## Lecture 2: Functions

Brian Hou<br>June 21, 2016

## Announcements

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## Expressions

Primitive expressions, names, and environments

## Primitive expressions

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- Functions (later today!)


## Primitive expressions

- Expressions in programs evaluate to values
- Primitive expressions evaluate directly to values with minimal work needed
- Numbers (e.g. 42, 3.14, 0)
- Names (e.g. pi, add)
- Functions (later today!)
- Some non-primitive expressions: $1 * 2$, add(3, 4)

Names

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Execution Rule for Assignment Statements:

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2. Bind all names to the left of $=$ to those resulting values in the current environment frame.

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## (demo)

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## Environment diagrams

- Environment diagrams visualize the interpreter's progress


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$$
\begin{array}{ll}
1 & x=1 \\
2 & y=x
\end{array}
$$

## Global frame

x 1

## Environment diagrams

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$$
\begin{array}{rl}
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\Rightarrow 2 & y=x
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## Global frame

x 1

Code (left)

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Code (left)
Frames (right)

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## Global frame

x 1

Code (left)
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Next to execute

## Global frame <br> x 1

Statements and expressions

## Environment diagrams

- Environment diagrams visualize the interpreter's progress


## Global frame

Code (left)

```
Assignment statement
Next to execute Assignment statement
```

Next to execute Assignment statement

```

Frames (right)

Statements and expressions

\section*{Environment diagrams}
- Environment diagrams visualize the interpreter's progress


\section*{Code (left)}

Statements and expressions

\section*{Global frame}

Frames (right)
Each name is bound to a value

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\section*{Just executed}
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Each name is bound to a value

A name cannot be repeated in a frame

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\section*{Functions}

Call expressions, functions, and def statements

\section*{Call expressions}

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\[
\operatorname{add}(2,3)
\]

\section*{Call expressions}

\section*{add ( 2 , 3 ) \\ operator}

\section*{Call expressions}


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- Call expressions use functions to compute a value
- The operator and operands themselves are expressions
- To evaluate this call expression:
1. Evaluate the operator to get a function value
2. Evaluate the operands to get its values
3. Apply the function to the values of the operands to get the final value

\section*{Defining functions}

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- Functions have inputs and outputs

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def <name>(<parameters>):
return <return expression>

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def square(x):
return x * x
y = square(-2)

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Frames
Global frame \(\longrightarrow\) func square \((x)\) square
f1: square

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f1: square
x \(\quad-2\)
Return value

Return value
(not a binding!)

Break!

Environments

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Rules for looking up names in user-defined functions (version 1):
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Rules for looking up names in user-defined functions (version 1):
1. Look it up in the local frame
2. If name isn't in local frame, look it up in the global frame
3. If name isn't in either frame, NameError

\section*{Looking up names in environments}

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f1: square
\[
x \quad-2
\]

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>>> def square(x):
... return x * x
>>> y = square(square(-2))

```

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(demo)
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Frames

f1: square
\begin{tabular}{r|r}
\(x\) & -2 \\
\begin{tabular}{r|r} 
Return \\
value
\end{tabular} & 4
\end{tabular}
f2: square


\section*{Multiple environments}
(demo)
>>> def square(x):
•• \(\quad\) return \(x * x\)
\(\ggg\)
square \((\) square \((-2))\)

Frames

f1: square
\begin{tabular}{r|r}
x & \(\mathbf{- 2}\) \\
\(\begin{aligned} \text { Return } \\
\text { value }\end{aligned}\) & 4
\end{tabular}
f2: square


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f1: square
\begin{tabular}{r|r}
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None and Print

None means that nothing is returned

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>>> def does_not_square(x):
... X * X

```

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>>> def does_not_square(x):
No return \(>\)... \(x\) * \(x\)

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No return > .. x * x
>>> does_not_square(-2)

```
>>> def does_not_square(x):

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\section*{Pure and non-pure functions}

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Pure functions
just return values

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Pure functions
just return values
\(2,3>\max\)


\section*{Pure and non-pure functions}


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Non-Pure functions have side effects

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Non-Pure functions have side effects


Python displays the output "-2"

\section*{Pure and non-pure functions}


Non-Pure functions have side effects


\section*{Pure and non-pure functions}


Non-Pure functions have side effects
```

A side effect isn't a
value; it's anything
that happens as a
consequence of
calling a function

```
-2 print

Python displays the output "-2"

Nested expressions with print

Nested expressions with print
```

>>> print(print(1), print(2))

```

Nested expressions with print
>>> print(print(1)), print(2))

Nested expressions with print


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More Functions

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\text { >>> four = describe(square, }-2 \text { ) }
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\section*{More Functions}
- The operands of a call expression can be any expression
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\[
\begin{aligned}
& \text { >>> four }=\text { describe(square, }-2) \\
& \text { Calling function with argument }-2 \\
& \text { Result was } 4
\end{aligned}
\]

\section*{More Functions}
- The operands of a call expression can be any expression
- What about the expression square?
```

>>> four = describe(square, -2)
Calling function with argument -2
Result was 4
>>> four

```

\section*{More Functions}
- The operands of a call expression can be any expression
- What about the expression square?
```

>>> four = describe(square, -2)
Calling function with argument -2
Result was 4
>>> four
4

```

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- The operands of a call expression can be any expression
- What about the expression square?
```

>>> four = describe(square, -2)
Calling function with argument -2
Result was 4
>>> four
4
>>> sixteen = describe(square, four)

```

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- The operands of a call expression can be any expression
- What about the expression square?
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>>> four = describe(square, -2)
Calling function with argument -2
Result was 4
>>> four
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>>> sixteen = describe(square, four)
Calling function with argument 4
Result was 16

```

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- The operands of a call expression can be any expression
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Calling function with argument -2
Result was 4
>>> four
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>>> sixteen = describe(square, four)
Calling function with argument 4
Result was 16
>>> sixteen

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Result was 4
>>> four
4
>>> sixteen = describe(square, four)
Calling function with argument 4
Result was 16
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1 6

```

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Result was 4
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