# RECURSION: *n.* SEE RECURSION 3

COMPUTER SCIENCE 61A

June 30, 2015

## 1 Recursion

A *recursive* function is a function that calls itself. Below is a recursive `factorial` function.

```
def factorial(n):
    if n == 0 or n == 1:
        return 1
    else:
        return n * factorial(n-1)
```

Although we haven't finished defining `factorial`, we are still able to call it since the function body is not evaluated until the function is called. We do have one *base case*: when `n` is `0` or `1`. Now we can compute `factorial(2)` in terms of `factorial(1)`, and `factorial(3)` in terms of `factorial(2)`, and `factorial(4)` – well, you get the idea.

There are *three* common steps in a recursive definition:

1. *Figure out your base case*: What is the simplest argument we could possibly get? For example, `factorial(0)` is 1 by definition.
2. *Make a recursive call with a simpler argument*: Simplify your problem, and assume that a recursive call for this new problem will simply work. This is called the "leap of faith". For `factorial`, we reduce the problem by calling `factorial(n-1)`.
3. *Use your recursive call to solve the full problem*: Remember that we are assuming your recursive call works. With the result of the recursive call, how can you solve the original problem you were asked? For `factorial`, we just multiply $(n-1)!$ by $n$.

## 1.1 Cool recursion questions!

1. Print out a countdown using recursion.

```
def countdown(n):
    """
    >>> countdown(3)
    3
    2
    1
    """
```

   First, think about a base case. What is the simplest input the problem could be given?

   After you've thought of a base case, think about a recursive call with a smaller argument that approches the base case. What happens if you call `countdown(n - 1)`?

   Then, put the base case and the recursive call together, and think about where a print statement would be needed.

2. Is there an easy way to change `countdown` to count up instead?

3. Write a function `recursive_mul(m, n)` that multiplies two numbers `m` and `n`. Assume `m` and `n` are positive integers. Use recursion, not `mul` or `*`!

   Hint: `5*3 = 5 + 5*2 = 5 + 5 + 5*1`.

   For the base case, what is the simplest possible input for `recursive_mul`?

   For the recursive case, what does calling `multiply(m - 1, n)` do? What does calling `multiply(m, n - 1)` do? Which one do we want to use?

```
def multiply(m, n):
    """
    >>> multiply(5, 3)
    15
    """
```

4. Write a procedure `expt(base, power)`, which implements the exponent function. For example, `expt(3, 2)` returns 9, and `expt(2, 3)` returns 8. Assume `power` is always a non-negative integer. Use recursion, not `pow`!

```
def expt(base, power):
```

5. Write a recursive function that sums the digits of a number `n`. Assume `n` is positive. You might find the operators `//` and `%` useful.

```
def sum_digits(n):
    """
    >>> sum_digits(7)
    7
    >>> sum_digits(30)
    3
    >>> sum_digits(228)
    12
    """
```

6. Below is the iterative version of `is_prime`, which returns `True` if positive integer `n` is a prime number and `False` otherwise:

```
def is_prime(n):
    if n == 1:
        return False
    k = 2
    while k < n:
        if n % k == 0:
            return False
        k += 1
    return True
```

Implement the recursive `is_prime` function. Do not use a while loop, use recursion.

```
def is_prime(n):
```

7. Write `sum_primes_up_to(n)`, which sums up every prime up to and including `n`. Assume you have an `is_prime(n)` predicate.

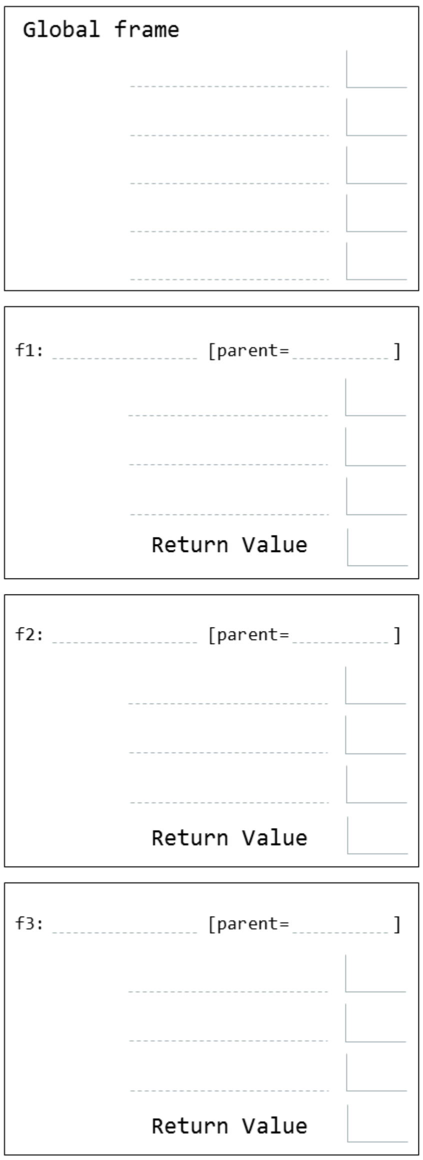
```
def sum_primes_up_to(n):
```

## 1.2 Recursive Environment Diagram!

1. Draw an environment diagram for the following code:

```
def rec(x, y):
    if y > 0:
        return x * rec(x, y - 1)
    return 1
rec(3, 2)
```

Bonus question: what does this function do?



## 2 Iteration vs. Recursion

We've written `factorial` recursively. Let's compare the iterative and recursive versions:

```
def factorial_recursive(n):
    if n == 0 or n == 1:
        return 1
    else:
        return n * factorial_recursive(n-1)

def factorial_iterative(n):
    total = 1
    while n > 1:
        total = total * n
        n = n - 1
    return total
```

Notice, while the recursive function "works" until n is less than or equal to 0, the iterative function "works" while n is greater than 0. They're essentially the same.

Let's also compare `fibonacci`.

```
def fib_recursive(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib_recursive(n - 1) + fib_recursive(n - 2)

def fib_iterative(n):
    current, next = 0, 1
    while n > 0:
        current, next = next, current + next
        n = n - 1
    return current
```

For the recursive version, we copied the definition of the Fibonacci sequence straight into code! The $n$th fibonacci number is simply the sum of the two before it. Iteratively, you need to keep track of more numbers and have a better understanding of the code.

Some code is easier to write iteratively and some recursively. Have fun experimenting with both!