Lecture 5: Higher-Order Functions

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Announcements

- Homework 2 is due Wednesday 6/29
- Project 1 is due Thursday 6/30
    - Earn 1 EC point for completing it by Wednesday 6/29
- Quiz 2 is on Thursday 6/30 at the beginning of lecture
    - Environment Diagrams and Higher–Order Functions
- Group Tutoring is available! See Piazza for details

Roadmap

Introduction

Functions

Data

Mutability

Objects

Interpretation

Paradigms

Applications

- This week (Functions), the goals are:
  - To understand the idea of *functional abstraction*
  - To study this idea through:
    - higher-order functions
    - recursion
    - orders of growth

Higher-Order Functions

Generalizing Computations (demo)

$$\sum_{k=1}^{5} \boxed{k} = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} \boxed{k^3} = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 \qquad = 225$$

$$\sum_{k=1}^{5} \boxed{\frac{8}{(4k-3)\cdot(4k-1)}} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

Generalizing Computations (demo)

```
def sum_naturals(n):
    total, k = 0, 1
    while k <= n:
        total, k = total + k, k + 1
    return total

def sum_cubes(n):
    total, k = 0, 1
    while k <= n:
        total, k = total + pow(k, 3), k + 1
    return total
```

Summation Example

```
cube = lambda k: pow(k, 3)
```

Function of a single argument (*not called "term"*)

```
def summation(n, term):
    """Sum the first N terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

A parameter that will be bound to a function

The cube function is passed as an argument value

The function bound to term gets called here

Locally Defined Functions (demo)

- Functions defined within other function bodies are bound to names in a *local* frame

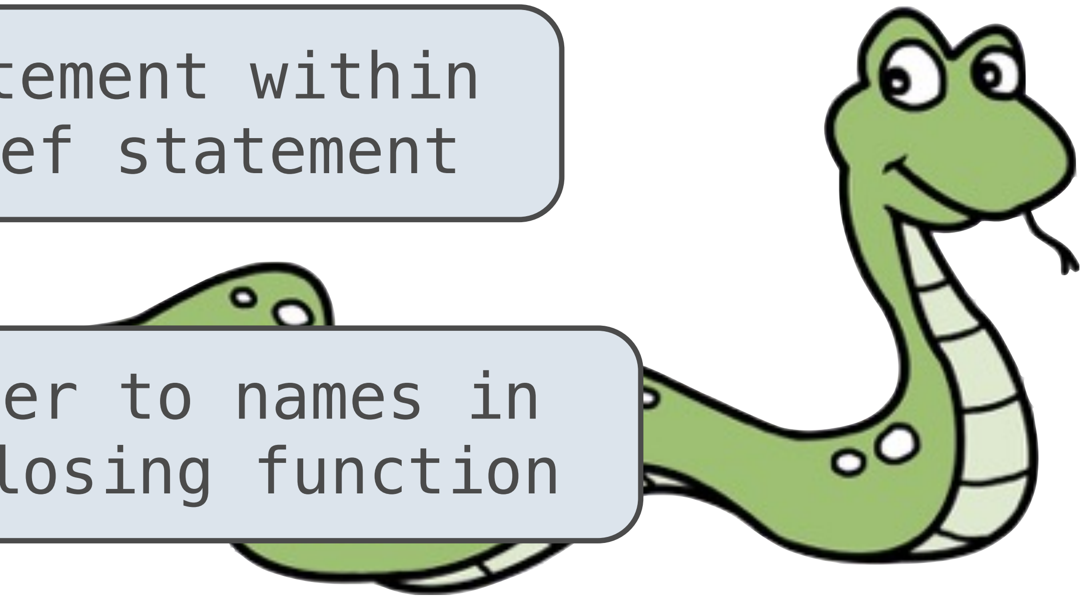
```
def make_adder(n):
    """Return a function that takes one argument K
    and returns K + N.
    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

A function that returns a function

The name add_three is bound to a function

A def statement within another def statement

Can refer to names in the enclosing function



Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language

Higher-order function:

1. A function that takes a function as an argument value **or**
2. A function that returns a function as a return value

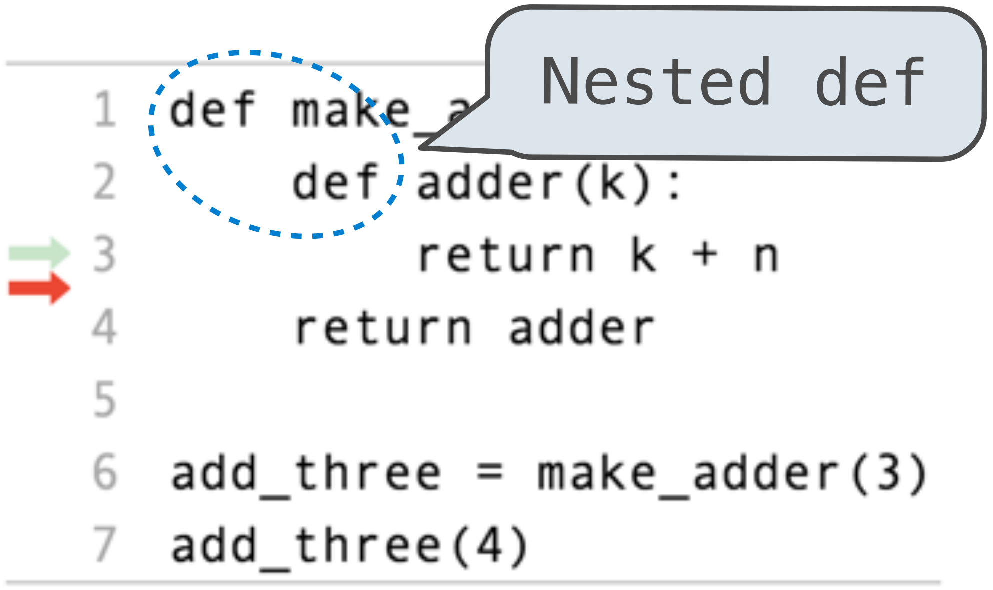
Higher-order functions:

- Express general methods of computation
- Remove repetition from programs
- Separate concerns among functions

Break!

Environments (Round 2)

Nested Definitions (demo)



Nested def

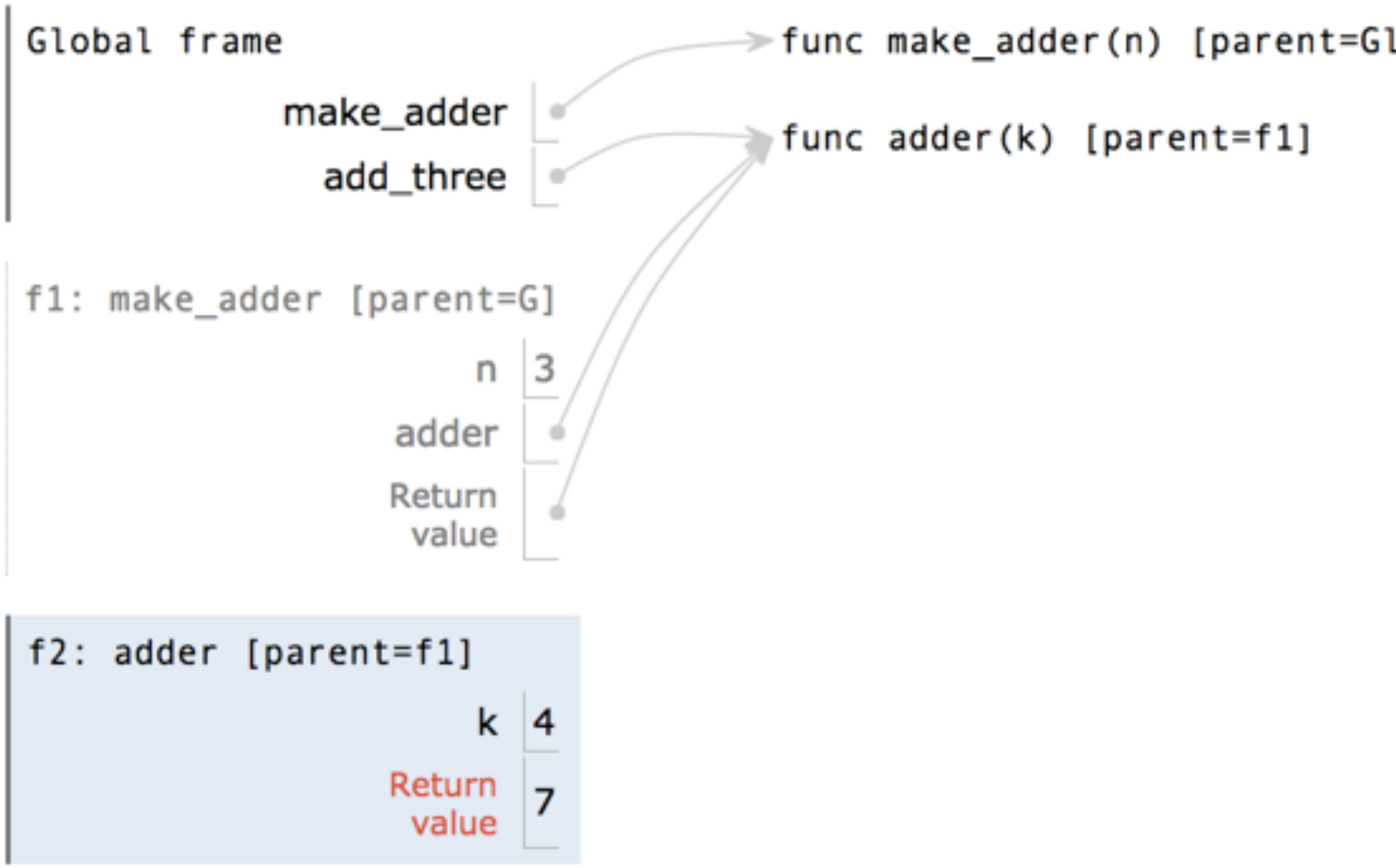
```
def make_a
    def adder(k):
        return k + n
    return adder

add_three = make_adder(3)
add_three(4)
```

- Every user-defined function has a parent frame

- The parent of a function is the frame in which it was defined

- Every local frame has a parent frame

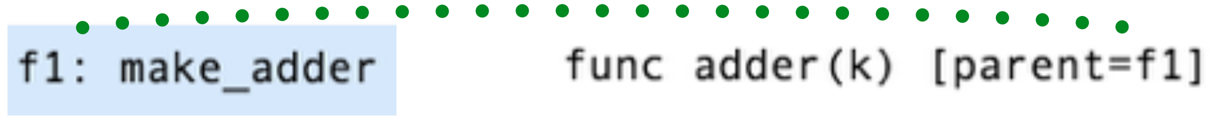
- The parent of a frame is the parent of the function called



Environment Diagram Rules (version 2)

Rules for def Statements:

1. Create a function with signature <name>(<parameters>) **and parent [parent=<label>]** (parent is the current frame)



f1: make_adder

func adder(k) [parent=f1]

2. Set the body of that function to be everything indented after the first line
3. Bind <name> to that function in the current frame

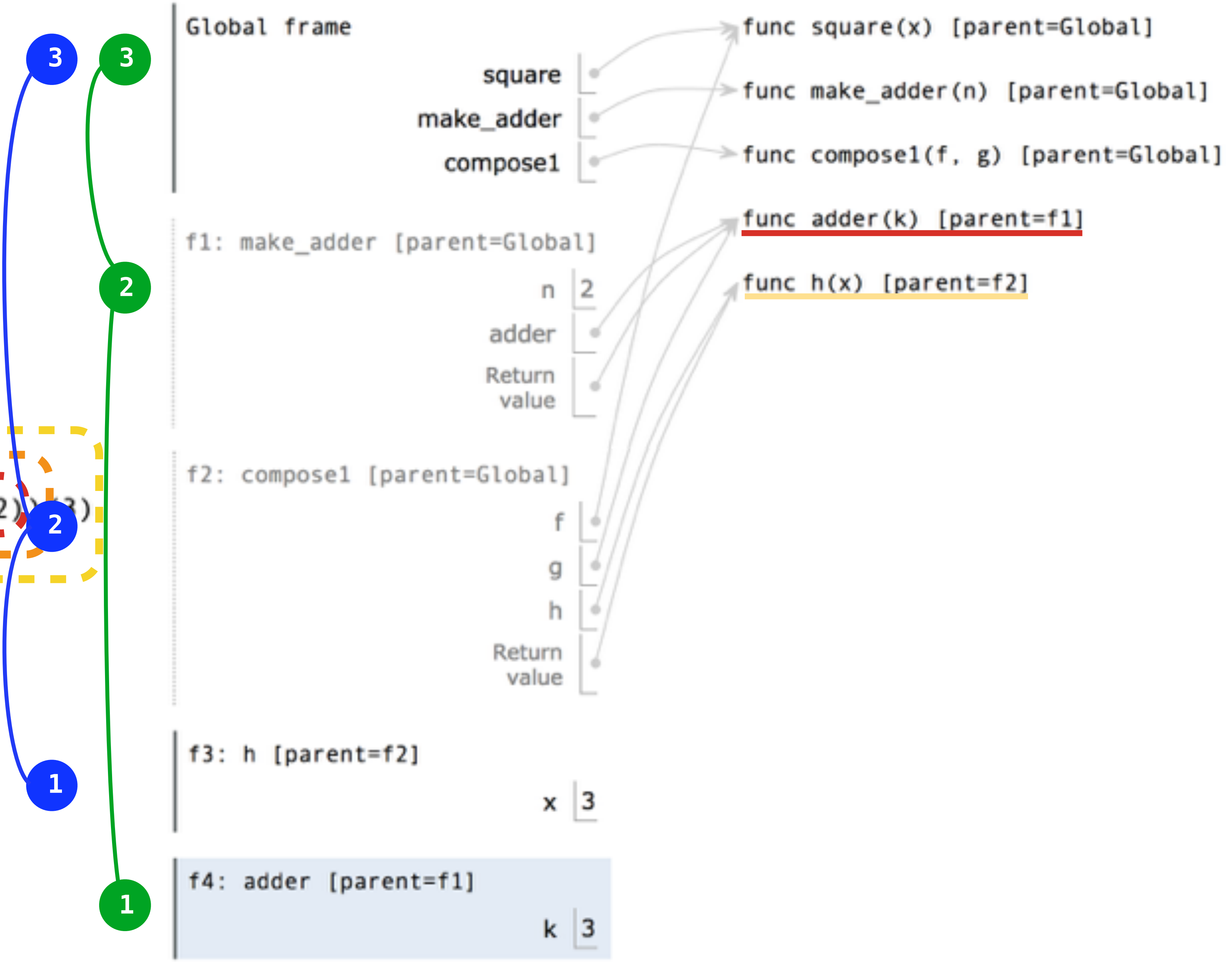
Rules for calling user-defined functions:

1. Create a new environment frame
2. **Copy the parent of the function to the local frame: [parent=<label>]**
3. Bind the function's parameters to its arguments in that frame
4. Execute the body of the function in the new environment

Function Composition

Environment Diagram

```
def square(x):
    return x * x

def make_adder(n):
    def adder(k):
        return k + n
    return adder

def compose1(f, g):
    def h(x):
        return f(g(x))
    return h

compose1(square, make_adder(2))(3)
```



```
Global frame
            square      -> func square(x) [parent=Global]
        make_adder      -> func make_adder(n) [parent=Global]
          compose1      -> func compose1(f, g) [parent=Global]
                           func adder(k) [parent=f1]
                           func h(x) [parent=f2]

f1: make_adder [parent=Global]
                 n   2
             adder
      Return value

f2: compose1 [parent=Global]
                 f
                 g
                 h
      Return value

f3: h [parent=f2]
                 x   3

f4: adder [parent=f1]
                 k   3
```

Return value of make_adder is an argument to compose1

Application: Currying

- add is a two-argument function that returns the sum of the two arguments
- make_adder is a one-argument function that returns a one-argument function that returns the sum of the two arguments
- Currying allows us to represent functions with multiple variables as chains of functions with single variables
- It is named after mathematician and logician Haskell Brooks Curry (who rediscovered it after Moses Schönfinkel)

```
(lambda x, y: x * y + 1)(3, 4)

(lambda x: lambda y: x * y + 1)(3)(4)
```