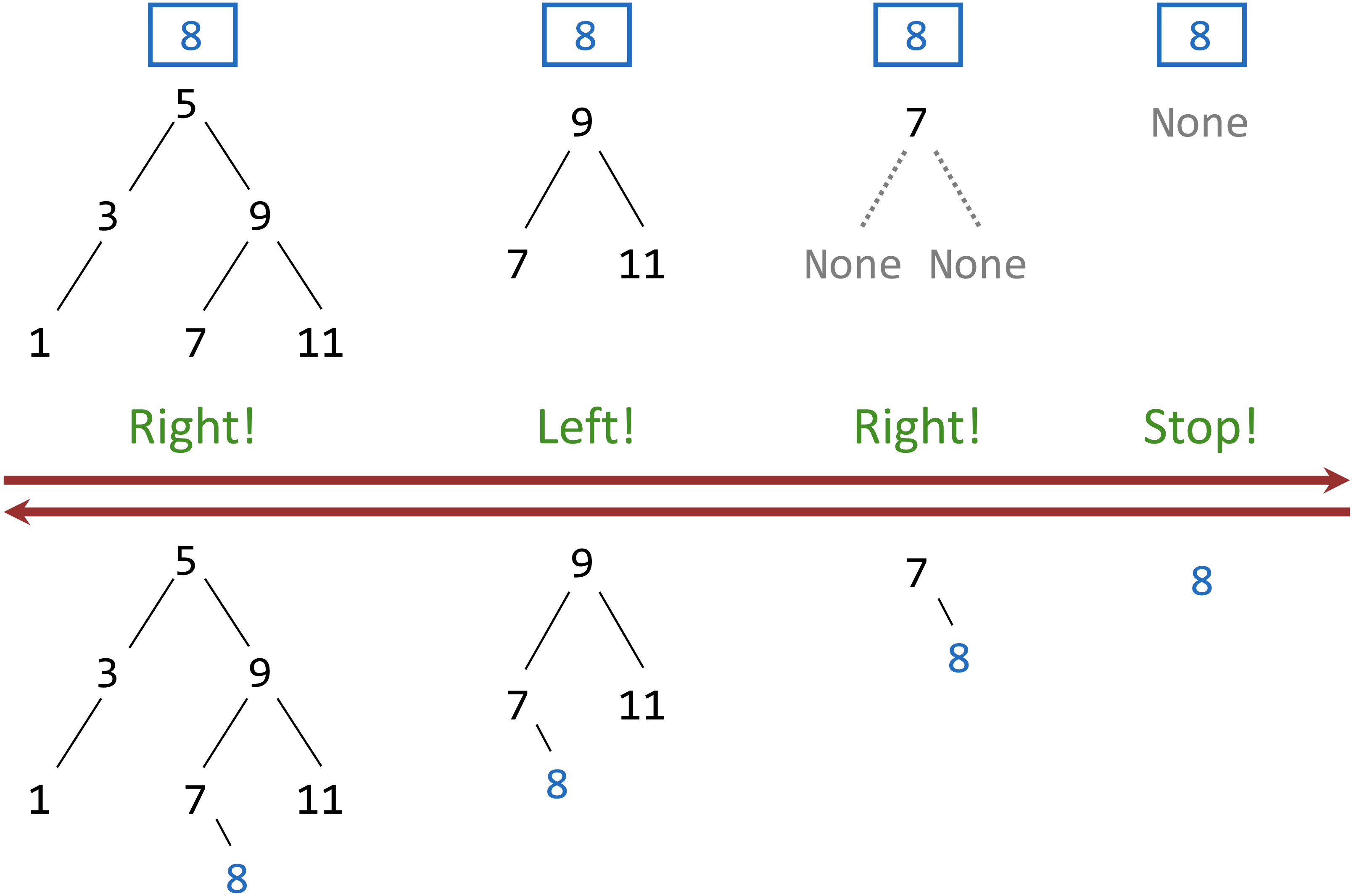
- The element is either in the left or right sub-branch
- By focusing on one branch, we reduce the set by about half

```
def set_contains3(s, v):
    if s is None:
        return False
    elif s.entry == v:
        return True
    elif s.entry < v:
        return set_contains3(s.right, v)
    elif s.entry > v:
        return set_contains3(s.left, v)
```



Order of growth?

Adjoining to a Tree Set



What Did I Leave Out?

Sets as ordered sequences:

- Adjoining an element to a set
- Union of two sets

Sets as binary trees:

- Intersection of two sets
- Union of two sets

That's homework 9!

Break

Handling Errors

Sometimes, computers don't do exactly what we expect

- A function receives unexpected argument types
- Some resource (such as a file) is not available
- A network connection is lost

September 9 1947: Moth found in a Mark II Computer

Methods

Methods are defined in the suite of a class statement

```
class Account(object):
    def __init__(self, account_holder):
        self.balance = 0
        self.holder = account_holder

    def deposit(self, amount):
        self.balance = self.balance + amount
        return self.balance

    def withdraw(self, amount):
        if amount > self.balance:
            return 'Insufficient funds'
        self.balance = self.balance - amount
        return self.balance
```

These def statements create function objects as always, but their names are bound as attributes of the class.

Exceptions

A built-in mechanism in a programming language to declare and respond to exceptional conditions

Python *raises* an exception whenever an error occurs

Exceptions can be *handled* by the program, preventing a crash

Unhandled exceptions will cause Python to halt execution

Mastering exceptions:

Exceptions are objects! They have classes with constructors

They enable non-local continuations of control:

If **`f`** calls **`g`** and **`g`** calls **`h`**, exceptions can shift control from **`h`** to **`f`** without waiting for **`g`** to return

However, exception handling tends to be slow

Assert Statements

Assert statements raise an exception of type **`AssertionError`**

```
assert <expression>, <string>
```

Assertions are designed to be used liberally and then disabled in production systems

```
python3 -O
```

"O" stands for optimized. Among other things, it disables assertions

Whether assertions are enabled is governed by the built-in bool **`__debug__`**

Raise Statements

Exceptions are raised with a *raise statement*

```
raise <expression>
```

`<expression>` must evaluate to an exception instance or class.

Exceptions are constructed like any other object; they are just instances of classes that inherit from **`BaseException`**

`TypeError` -- A function was passed the wrong number/type of argument

`NameError` -- A name wasn't found

`KeyError` -- A key wasn't found in a dictionary

`RuntimeError` -- Catch-all for troubles during interpretation

Try Statements

Try statements handle exceptions

```
try:
    <try suite>
except <exception class> as <name>:
    <except suite>
...
```

Execution rule:

- The **<try suite>** is executed first;
- If, during the course of executing the **<try suite>**, an exception is raised that is not handled otherwise, and
- If the class of the exception inherits from **<exception class>**, then
- The **<except suite>** is executed, with **<name>** bound to the exception

Handling Exceptions

Exception handling can prevent a program from terminating

```
>>> try:
        x = 1/0
    except ZeroDivisionError as e:
        print('handling a', type(e))
        x = 0

handling a <class 'ZeroDivisionError'>
>>> x
0
```

Multiple try statements: Control jumps to the except suite of the most recent try statement that handles that type of exception.

WWPD: What Would Python Do?

How will the Python interpreter respond?

```
def invert(x):
    result = 1/x  # Raises a ZeroDivisionError if x is 0
    print('Never printed if x is 0')
    return result

def invert_safe(x):
    try:
        return invert(x)
    except ZeroDivisionError as e:
        return str(e)
```

```
>>> invert_safe(1/0)
>>> try:
        invert_safe(0)
    except BaseException:
        print('Handled!')

>>> inverrrt_safe(1/0)
```

Quick Break!

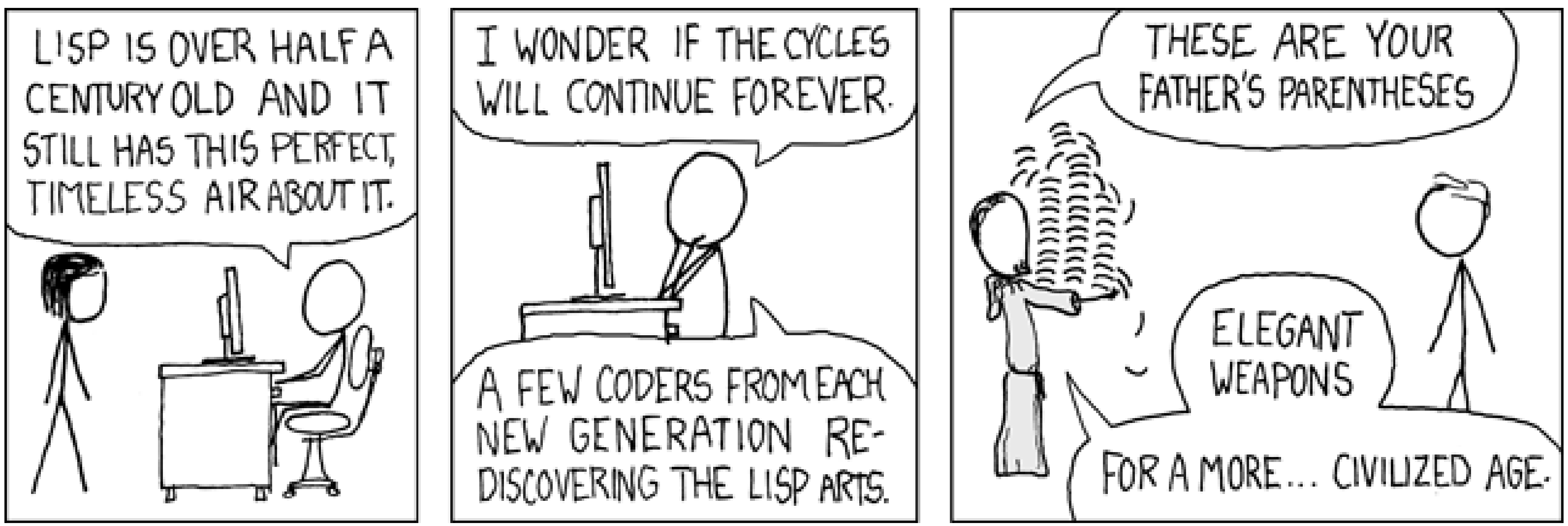
- We will start talking about Scheme today – Eric will dive more deeply into Scheme tomorrow!

Scheme Is a Dialect of Lisp

"The greatest single programming language ever designed."
-Alan Kay, co-inventor of OOP

"The most powerful programming language is Lisp. If you don't know Lisp (or its variant, Scheme), you don't appreciate what a powerful language is. Once you learn Lisp you will see what is missing in most other languages."
-Richard Stallman, founder of the Free Software movement

"Probably my favorite programming language."
-Eric Tzeng, CS61A Instructor



http://imgs.xkcd.com/comics/lisp_cycles.png

Scheme Fundamentals

Scheme programs consist of expressions, which can be:

- Primitive expressions: **`2`**, **`3.3`**, **`true`**, **`+`**, **`quotient`**, ...
- Combinations: **`(quotient 10 2)`**, **`(not true)`**, ...

Numbers are self-evaluating; symbols are bound to values

Call expressions have an operator and 0 or more operands

```
> (quotient 10 2)
5
> (quotient (+ 8 7) 5)
3
> (+ (* 3
        (+ (* 2 4)
           (+ 3 5)))
     (+ (- 10 7)
        6))
57
```

"quotient" names Scheme's built-in integer division procedure (i.e., function)

Combinations can span multiple lines
(spacing doesn't matter)

Special Forms

A combination that is not a call expression is a *special form*:

- **If** expression: (if <predicate> <consequent> <alternative>)
- **And** and **or**: (and <e_1> ... <e_n>), (or <e_1> ... <e_n>)
- Binding names: (define <name> <expression>)
- New procedures: (define (<name> <formal parameters>) <body>)

```
> (define pi 3.14)
> (* pi 2)
6.28
```

The name “pi” is bound to 3.14 in the global frame

```
> (define (abs x)
    (if (< x 0)
        (- x)
        x))
> (abs -3)
3
```

A procedure is created and bound to the name “abs”

Lambda Expressions

Lambda expressions evaluate to anonymous procedures

```
(lambda (<formal-parameters>) <body>)
```

Two equivalent expressions:

```
(define (plus4 x) (+ x 4))

(define plus4 (lambda (x) (+ x 4)))
```

An operator can be a combination too:

```
((lambda (x y z) (+ x y (square z))) 1 2 3)
```

Evaluates to the *add-x-&-y-&-*z^2 procedure

Pairs

We can implement pairs functionally:

```
(define (pair x y) (lambda (m) (if (= m 0) x y)))
(define (first p) (p 0))
(define (second p) (p 1))
```

Scheme also has built-in pairs that use weird names:

- **`cons`**: Two-argument procedure that **creates a pair**
- **`car`**: Procedure that returns the **first element** of a pair
- **`cdr`**: Procedure that returns the **second element** of a pair

A pair is represented by a dot between the elements, all in parens

```
> (cons 1 2)
(1 . 2)
> (car (cons 1 2))
1
> (cdr (cons 1 2))
2
```

Recursive Lists

A recursive list can be represented as a pair in which the second element is a recursive list or the empty list

Scheme lists are recursive lists:

- **`nil`** is the empty list
- A non-empty Scheme list is a pair in which the second element is **`nil`** or a Scheme list

Scheme lists are written as space-separated combinations

```
> (define x (cons 1 (cons 2 (cons 3 (cons 4 nil)))))
> x
(1 2 3 4)
> (cdr x)
(2 3 4)
> (cons 1 (cons 2 (cons 3 4)))
(1 2 3 . 4)
```

Not a well-formed list!