# Lecture 18: Mutable Trees

Mitas Ray
07/21/2016

# Announcements

# Trees

# Terminology

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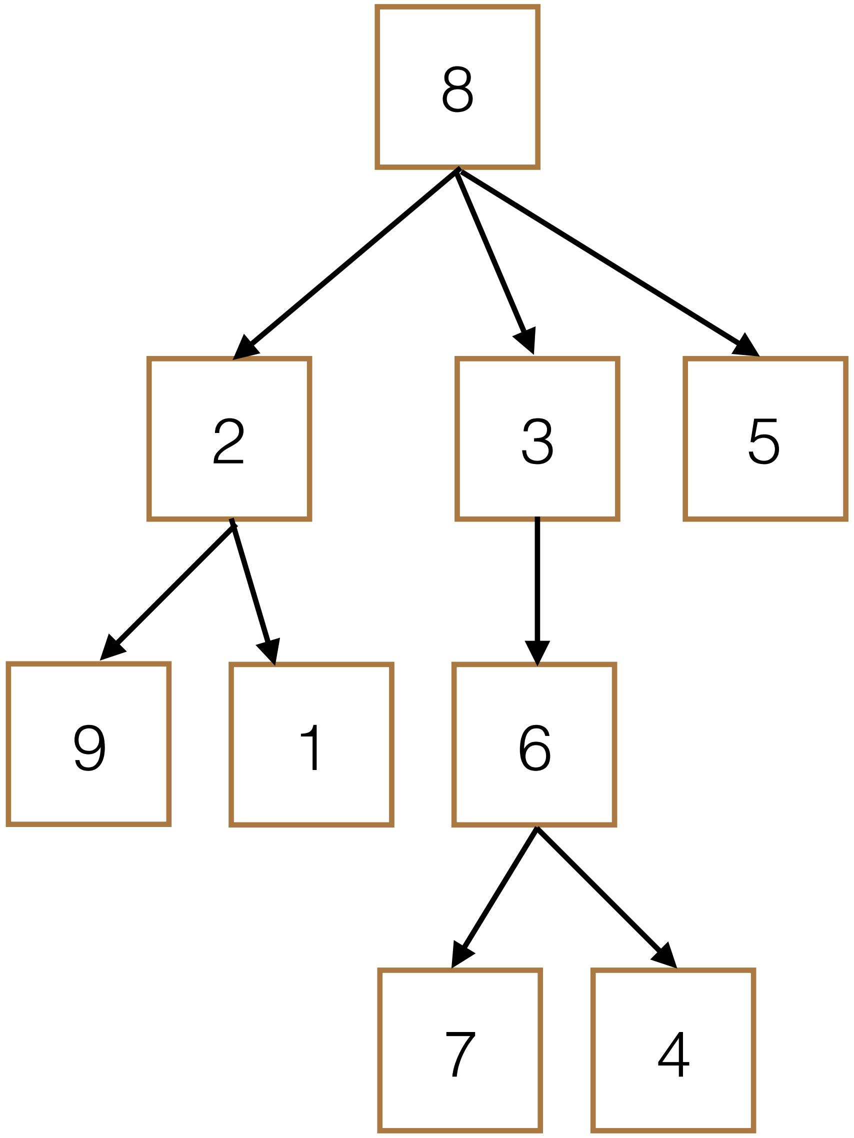
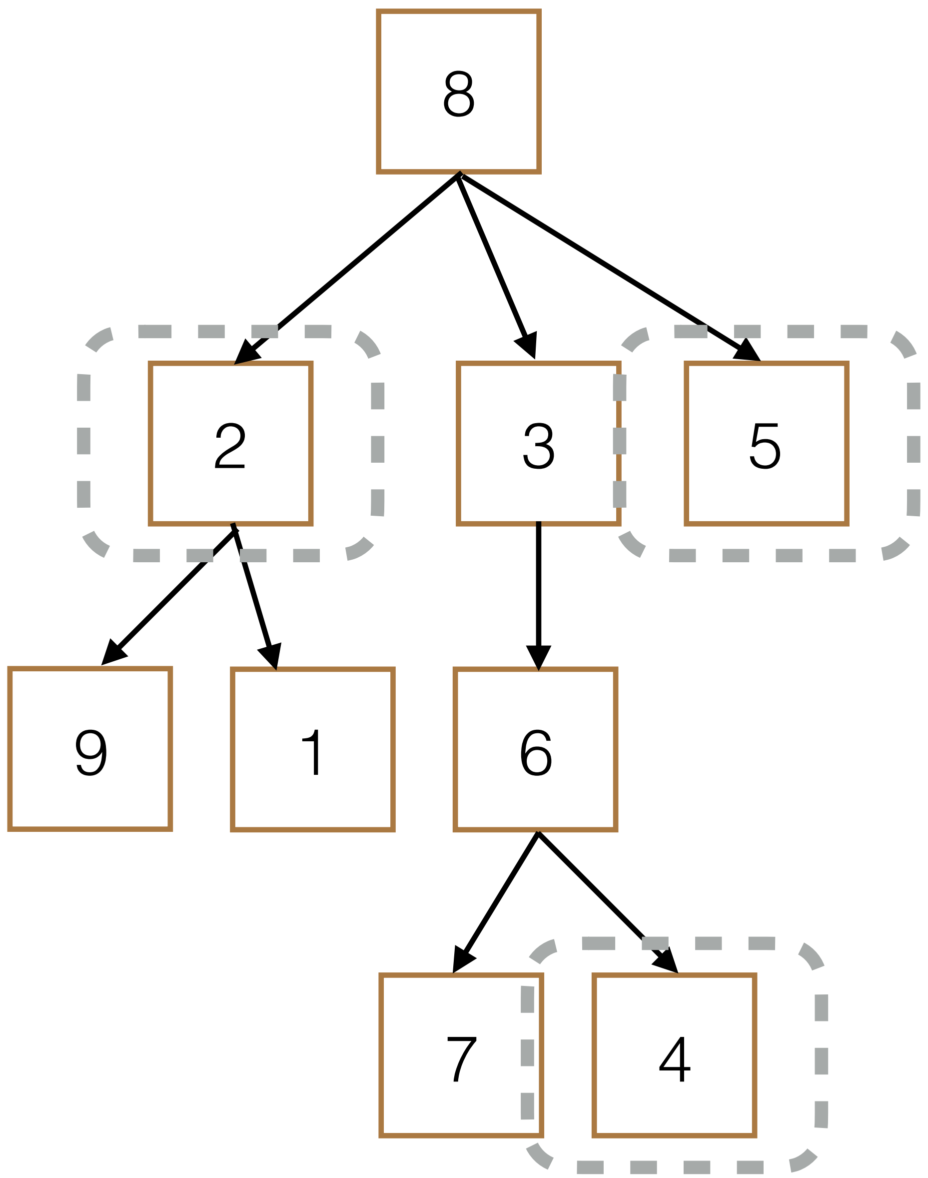


# Terminology

- **Node**: single unit containing an entry



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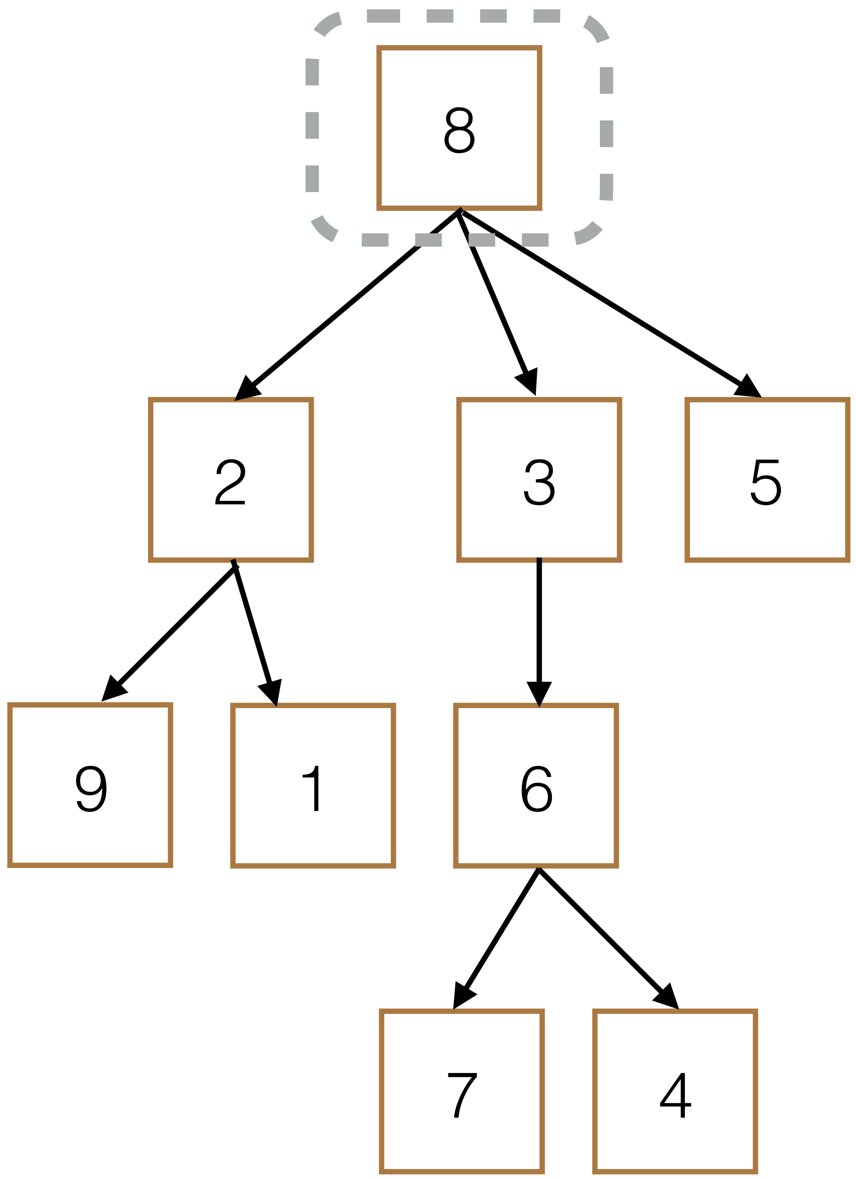

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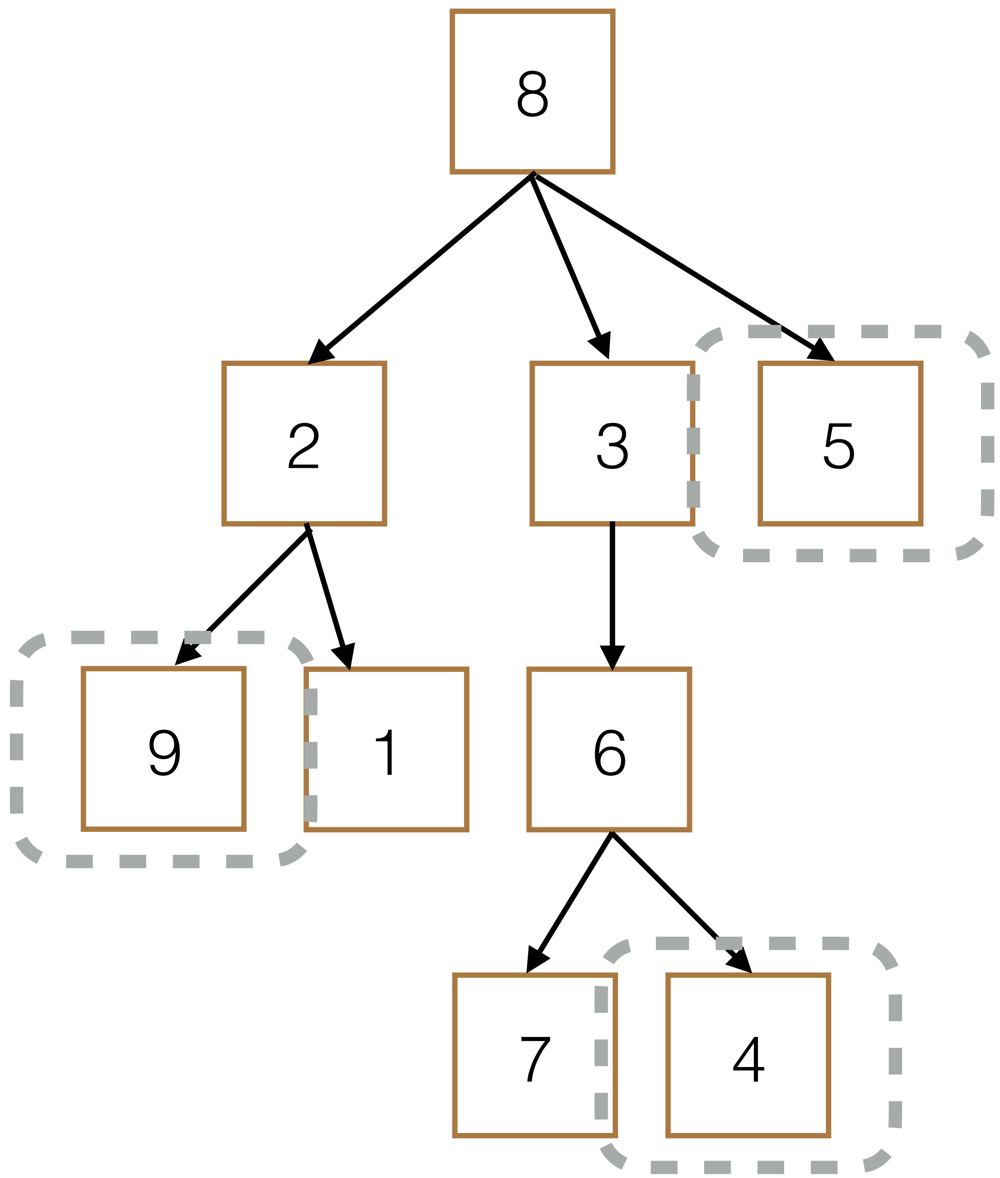
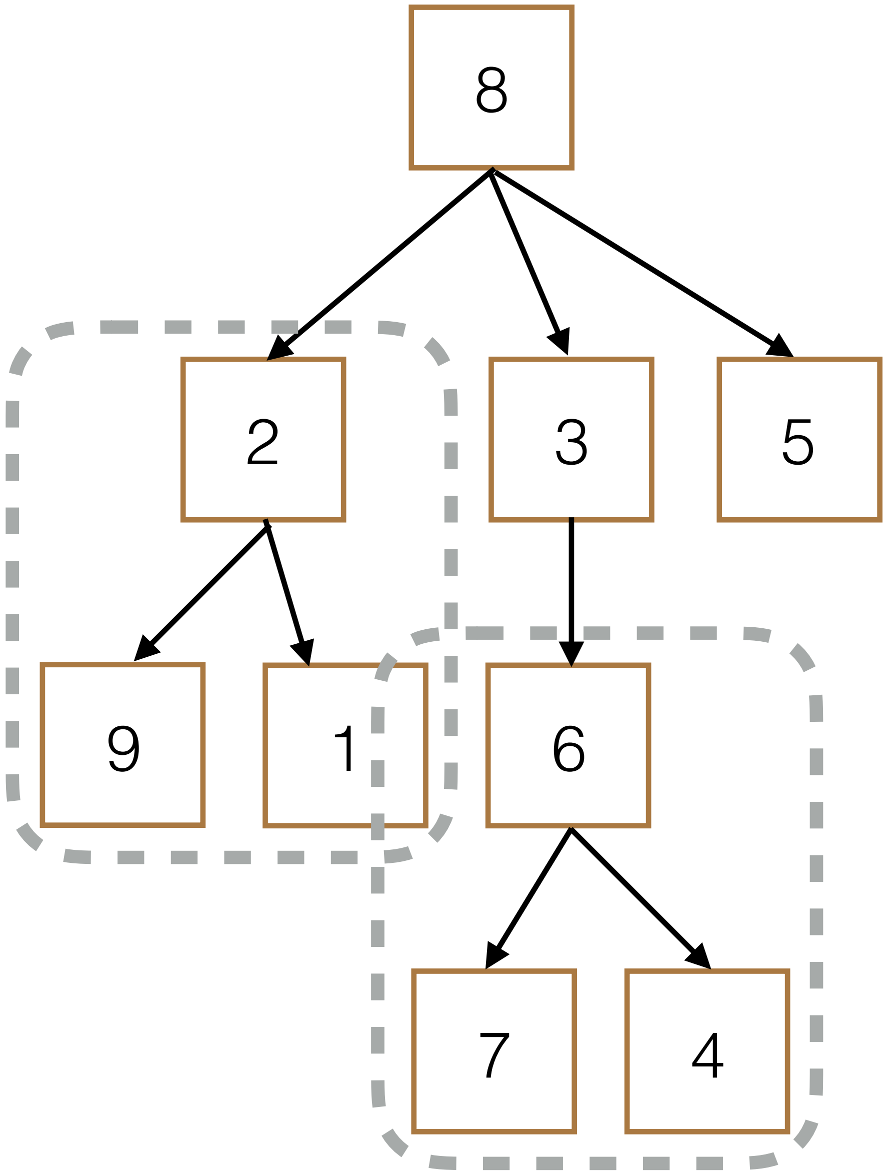
- **Leaf**: a node with no children



# Terminology

- **Node**: single unit containing an entry
- **Root**: top node
- **Leaf**: a node with no children
- **Children**: subtree with a parent



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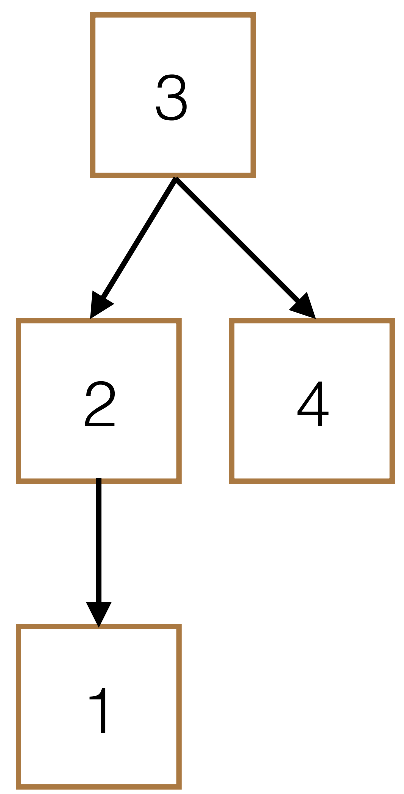

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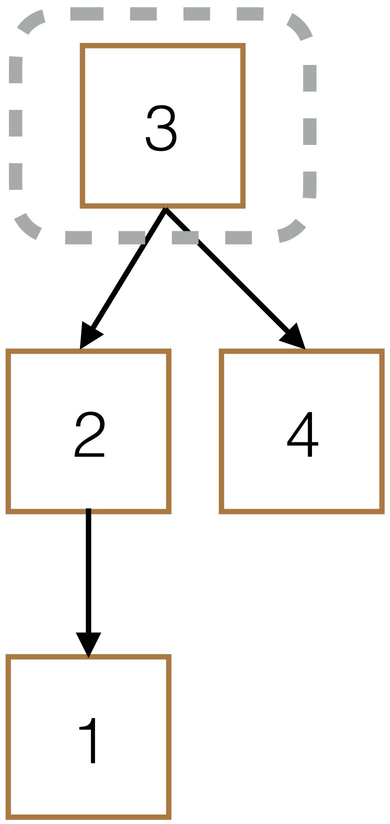

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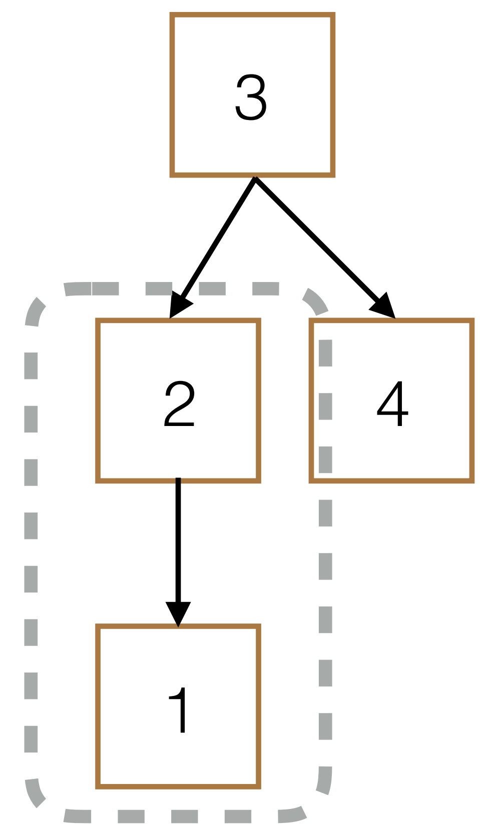

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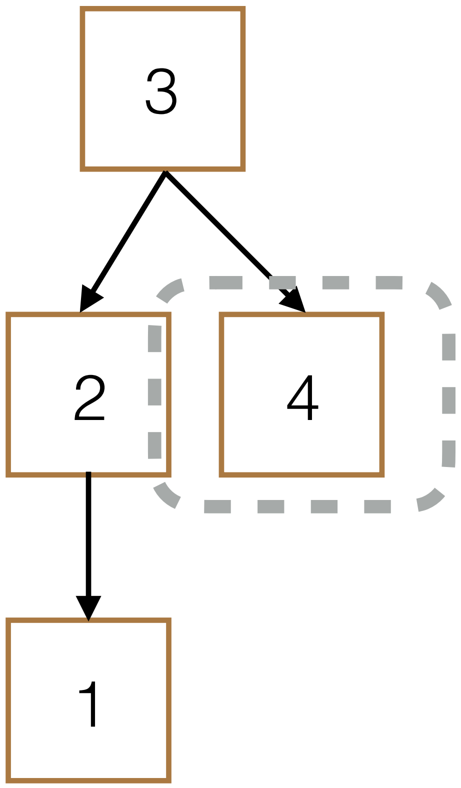

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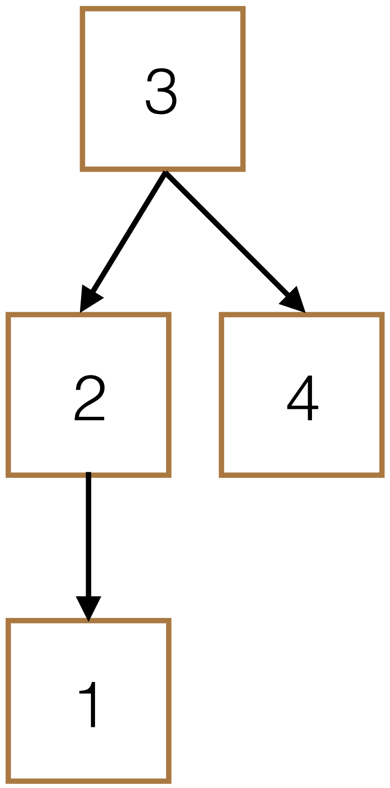

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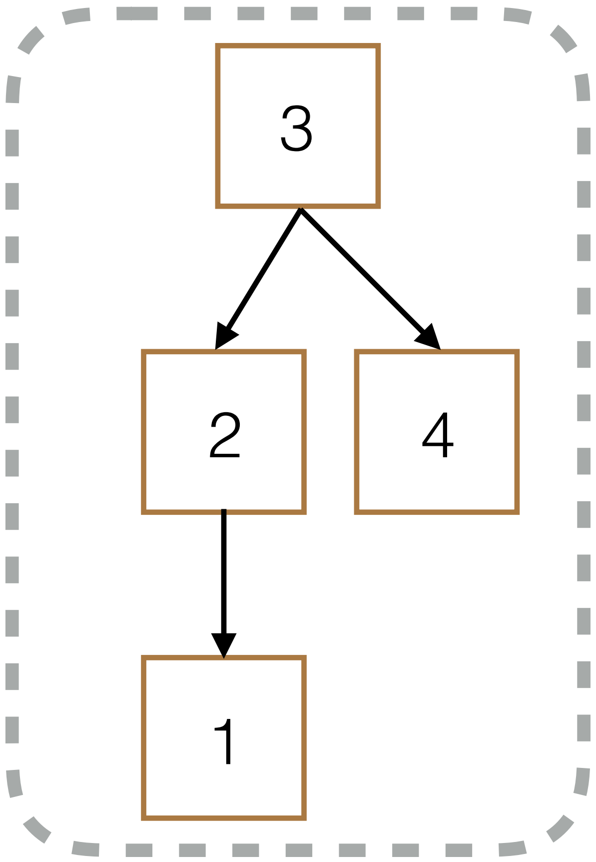

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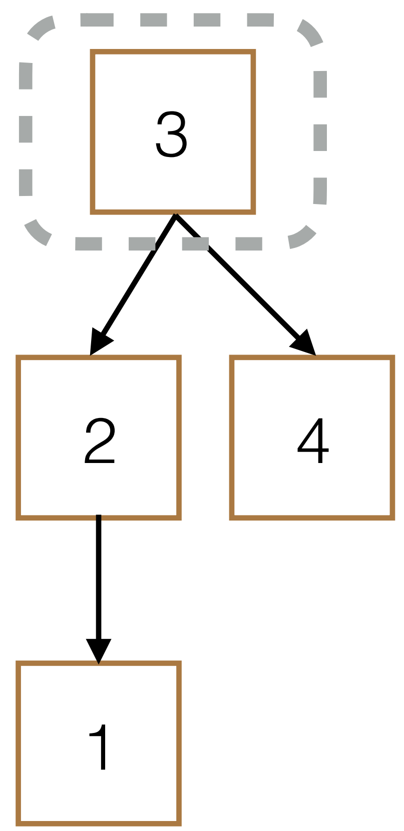

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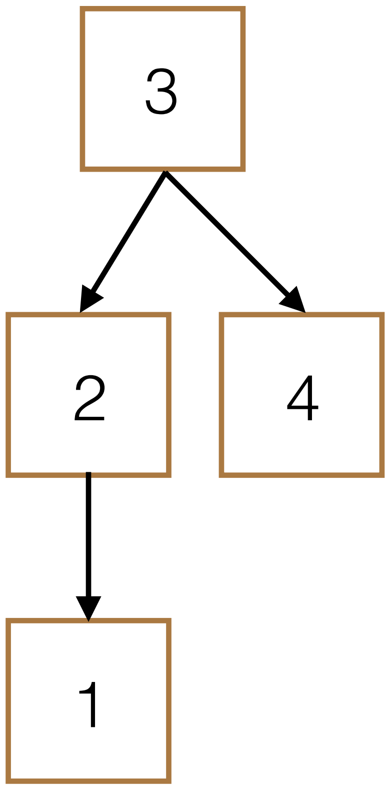

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>>> t.children[0].entry
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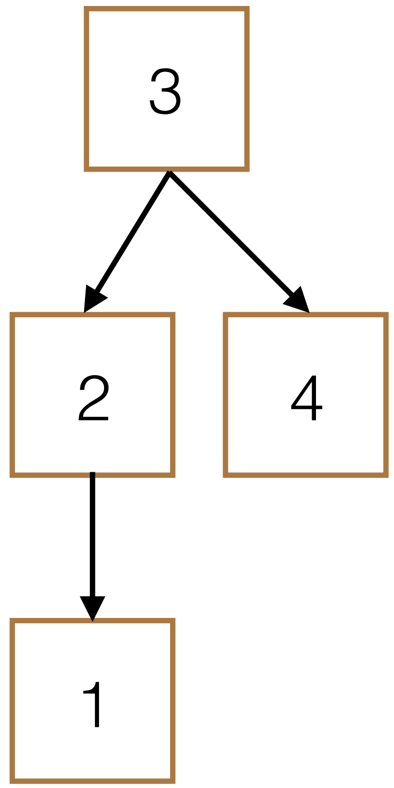

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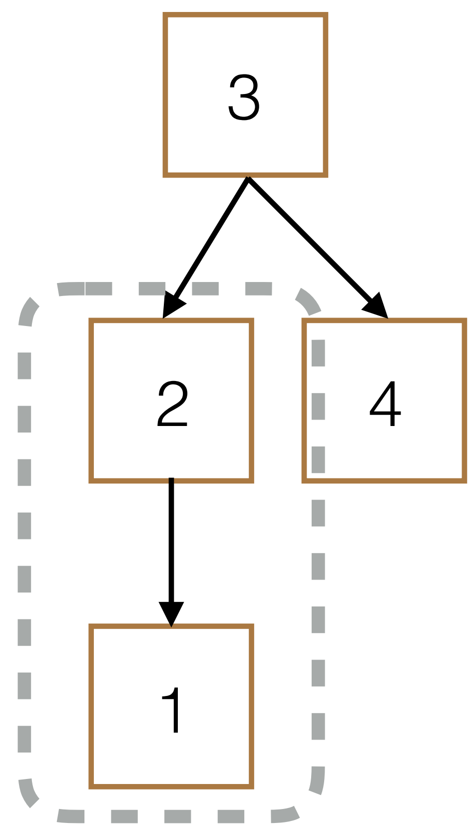

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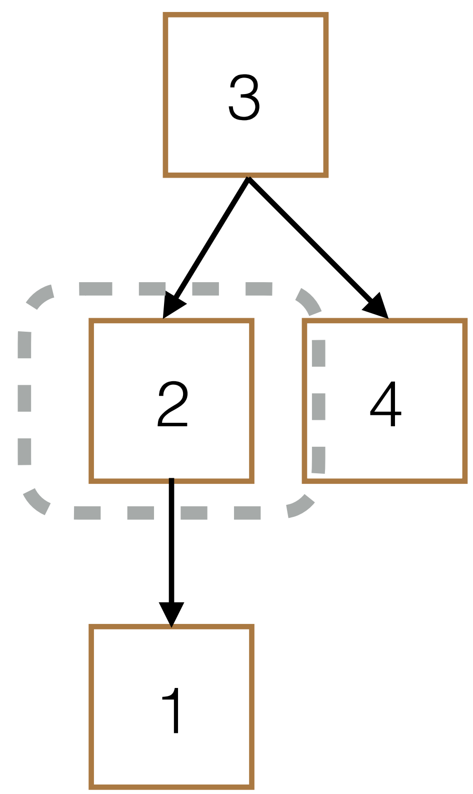

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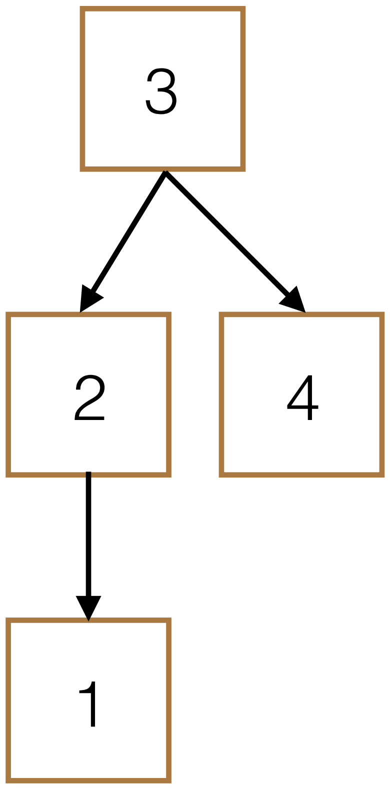

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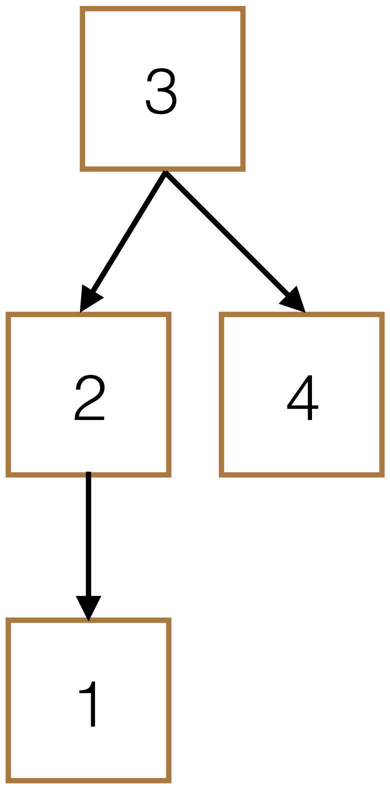

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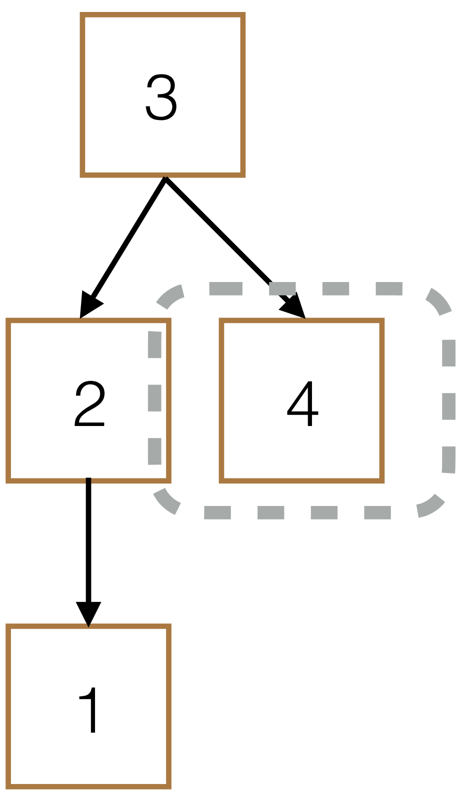

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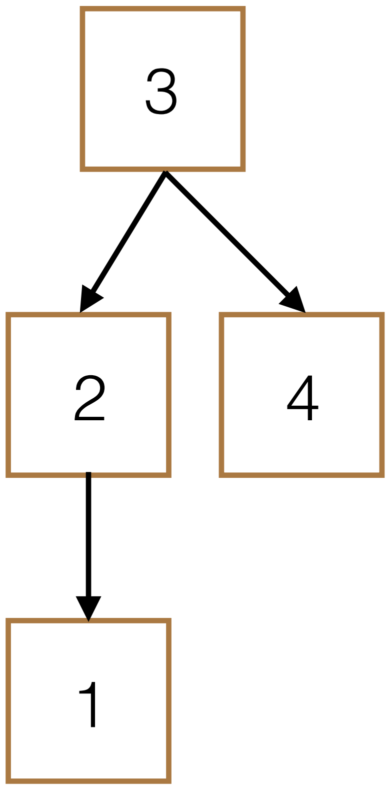

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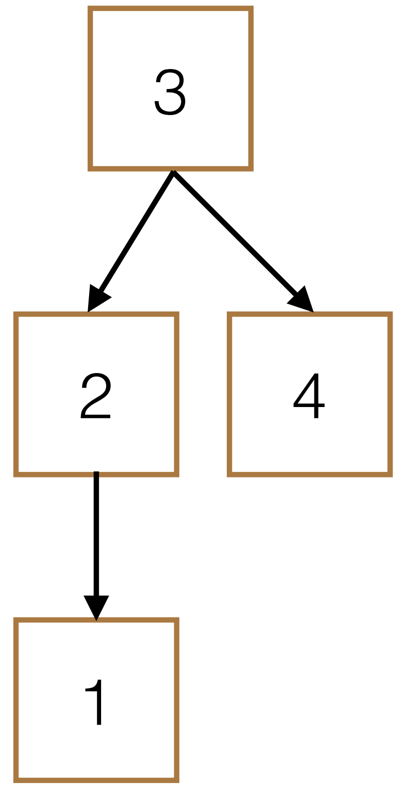

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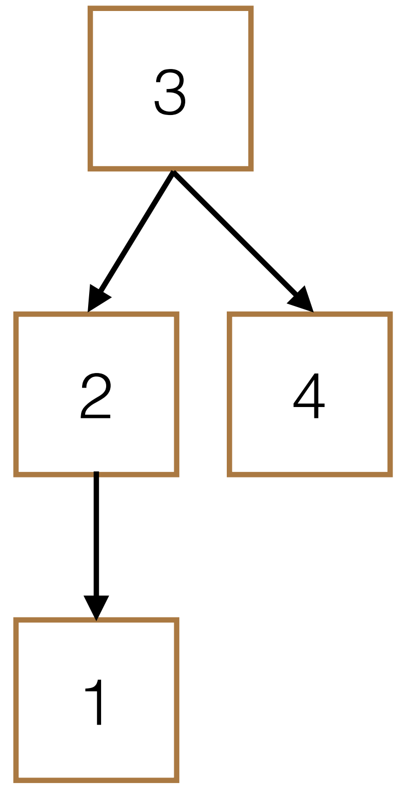

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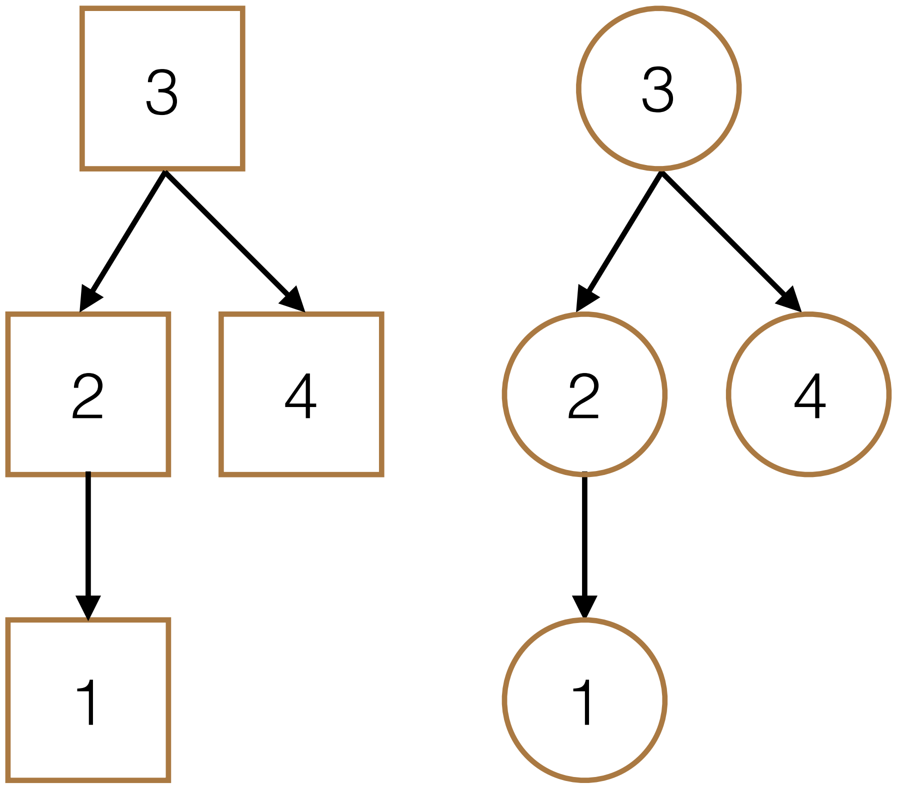

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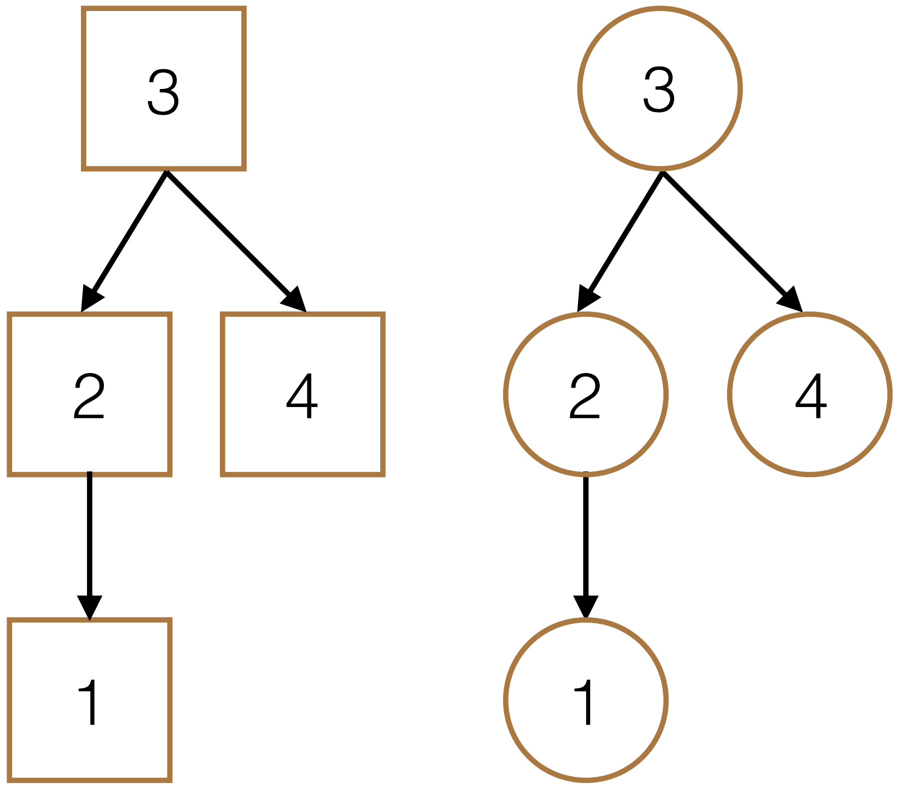

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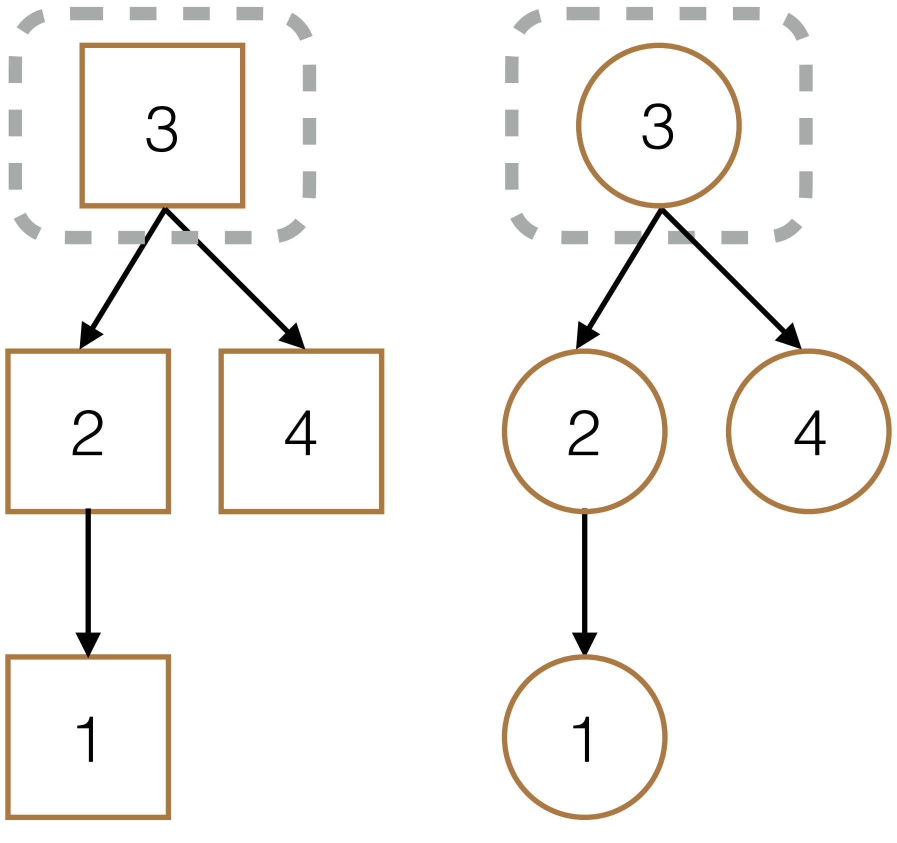

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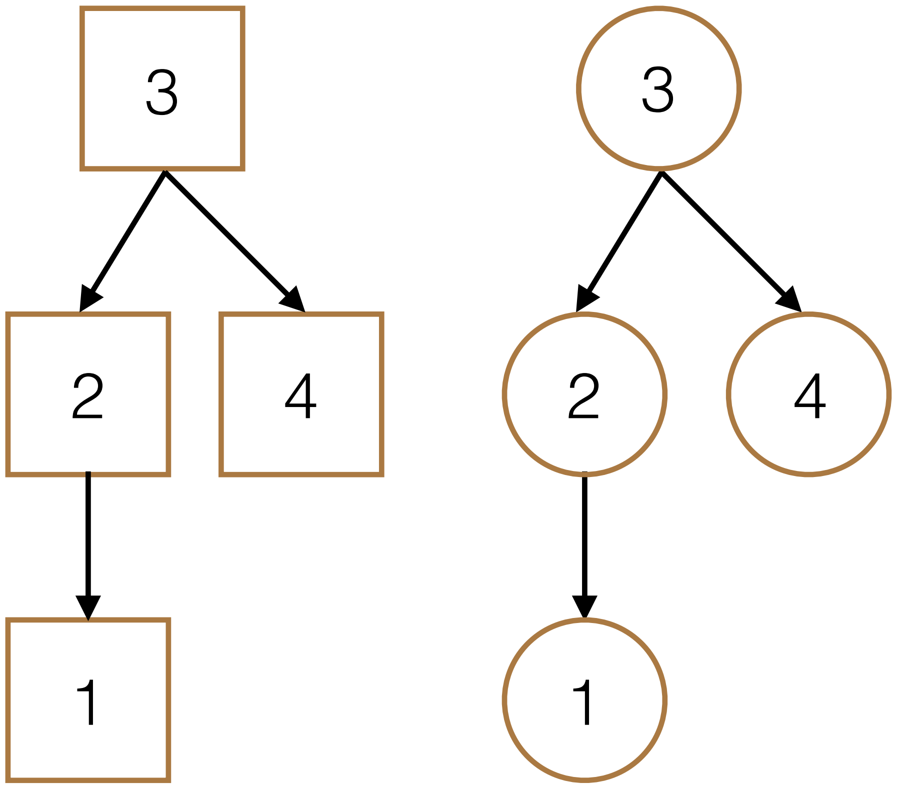

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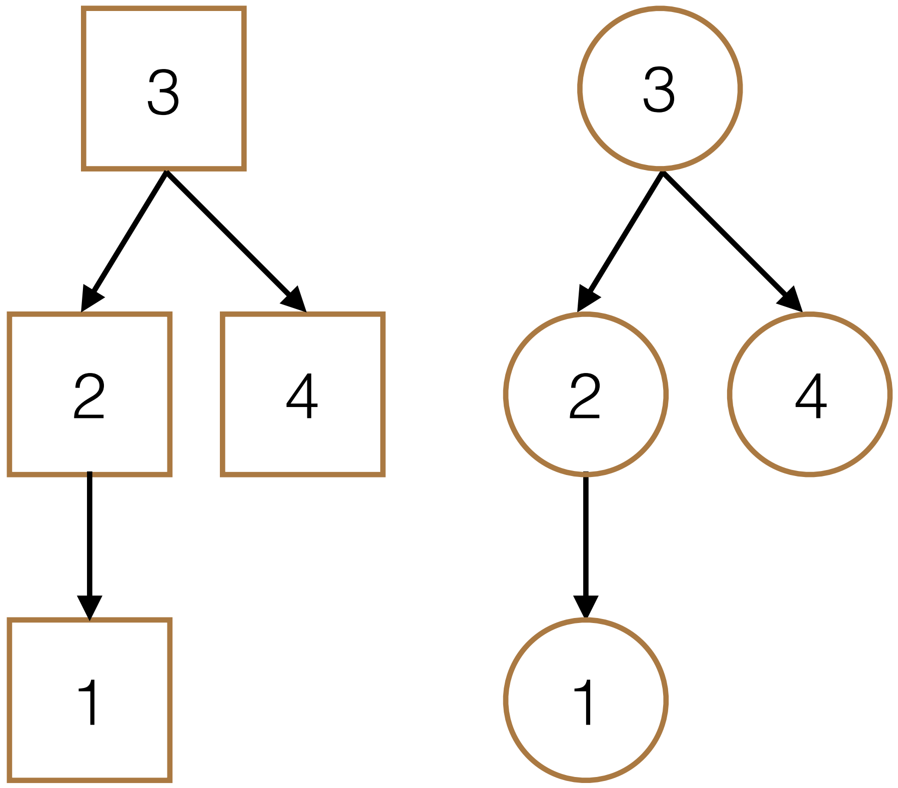

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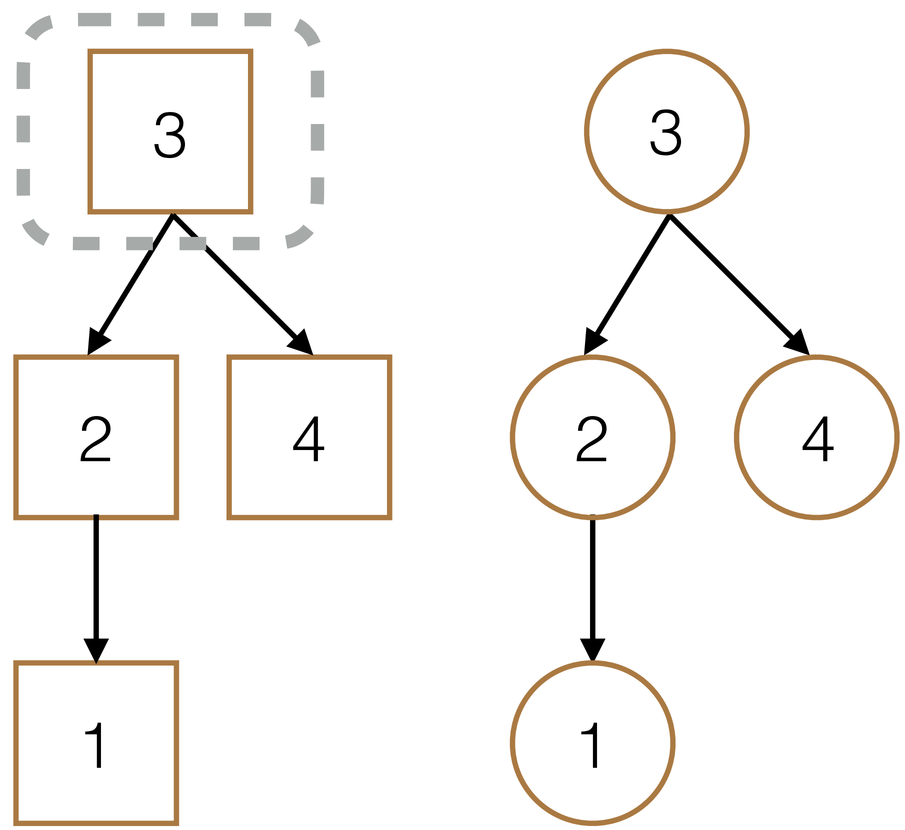

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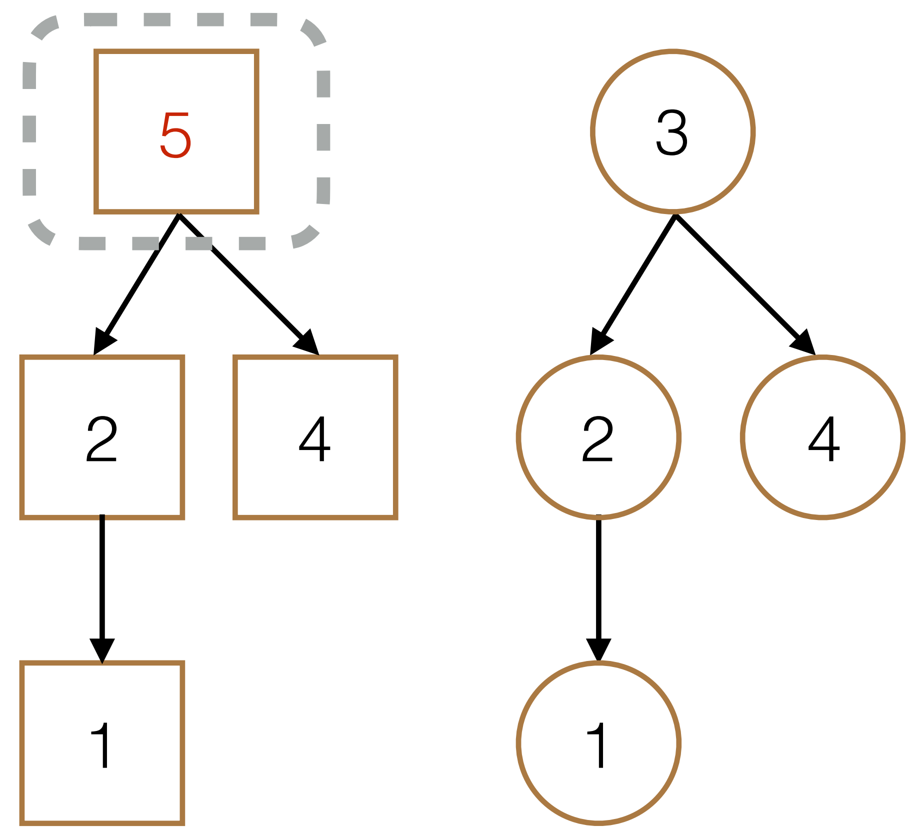

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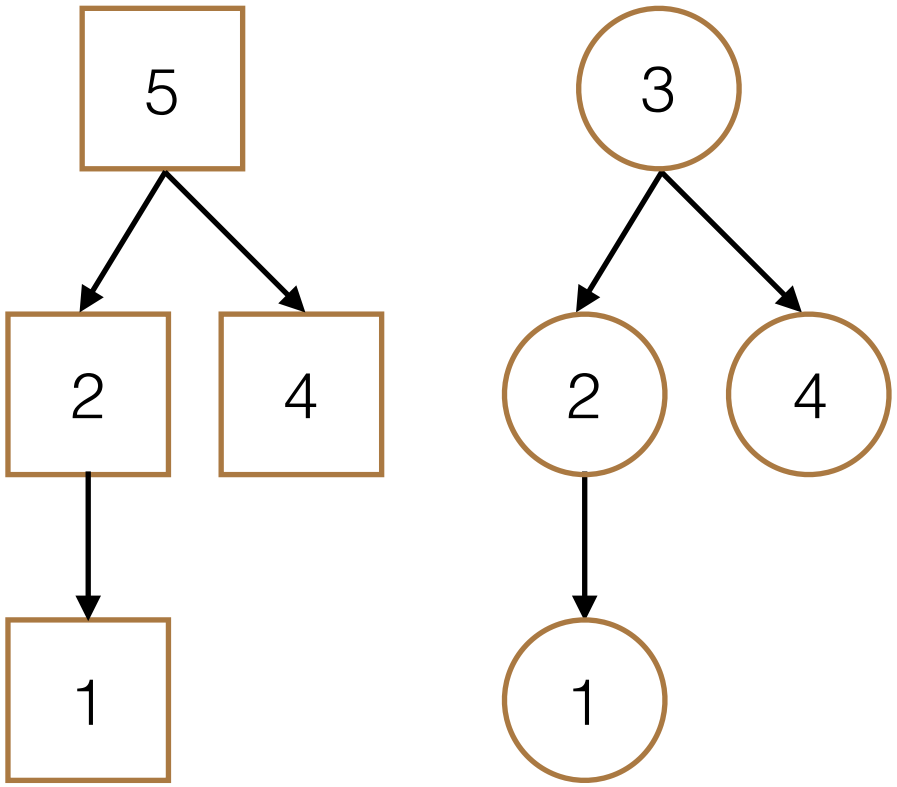

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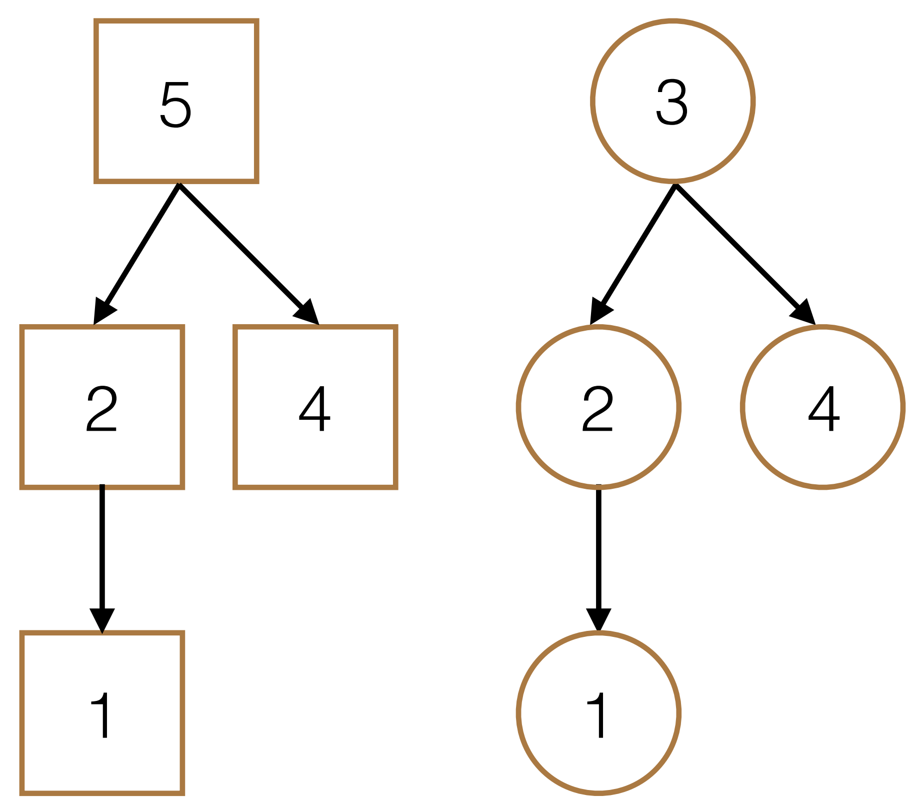
SyntaxError: can't assign ...
```



# Comparison to ADT

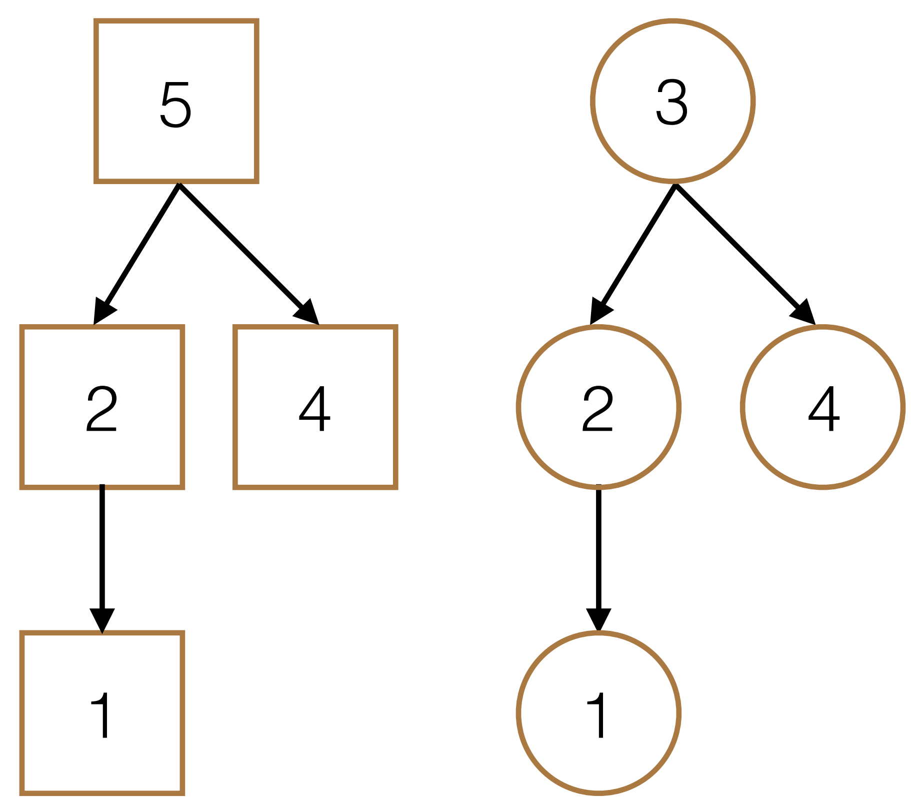
```
class Tree:
    def __init__(self, entry,
                       children=[]):
        for c in children:
            assert isinstance(c, Tree)
        self.entry = entry
        self.children = children
```

```
def tree(entry, children=[]):
    return [entry, children]

def entry(tree):
    return tree[0]

def children(tree):
    return tree[1]
```

```
>>> t_class = Tree(3, [Tree(2,
...       [Tree(1)]), Tree(4)])
>>> t_adt = tree(3, [tree(2,
...       [tree(1)]), tree(4)])
>>> t_class.entry == entry(t_adt)
True
>>> t_class.entry = 5
>>> entry(t_adt) = 5
SyntaxError: can't assign ...
>>> t_class.entry == entry(t_adt)
```



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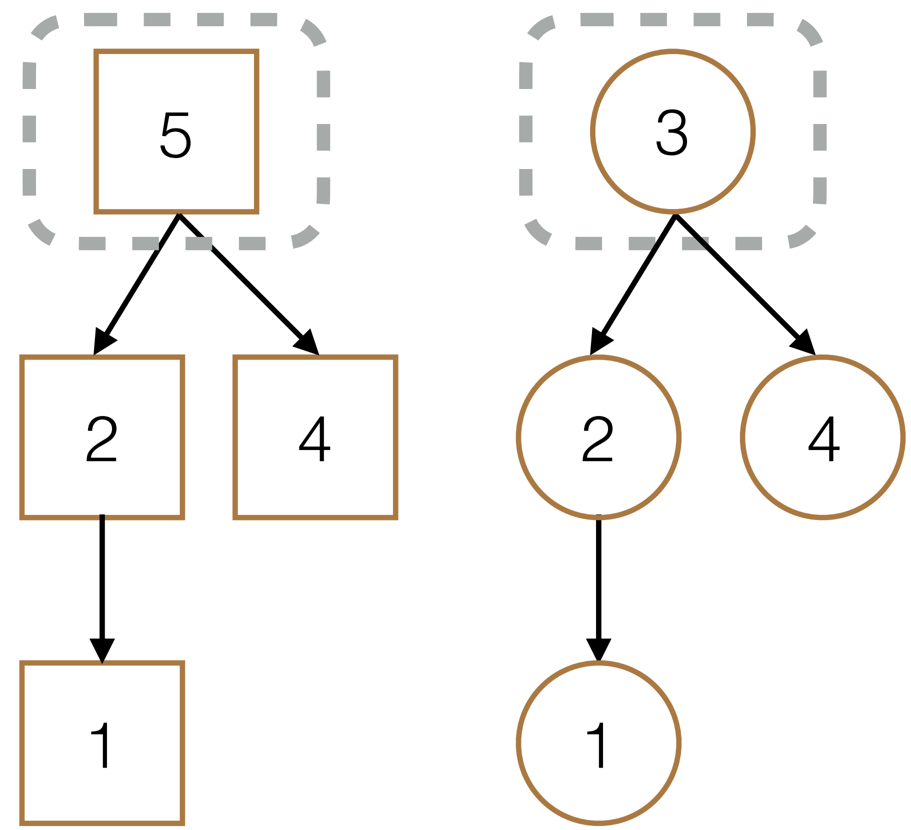

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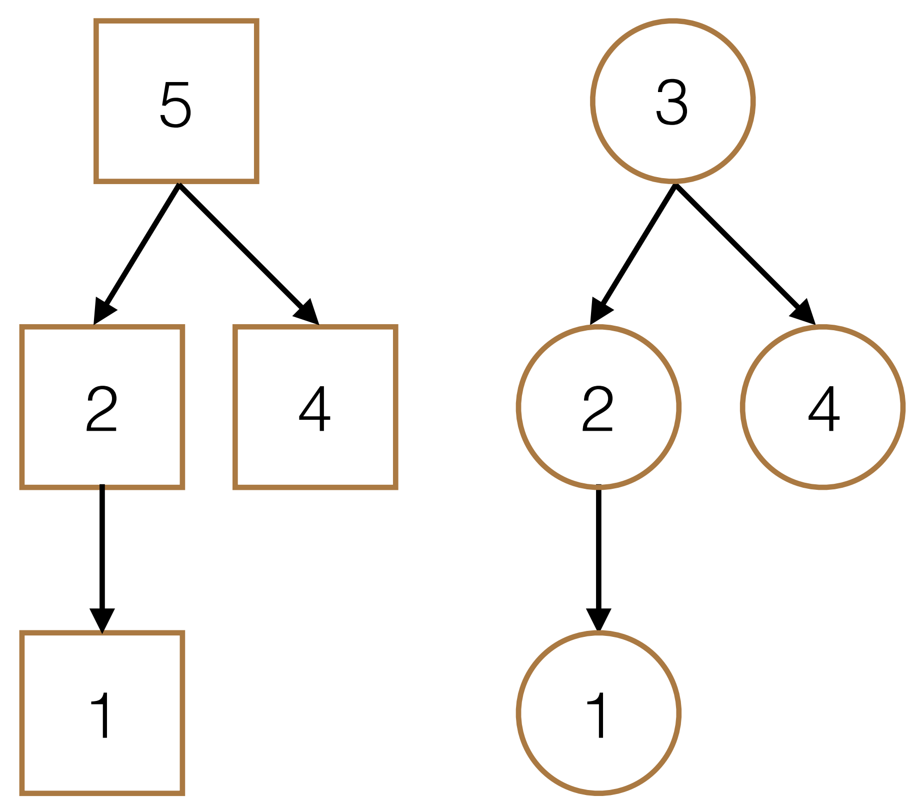

# Map

# Map

- Want to apply a function `fn` to each element in the tree

```
class Tree:
    def __init__(self, entry,
                 children=[]): ...

    def map(self, fn):
```

# Map

- Want to apply a function **`fn`** to each element in the tree

- Main Ideas

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class Tree:
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# Map

- Want to apply a function `fn` to each element in the tree
- Main Ideas
    - Apply `fn` to current node (mutate tree)

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- Want to apply a function `fn` to each element in the tree

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  - Apply `fn` to current node (mutate tree)
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                children=[]): ...

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        self.entry = fn(self.entry)
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  - Apply **`fn`** to current node (mutate tree)
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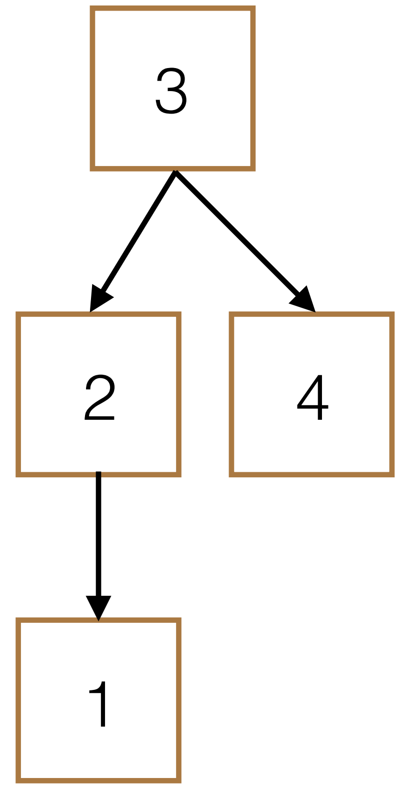

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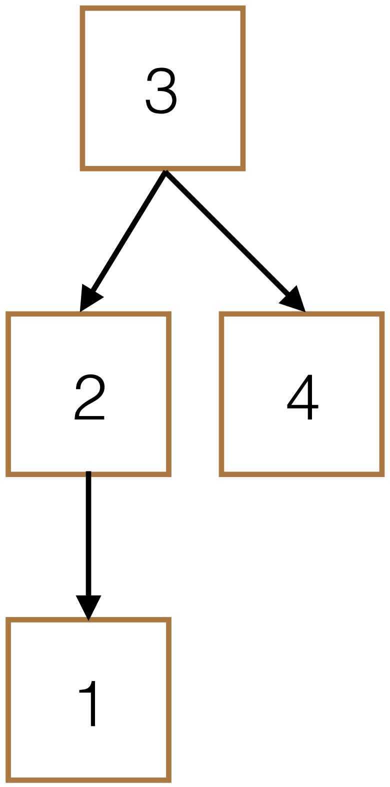

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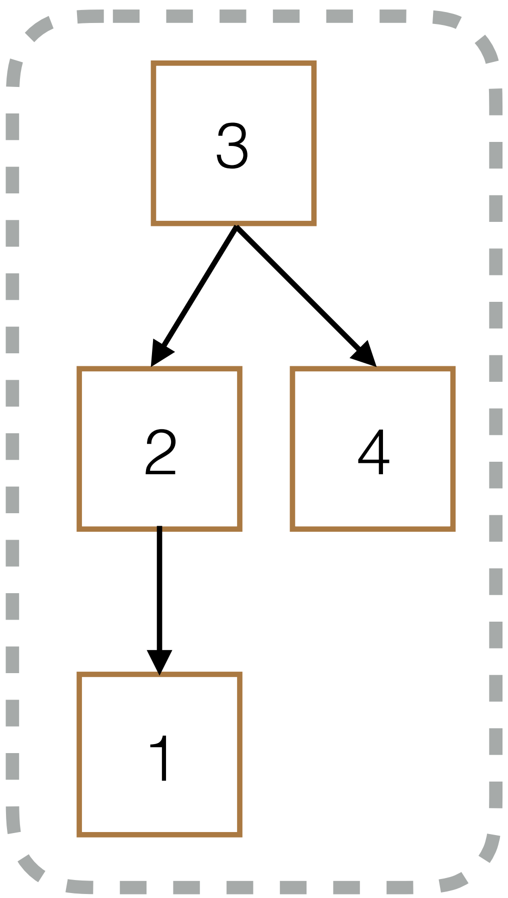

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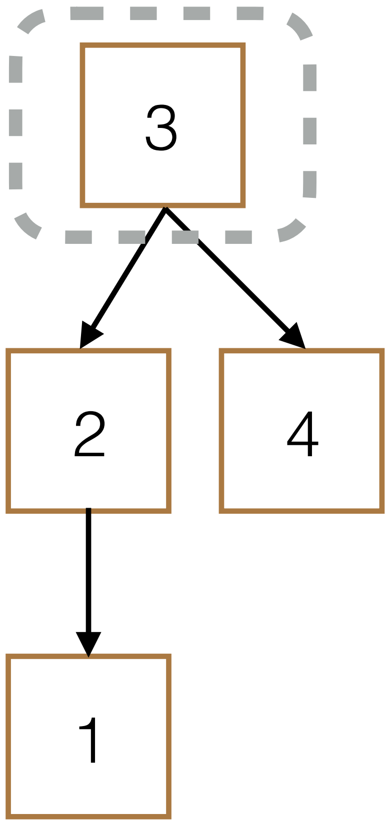

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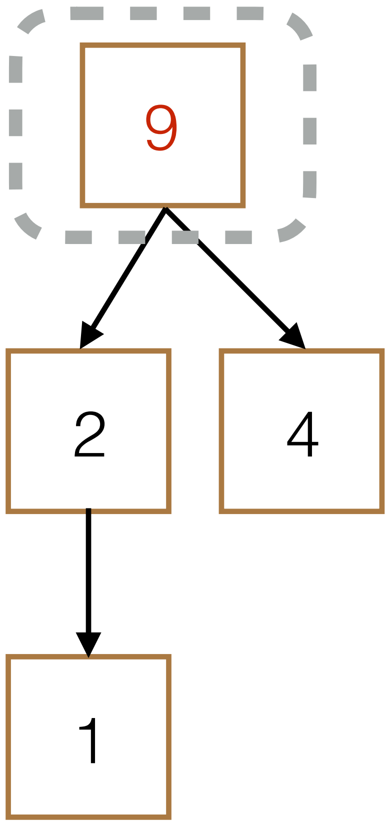

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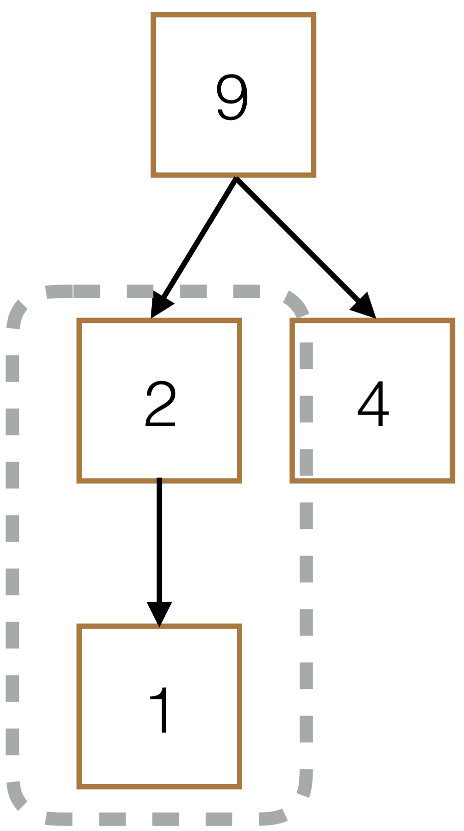

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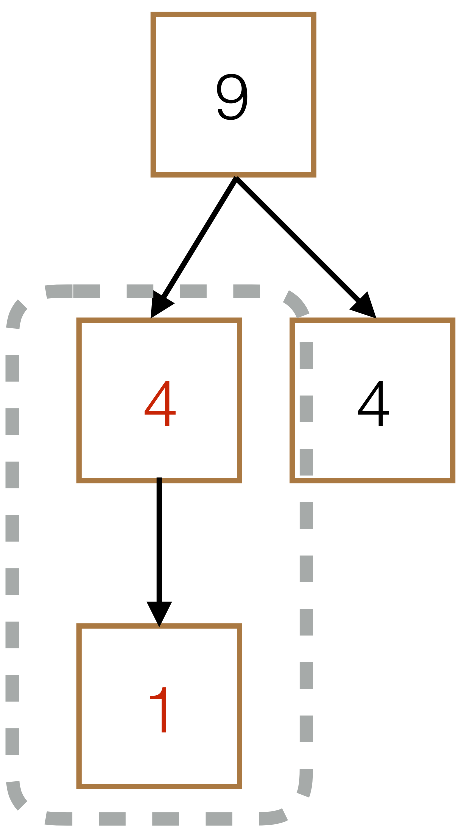

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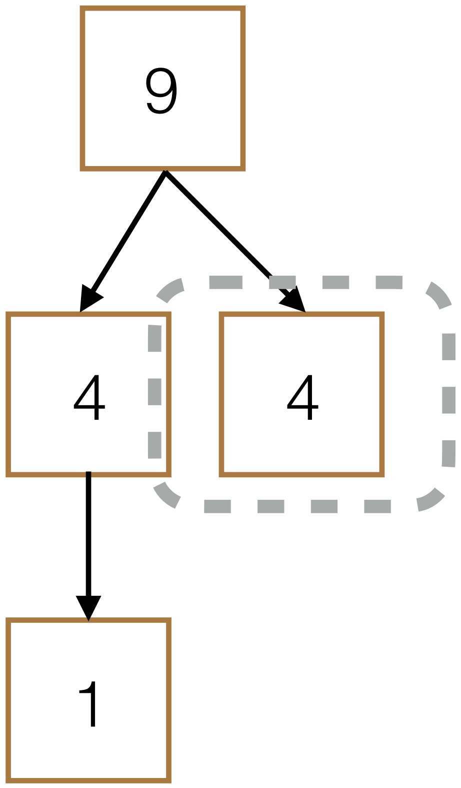

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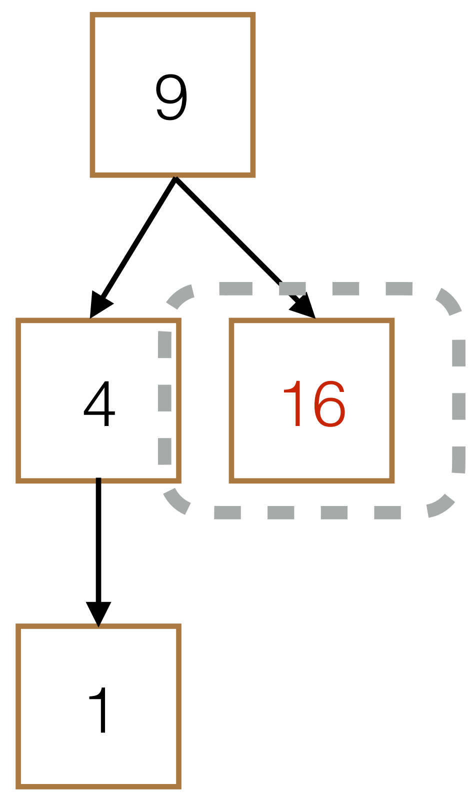

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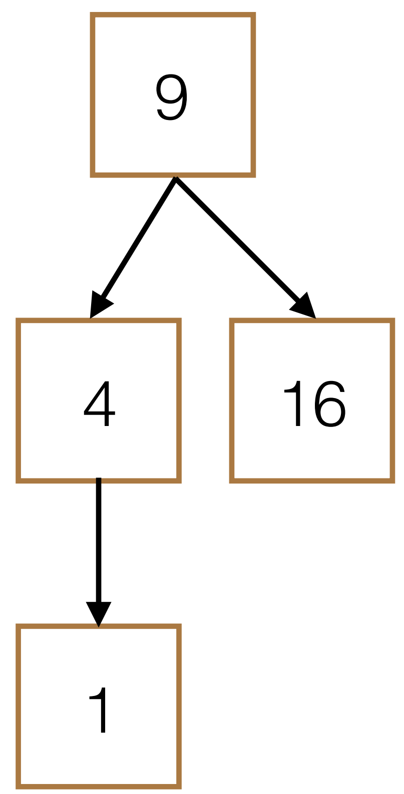

# Existence

# Existence

- Does the tree contain element `e`?

```
class Tree:
    def __init__(self, entry,
                 children=[]): ...

    def __contains__(self, e):
```

# Existence

- Does the tree contain element **e**?

- Main Ideas

```
class Tree:
    def __init__(self, entry,
                    children=[]): ...

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# Existence

- Does the tree contain element `e`?

- Main Ideas

    - Check entry of current node

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class Tree:
    def __init__(self, entry,
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# Existence

- Does the tree contain element `e`?

- Main Ideas
  - Check entry of current node
  - Otherwise, check children

```
class Tree:
    def __init__(self, entry,
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# Existence

- Does the tree contain element `e`?
- Main Ideas
  - Check entry of current node
  - Otherwise, check children
    - If no children to investigate, return False

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class Tree:
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- Does the tree contain element `e`?

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        if self.entry == e:
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        if self.entry == e:
            return True
        for c in self.children:
            if c.__contains__(e):
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```

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- Does the tree contain element **e**?
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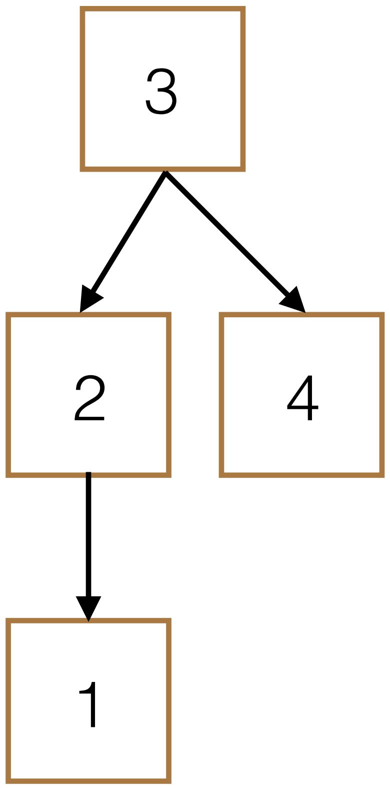

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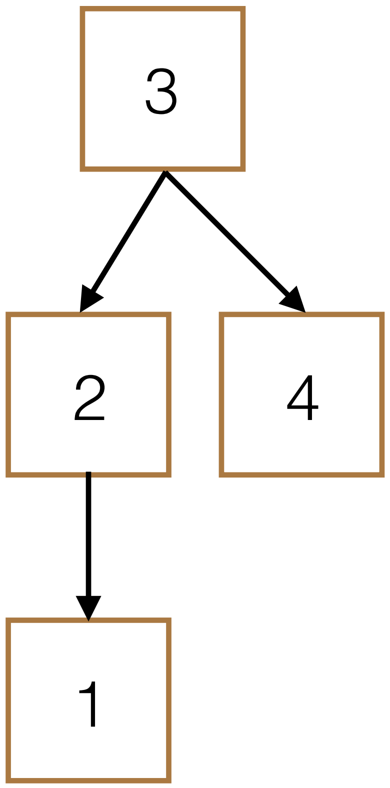
>>> 8 in t
```



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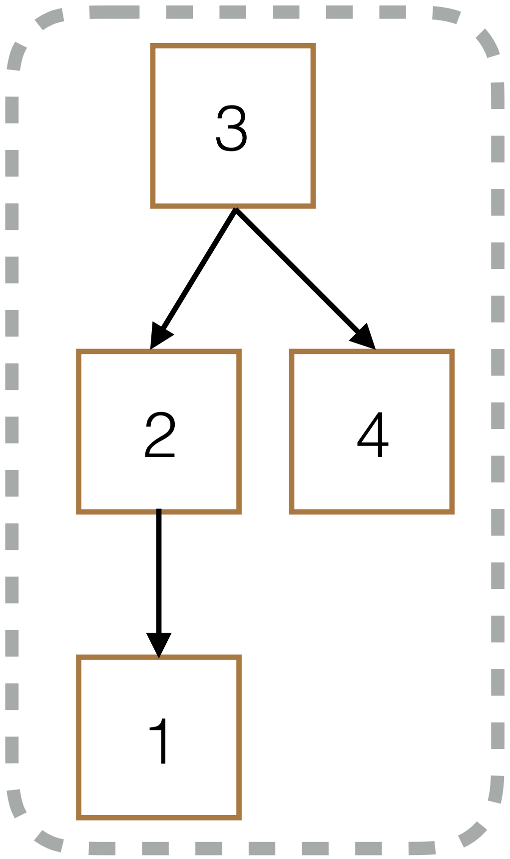

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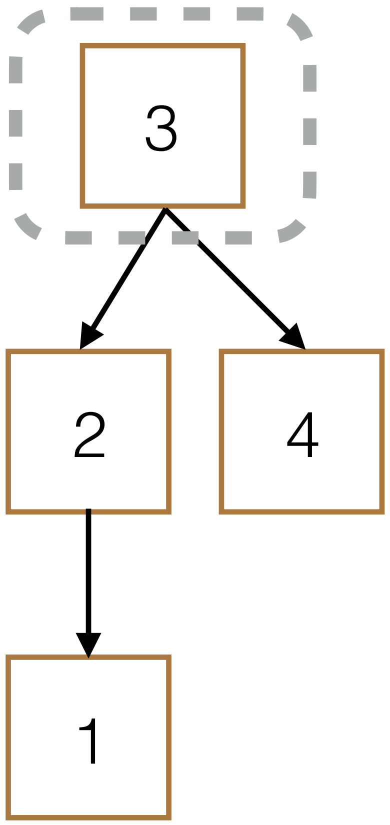

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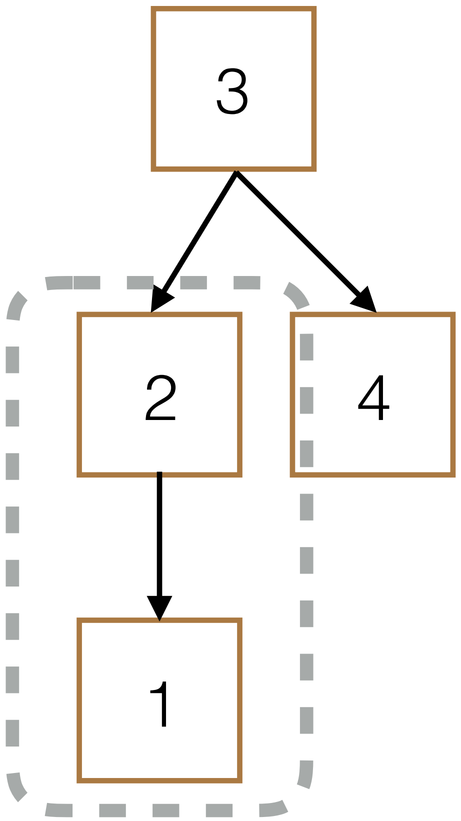

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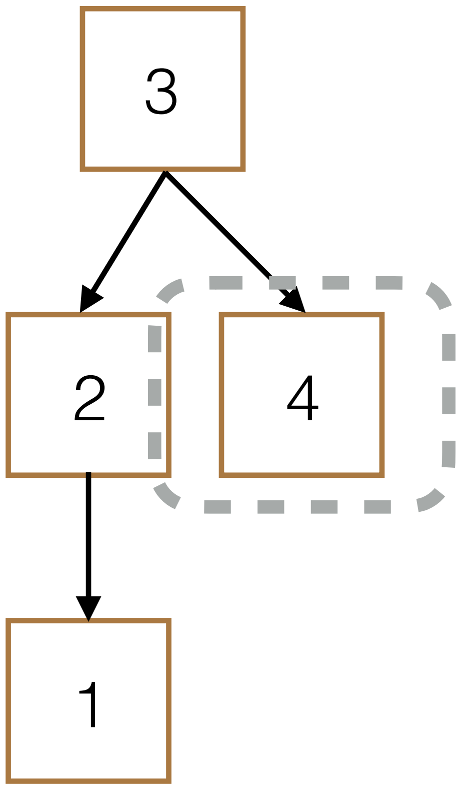

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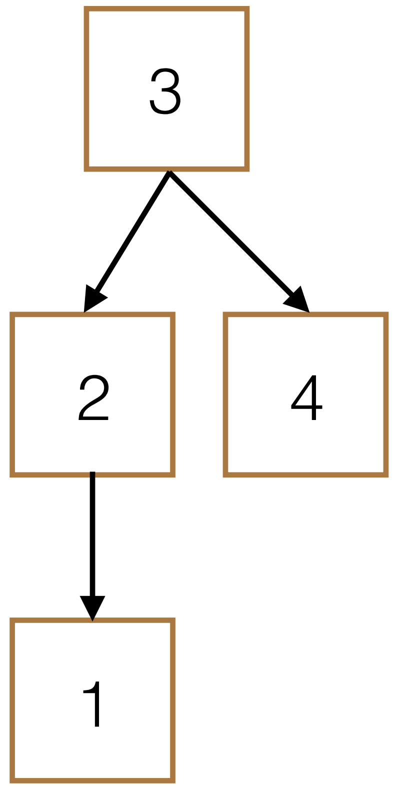

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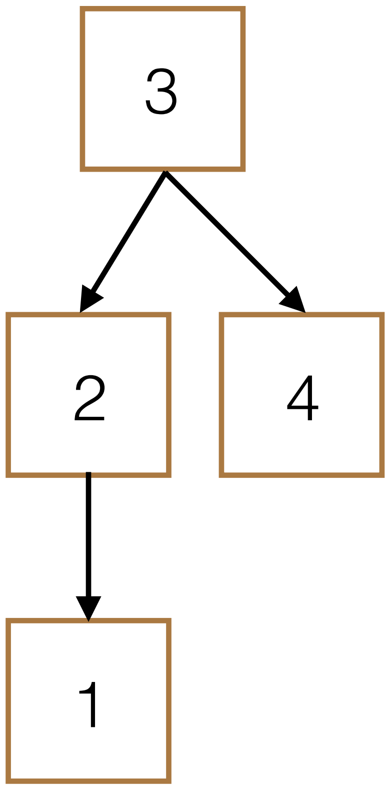

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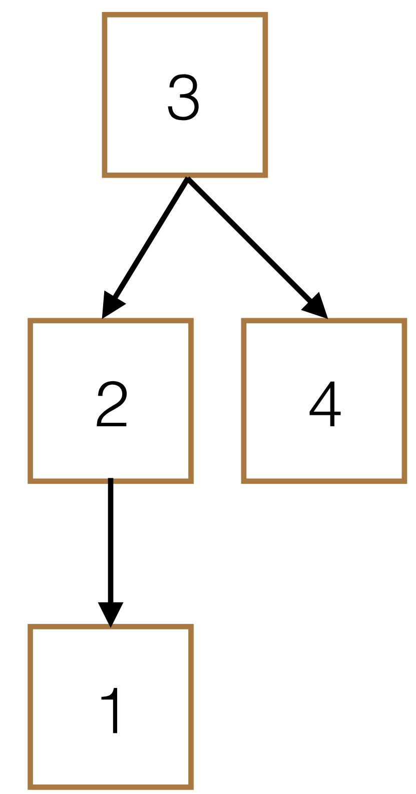

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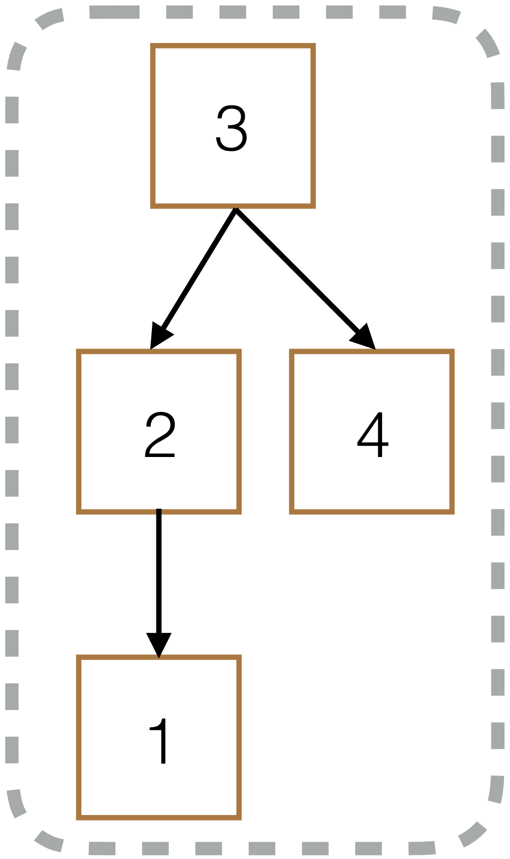

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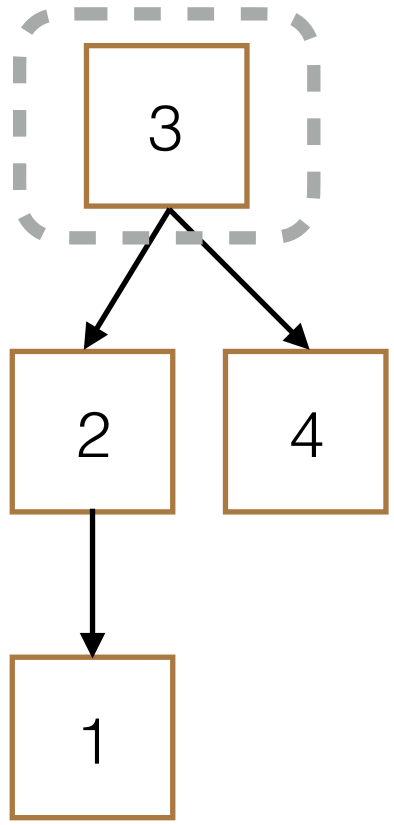

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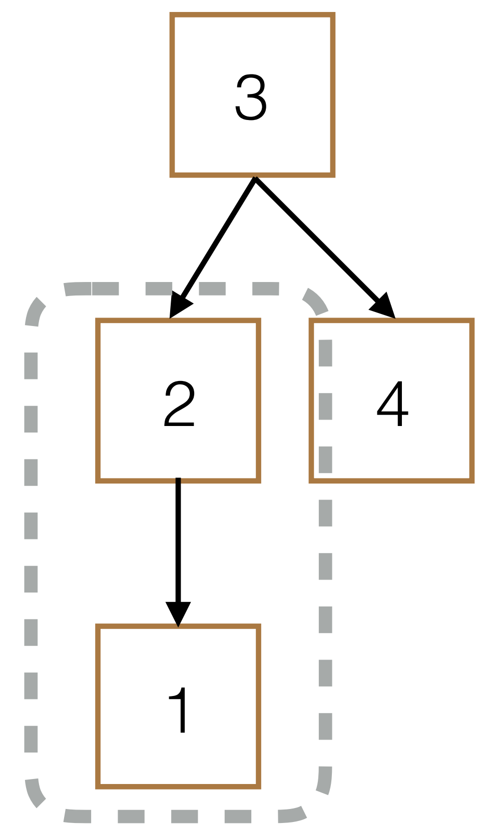

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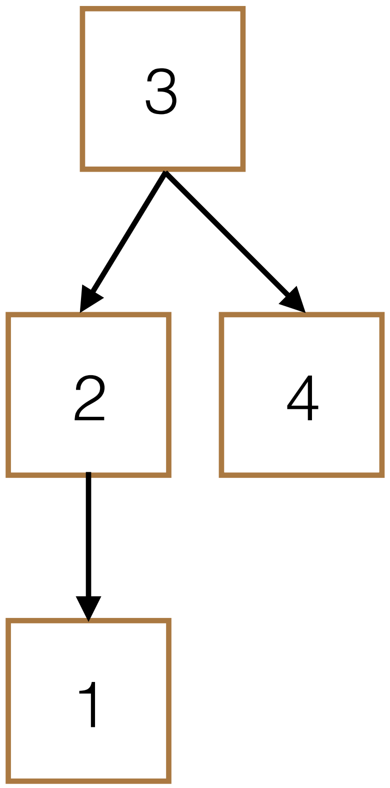

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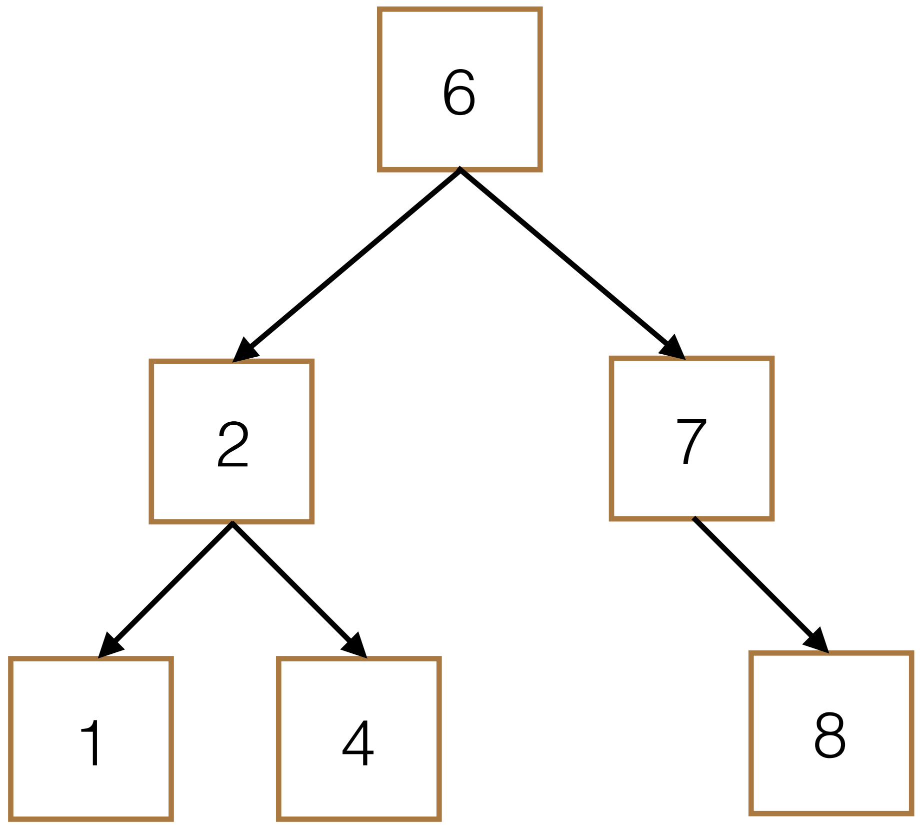

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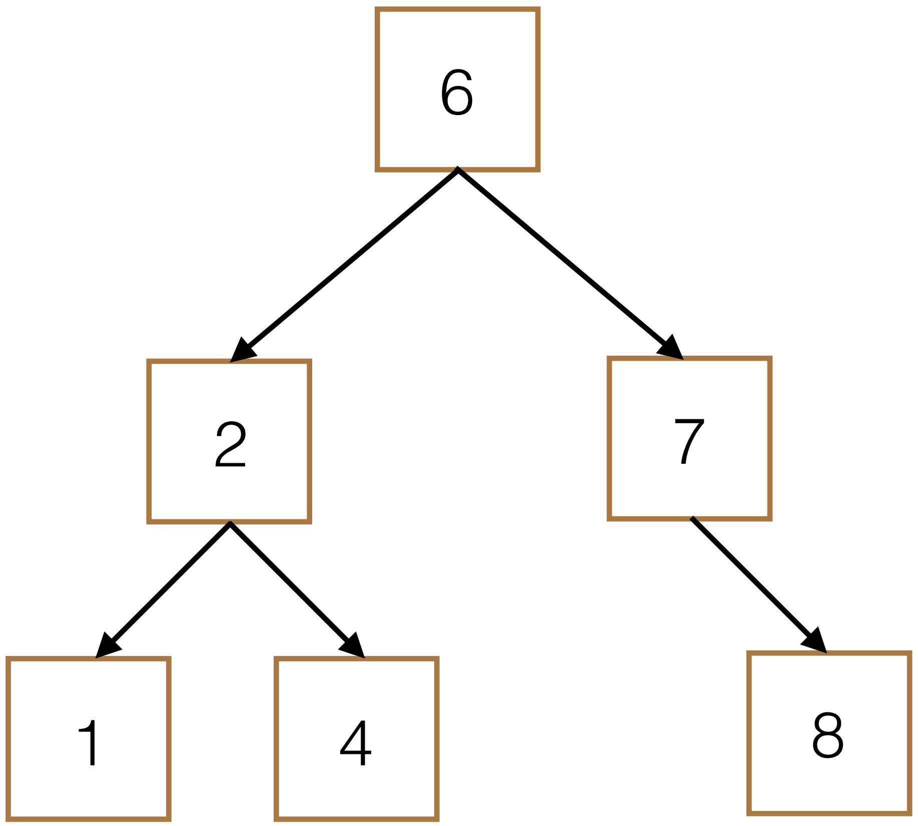
BST!



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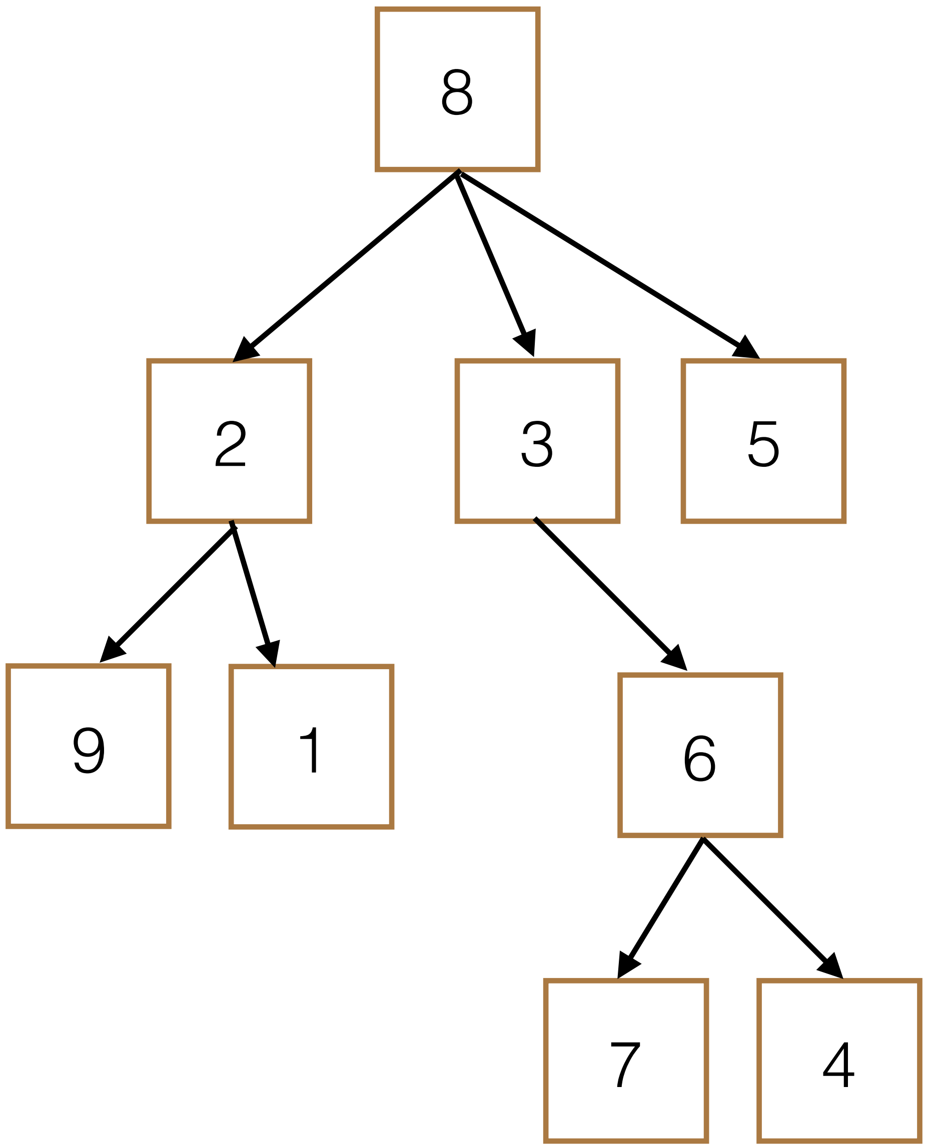

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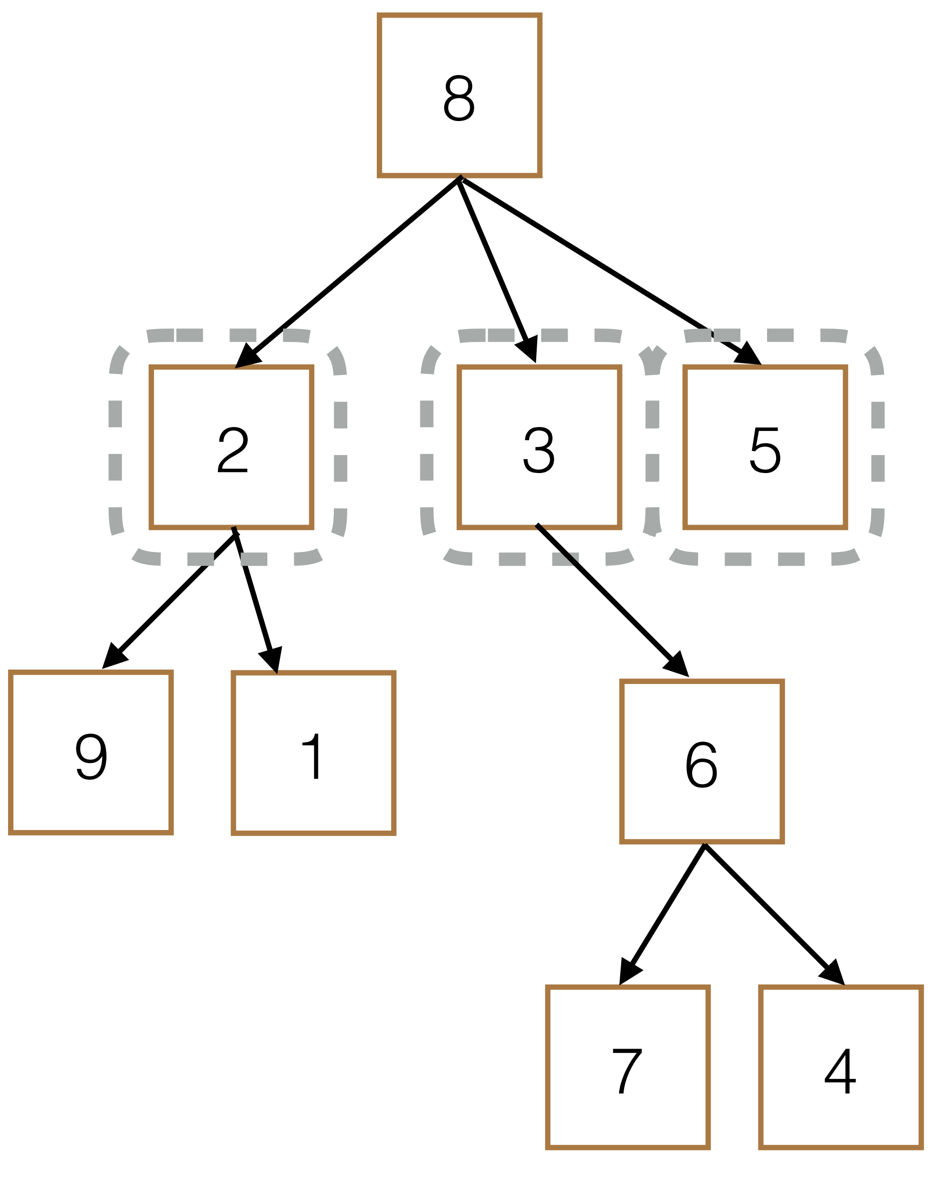

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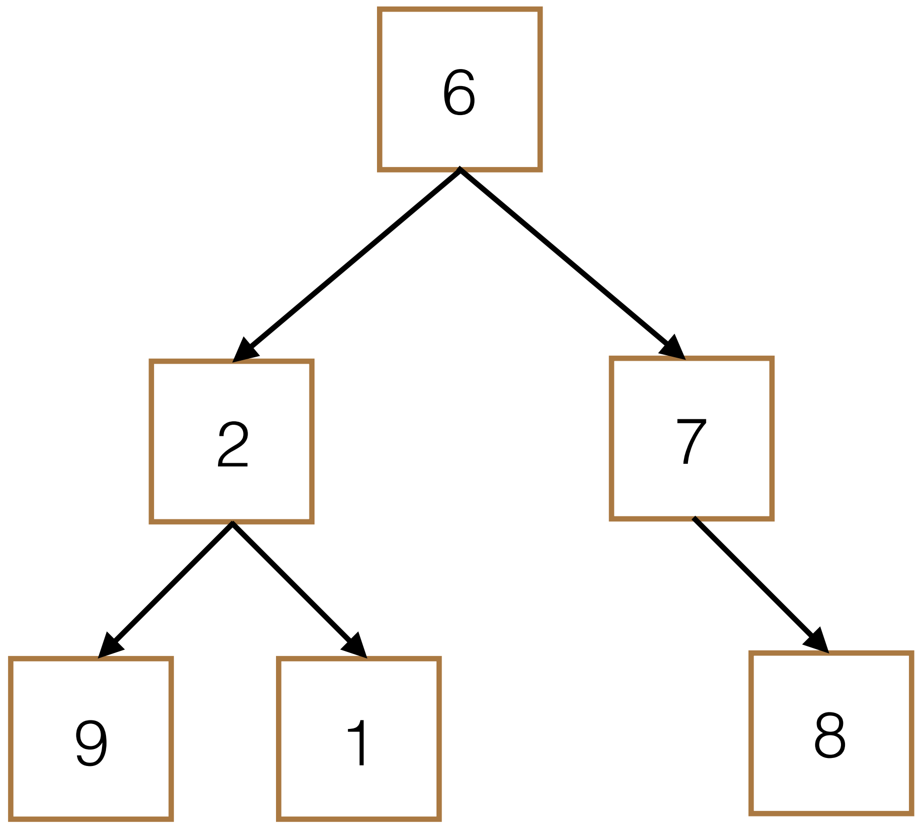

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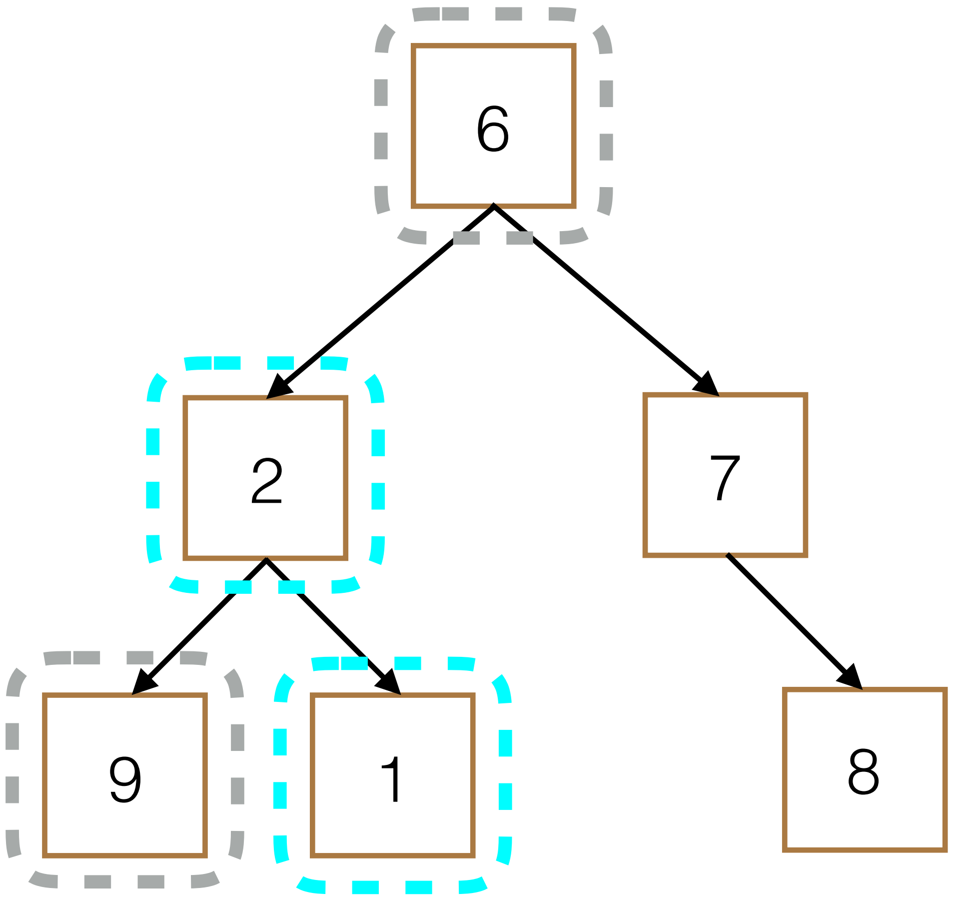

# Binary Search Tree Class

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class Tree:
    def __init__(self, entry, children=[]):
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            assert isinstance(c, Tree)
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class BST:
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class BST:
    empty = ()
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class BST:
    empty = ()
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        if left is not BST.empty:
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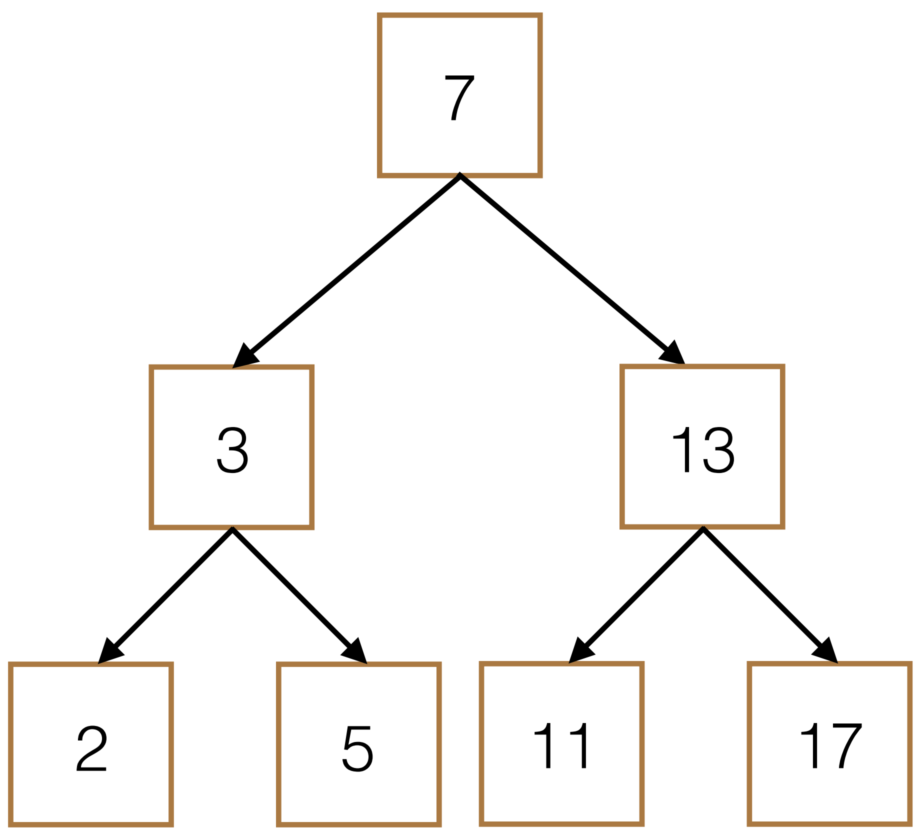

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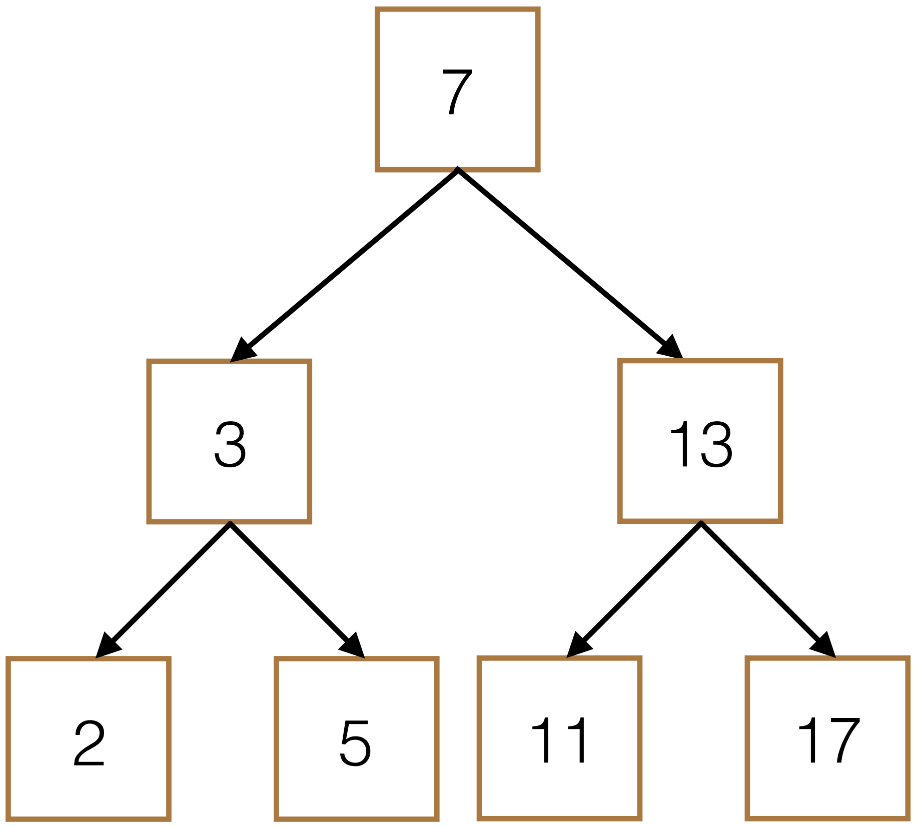

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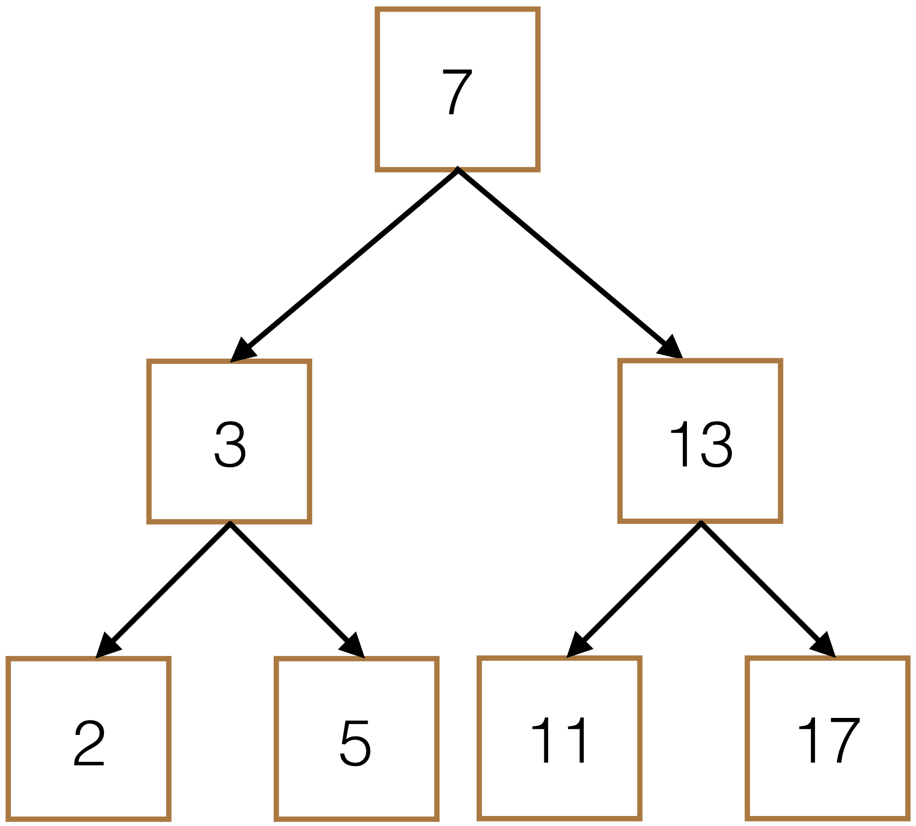

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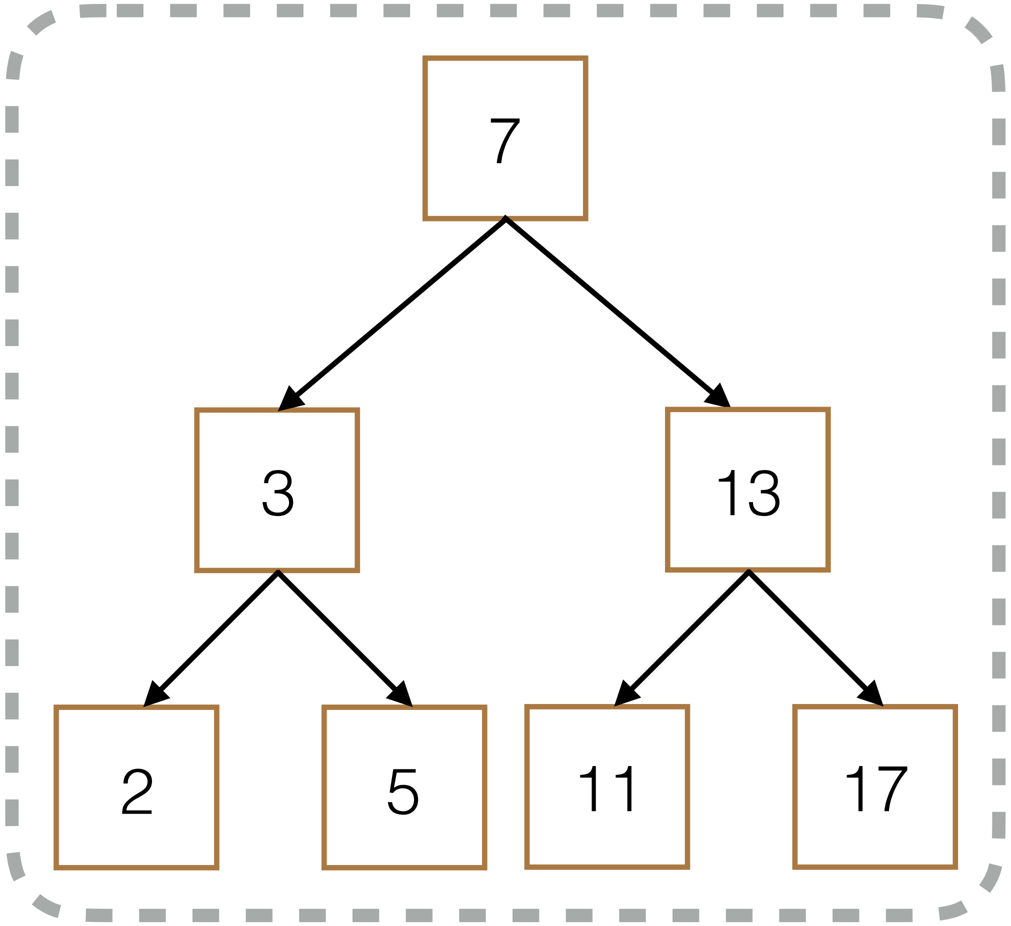

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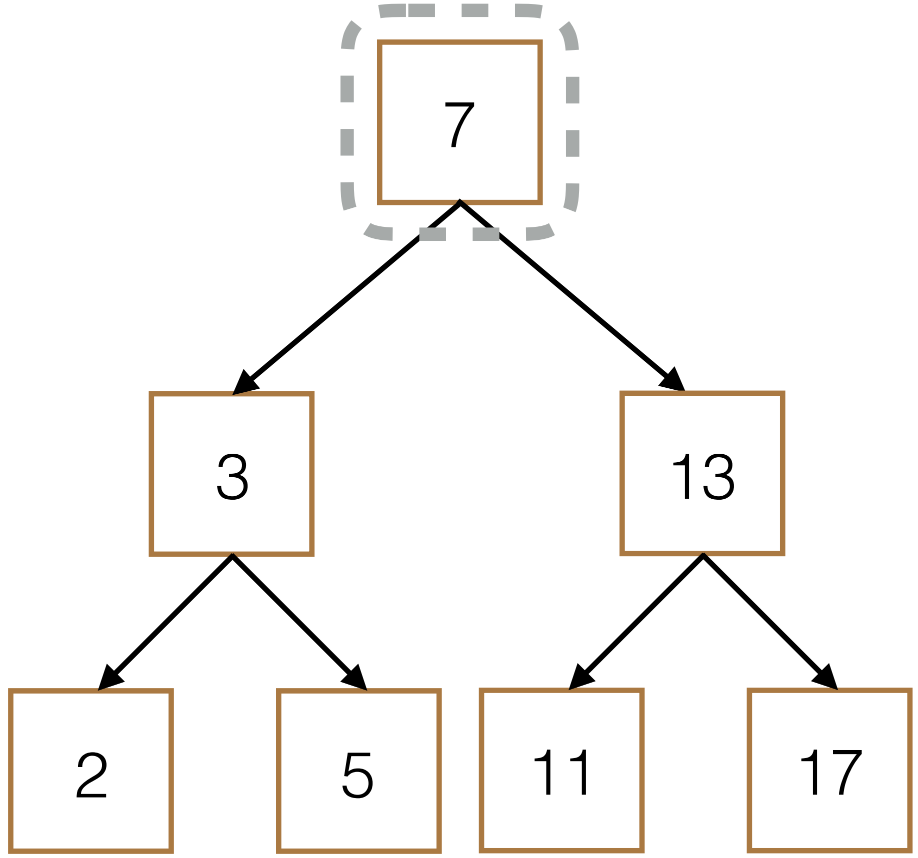

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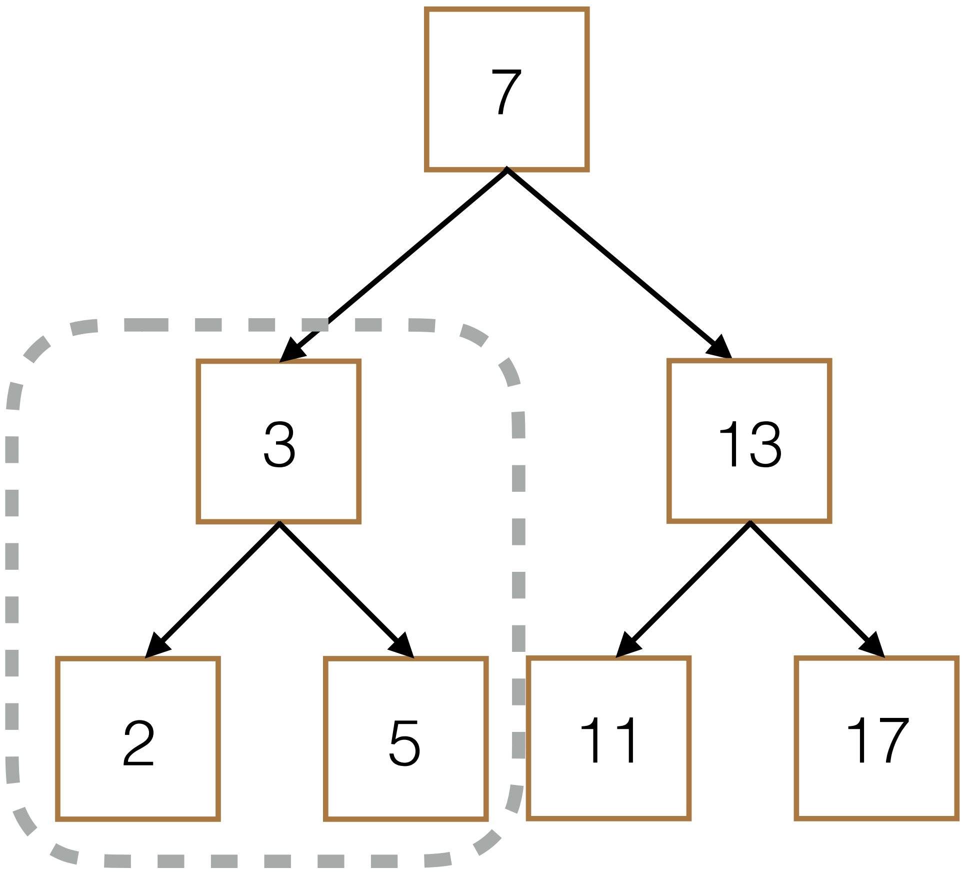

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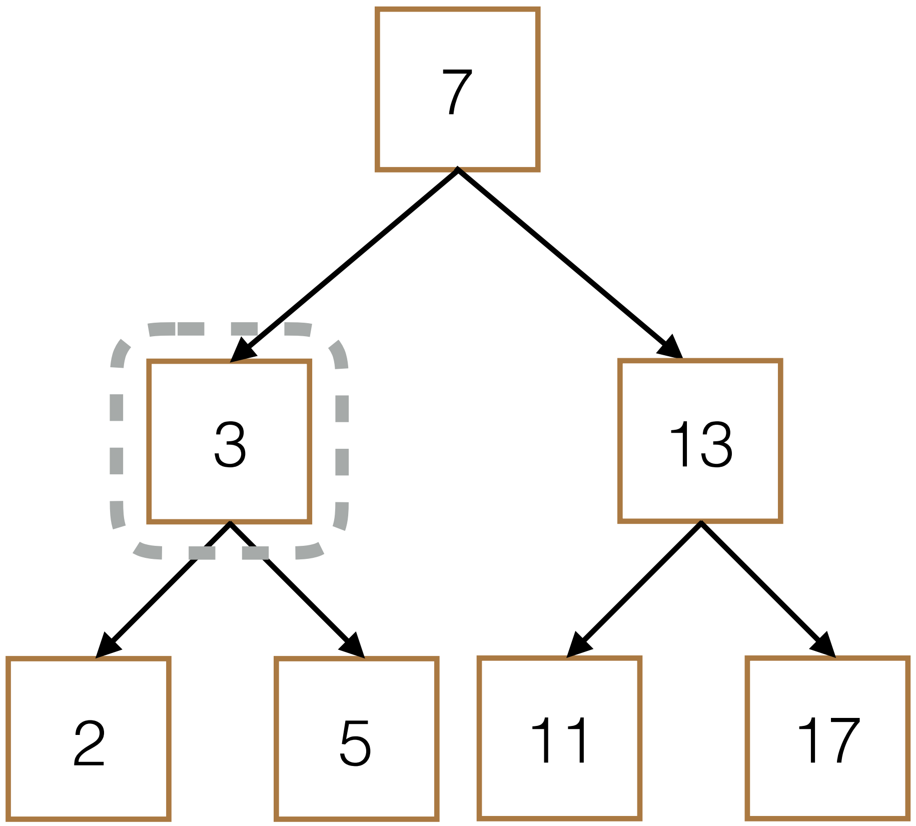

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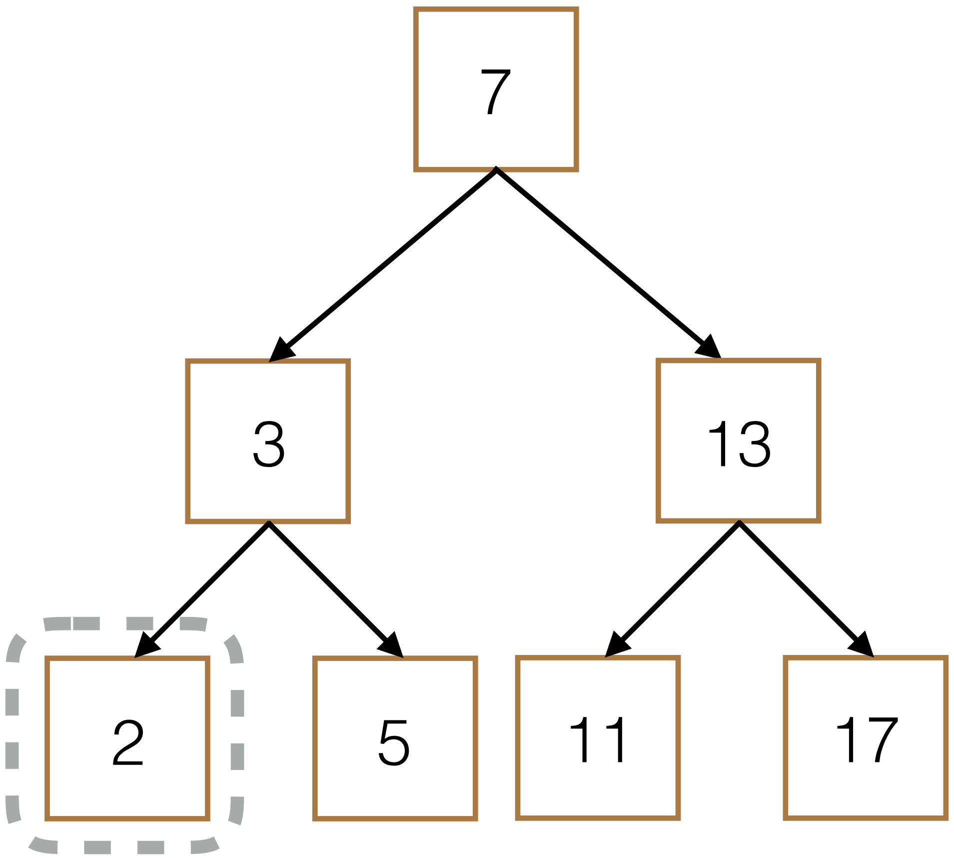

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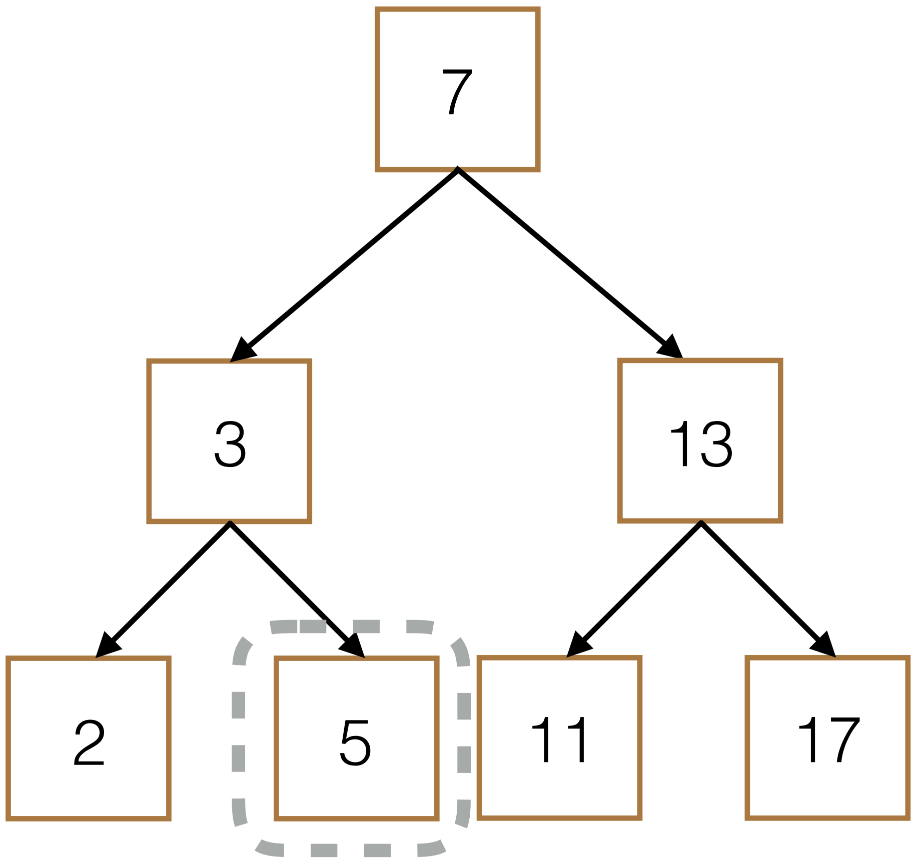

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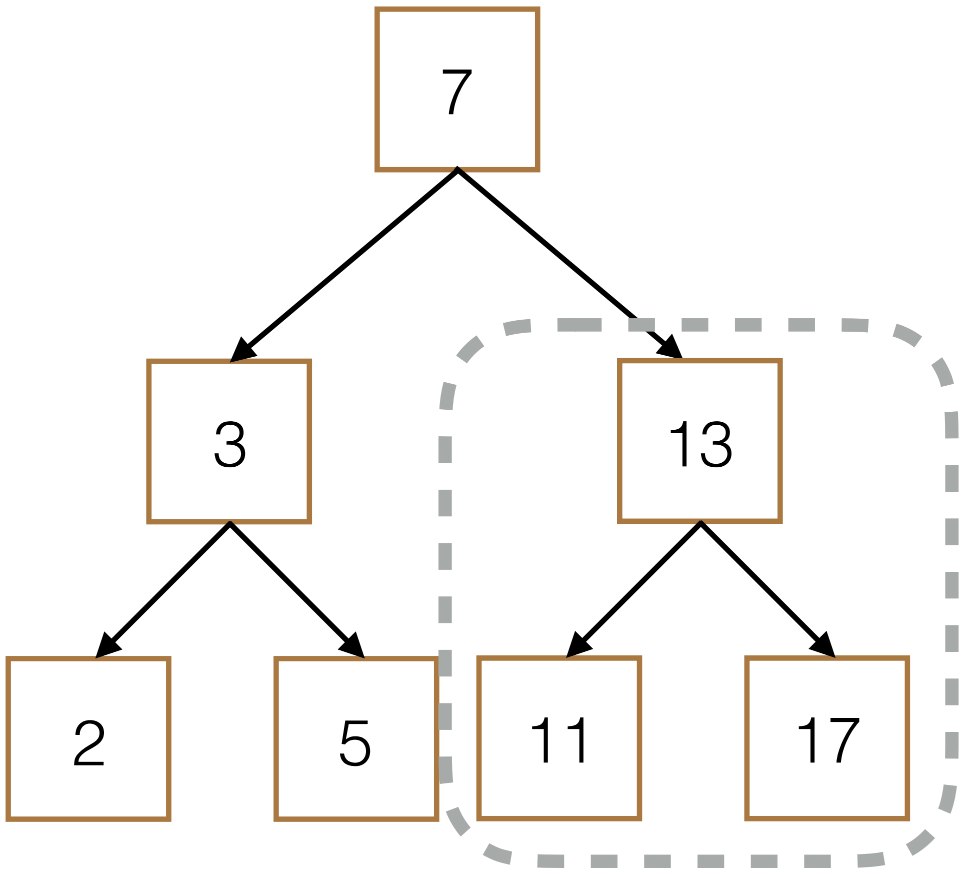

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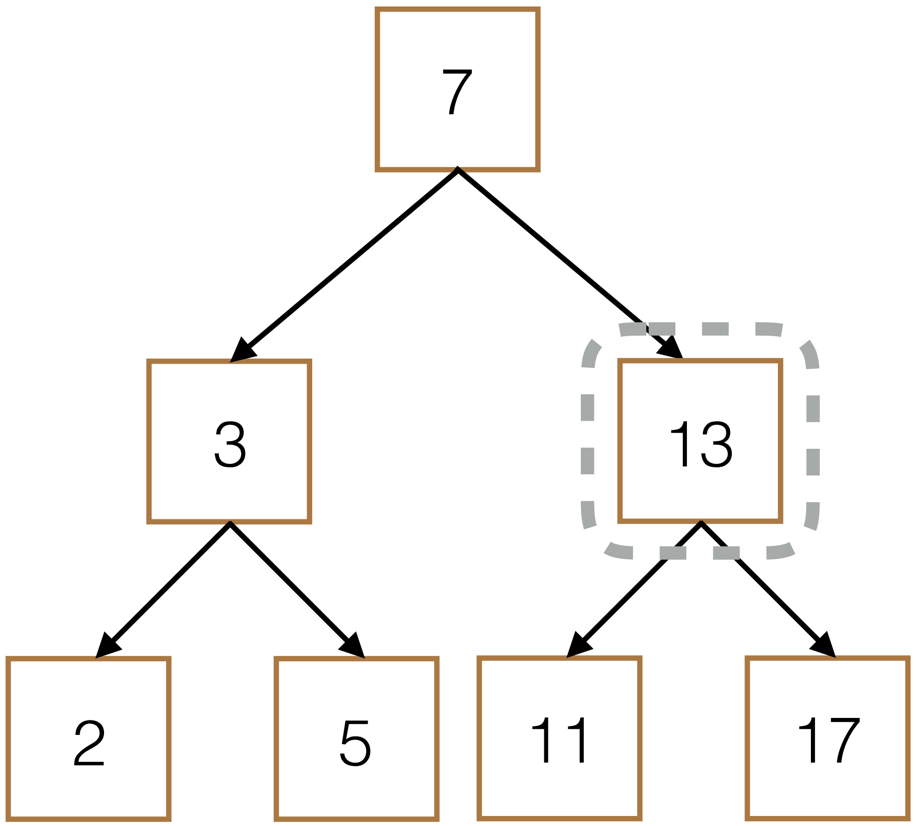

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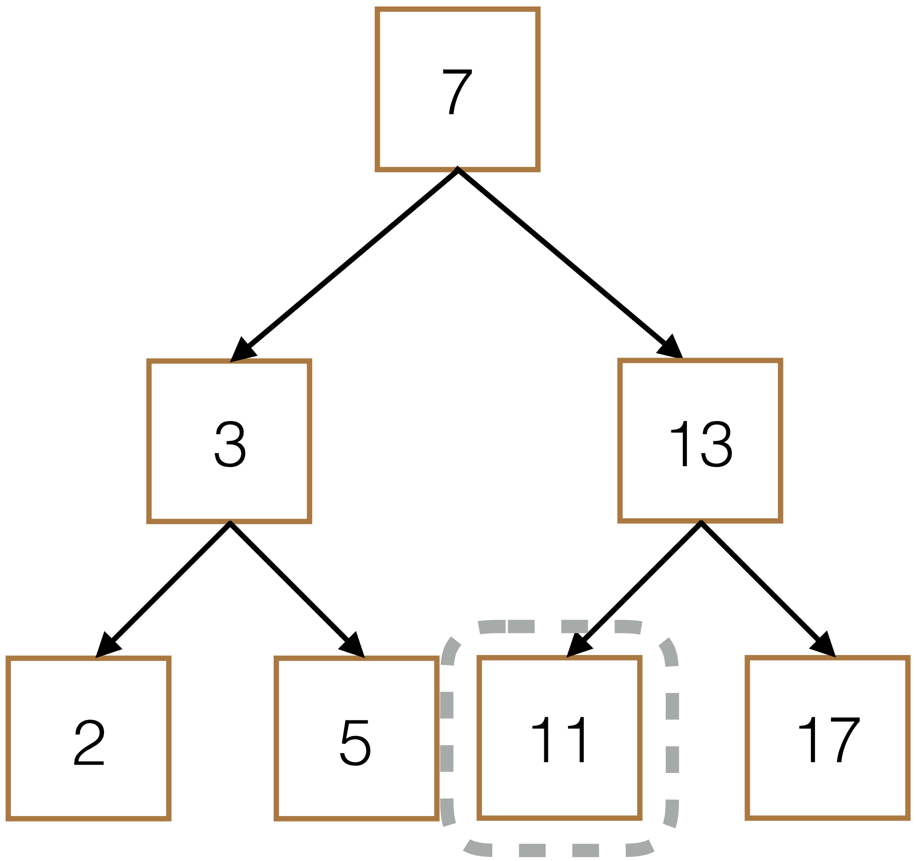

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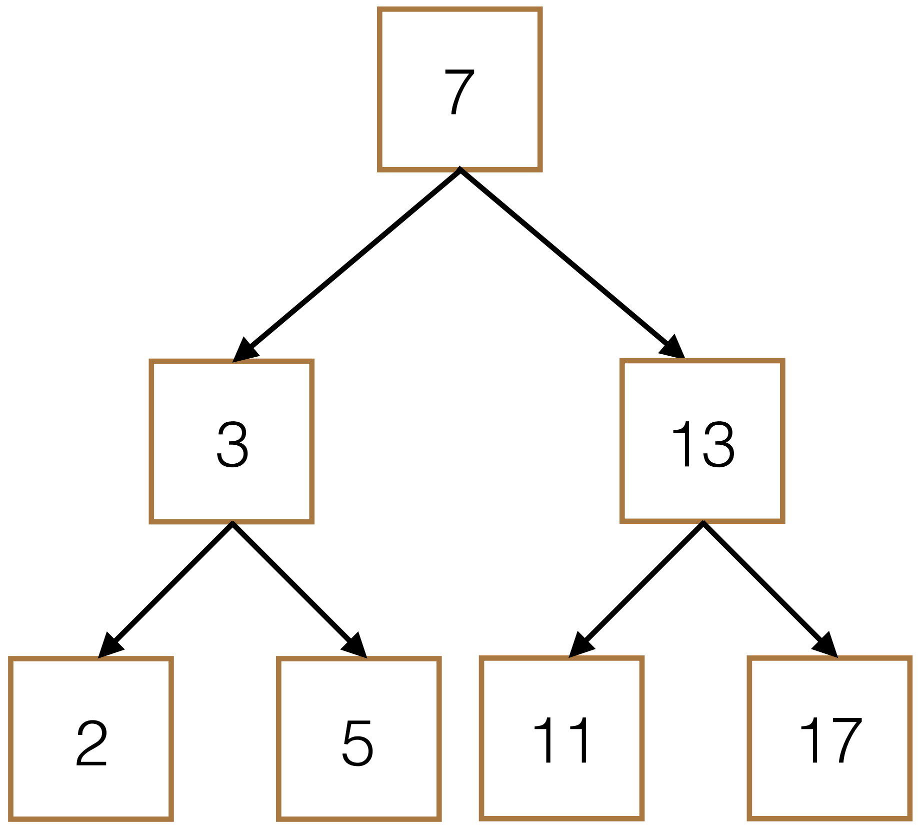

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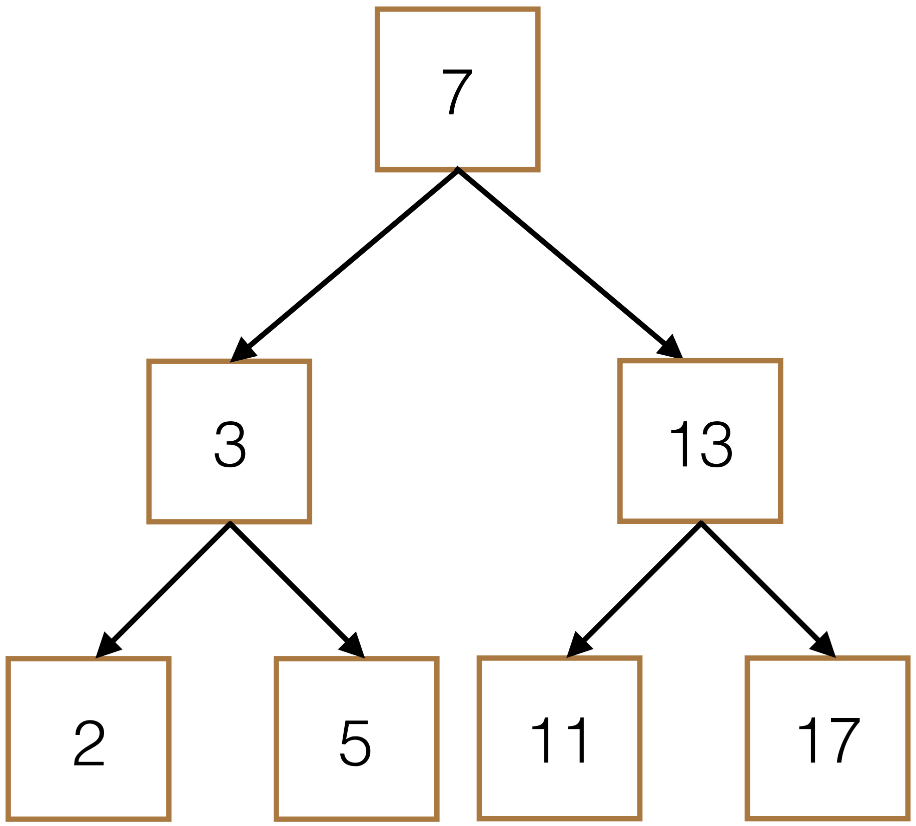


$\Theta(n)$

# Runtime Comparison

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class BST:
    def __init__(self, entry, left=empty, right=empty): ...

    def __contains__(self, e):
        if self.entry == e:
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                        is not BST.empty:
            return e in self.left
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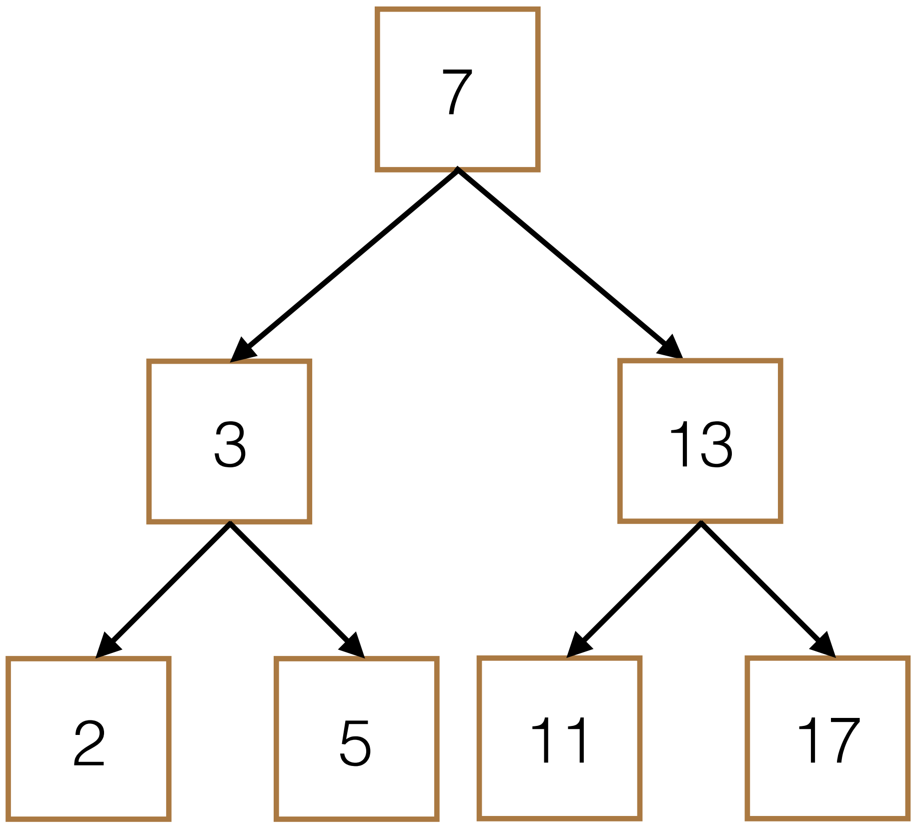

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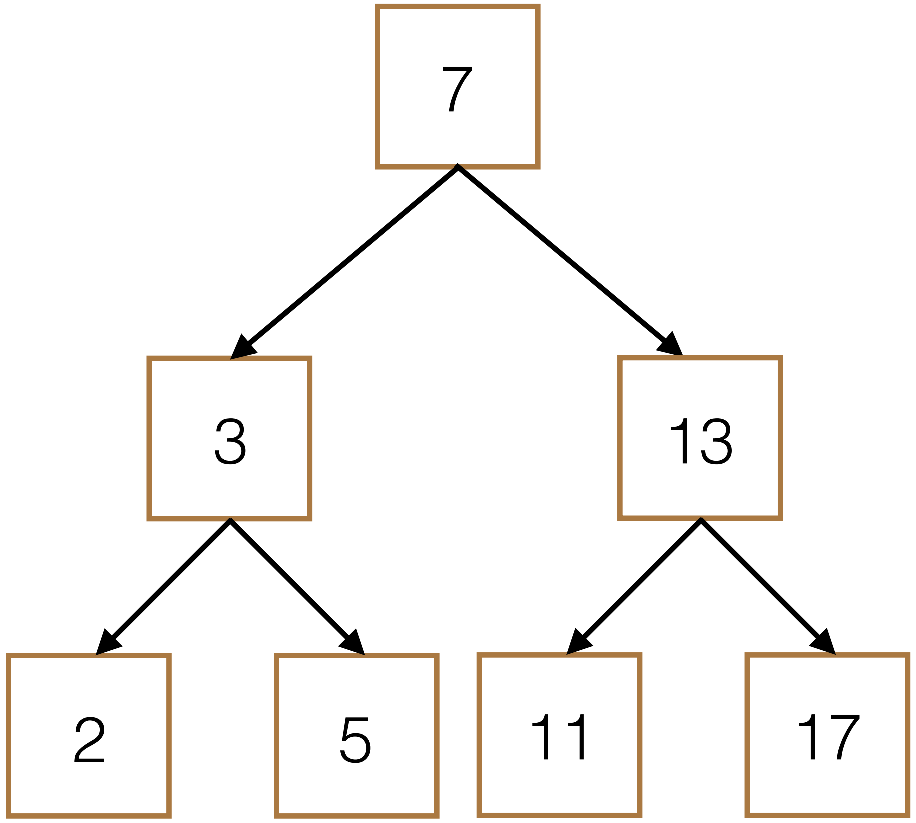


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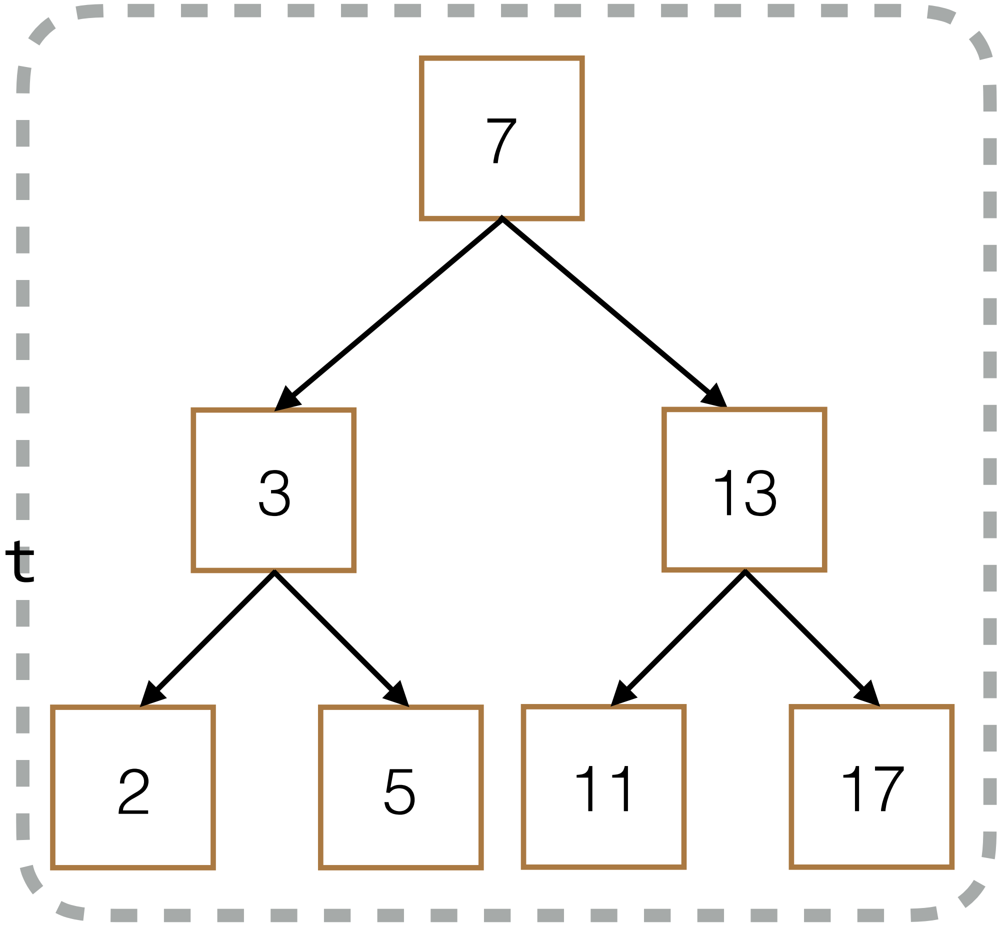

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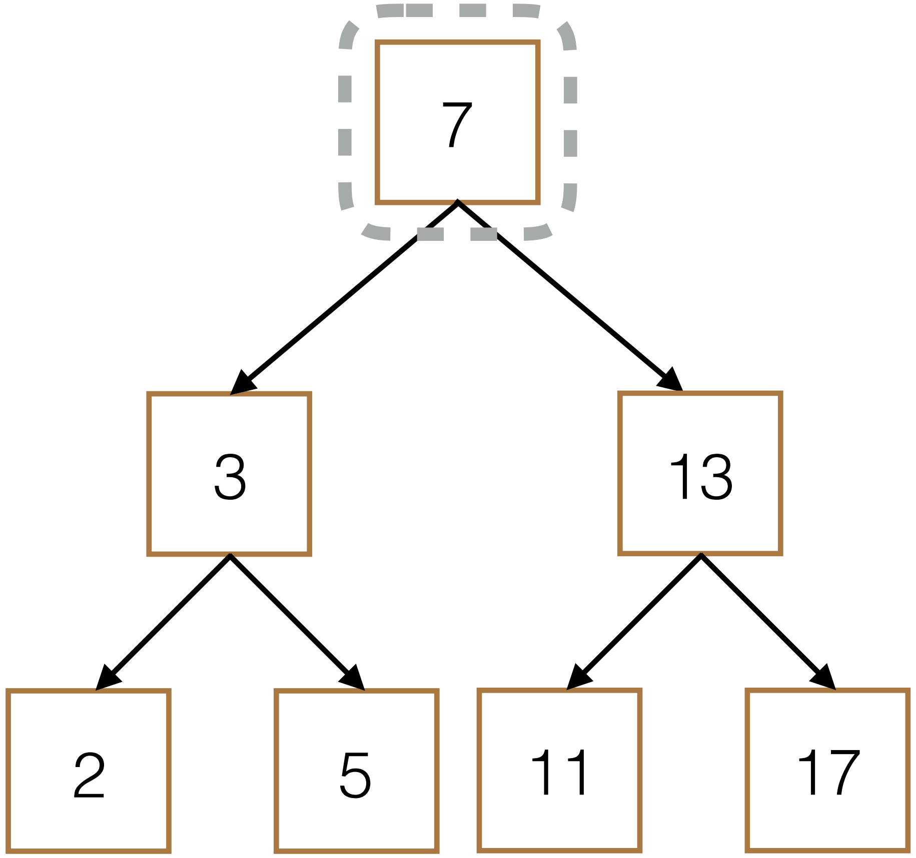


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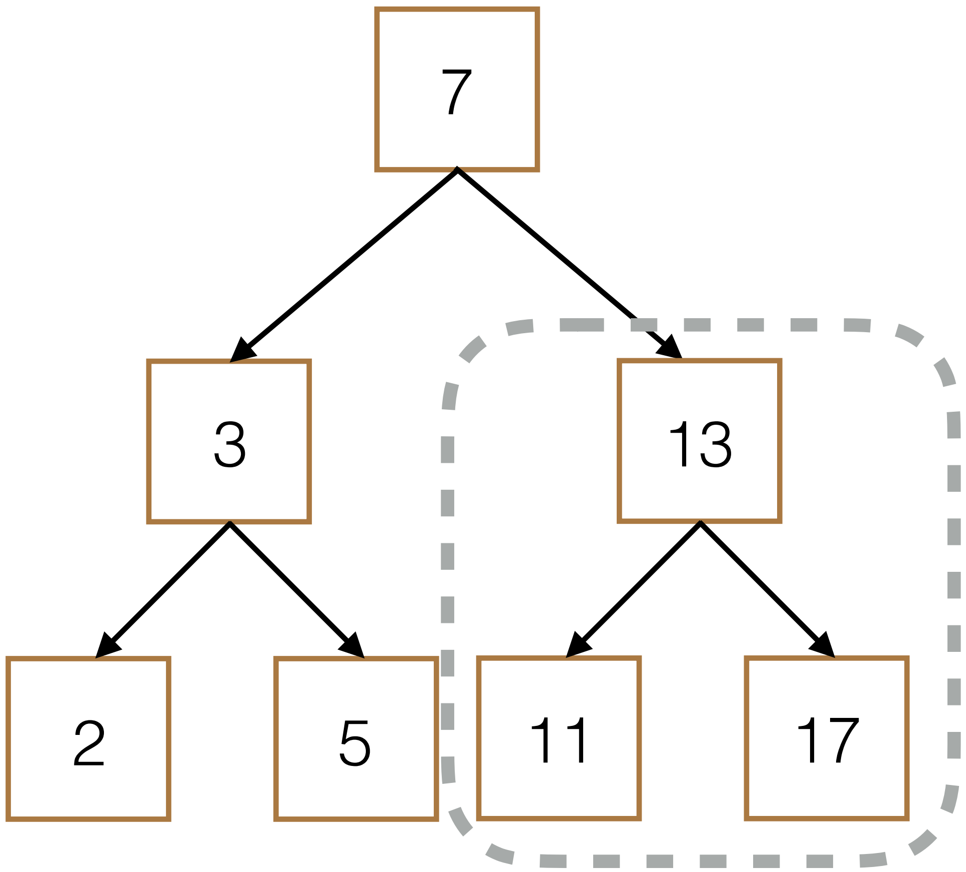

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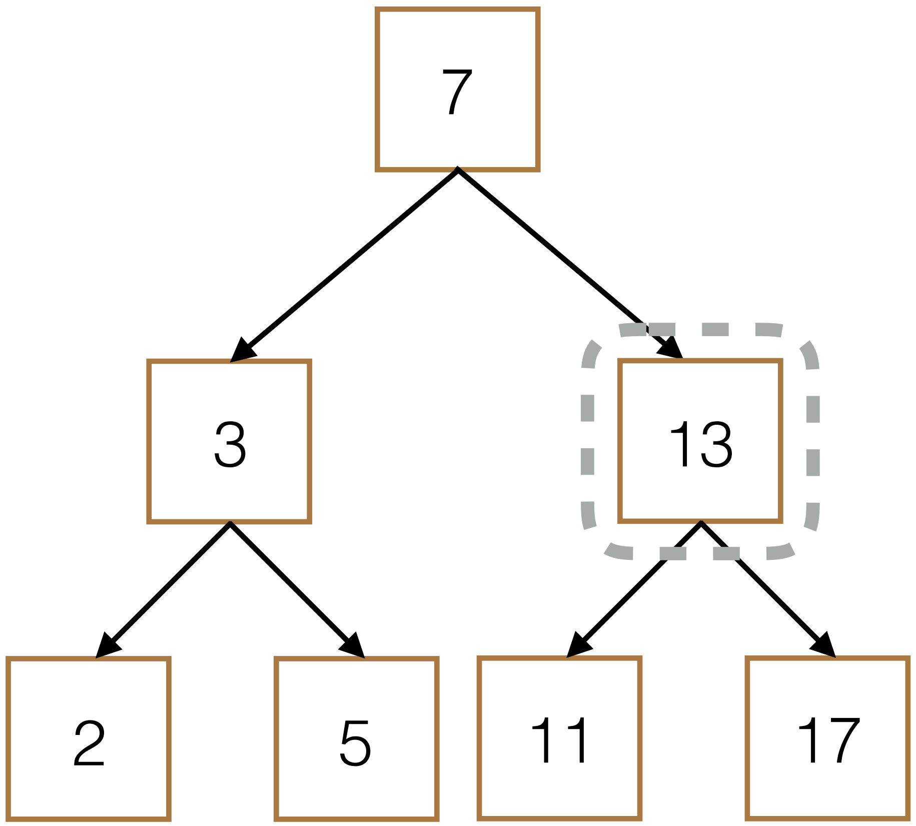


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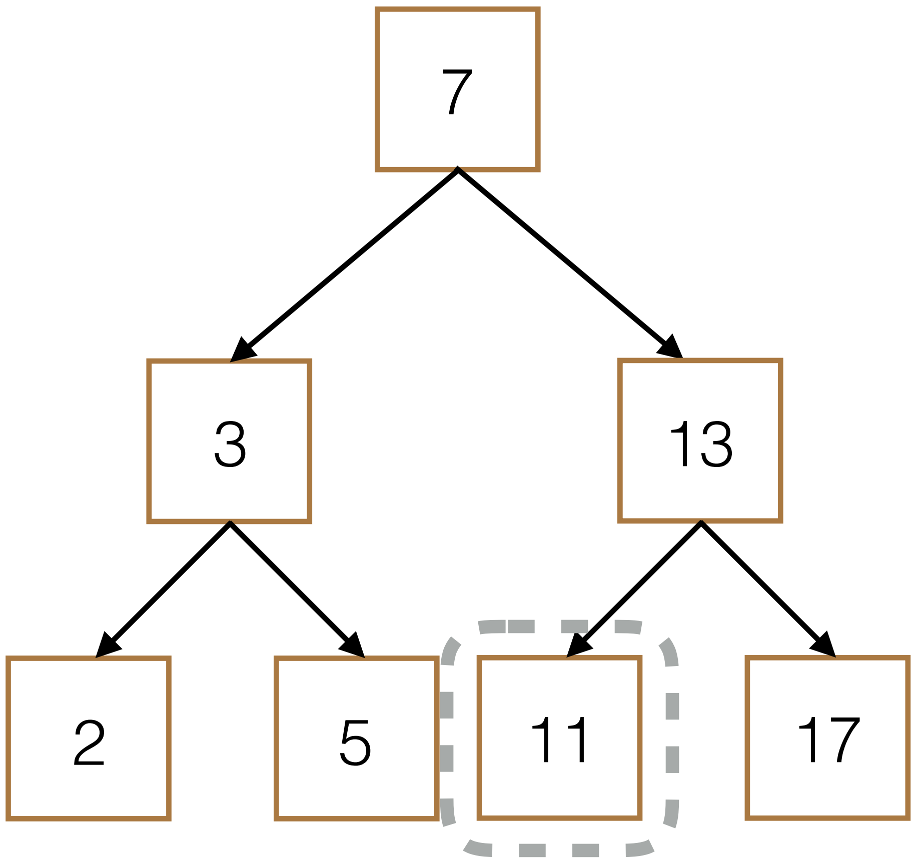

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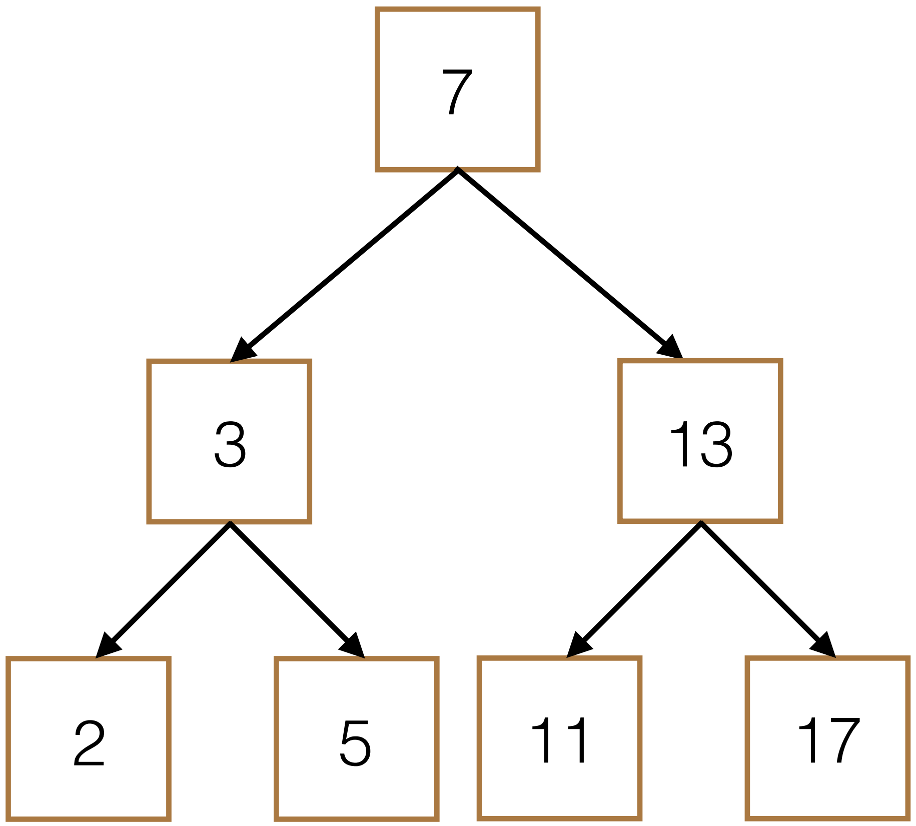


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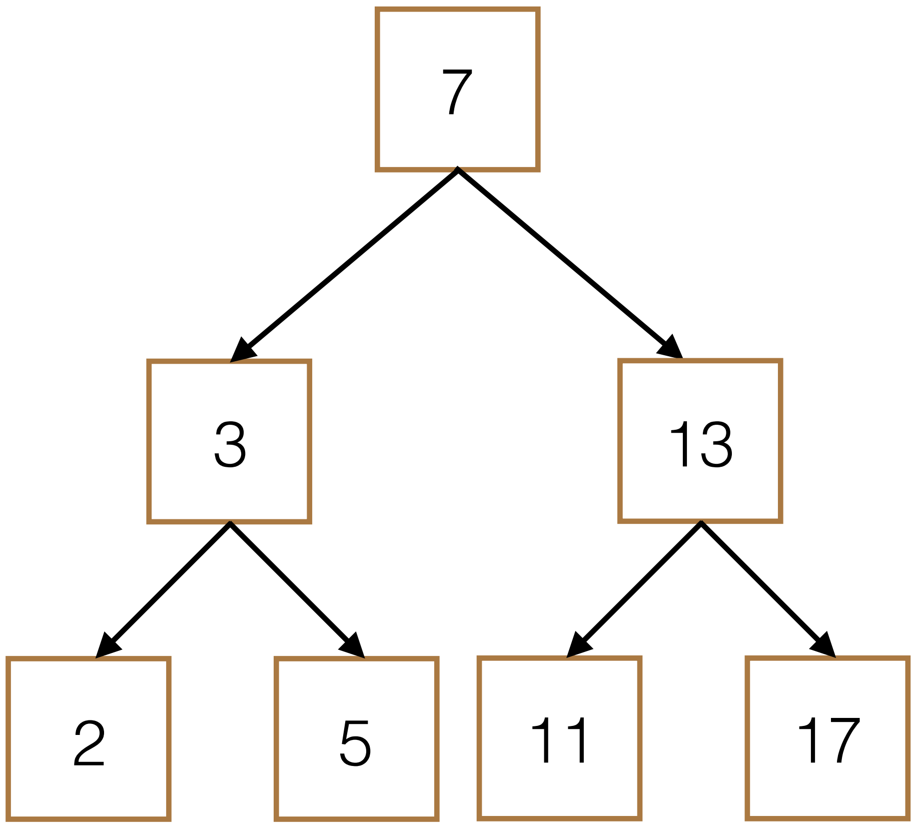


$\Theta(\log n)$

# Summary

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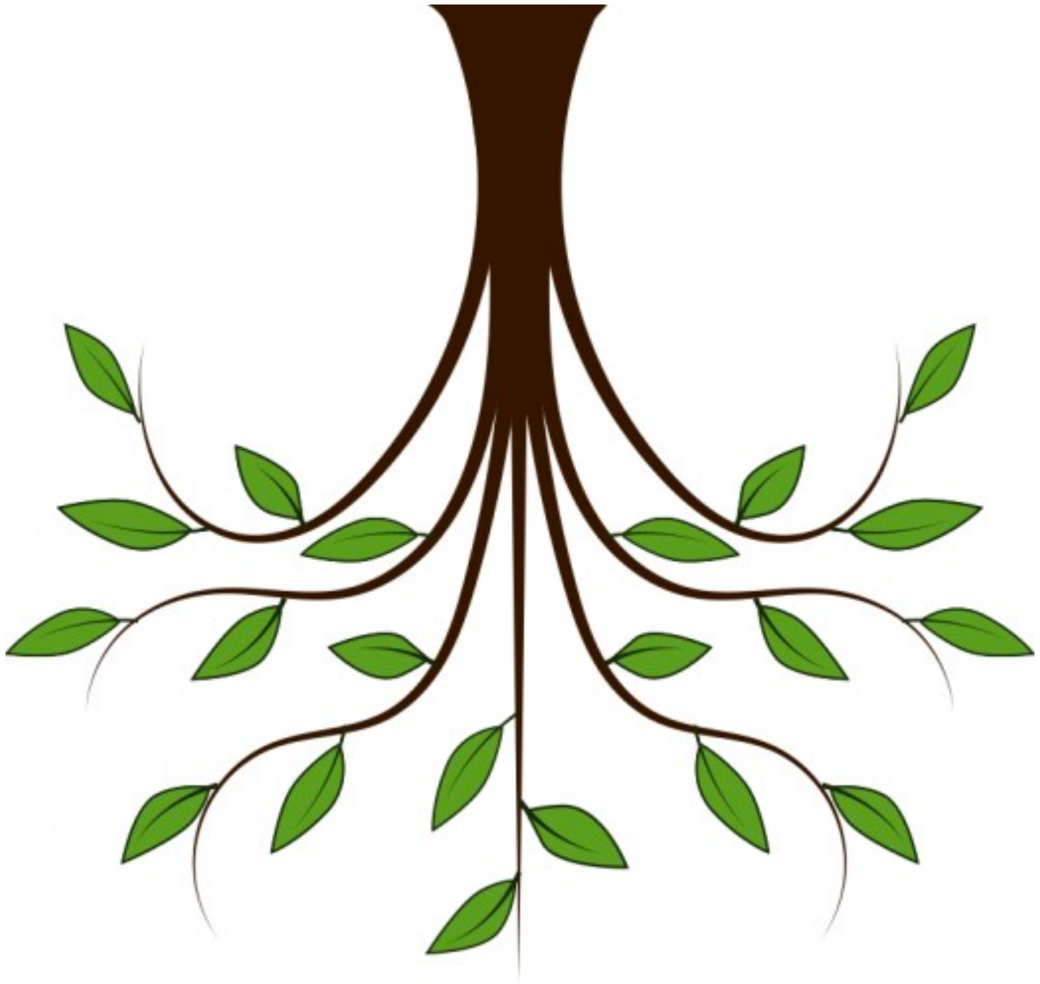
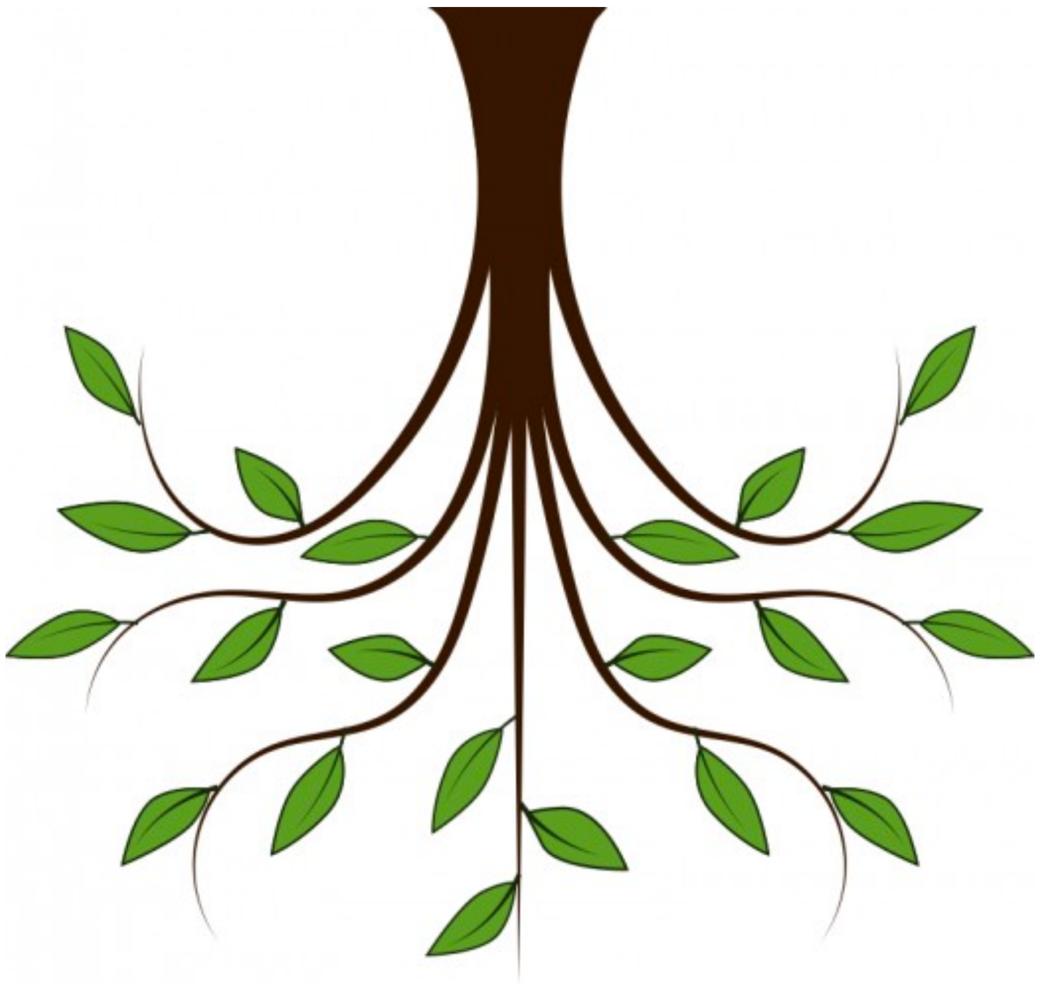


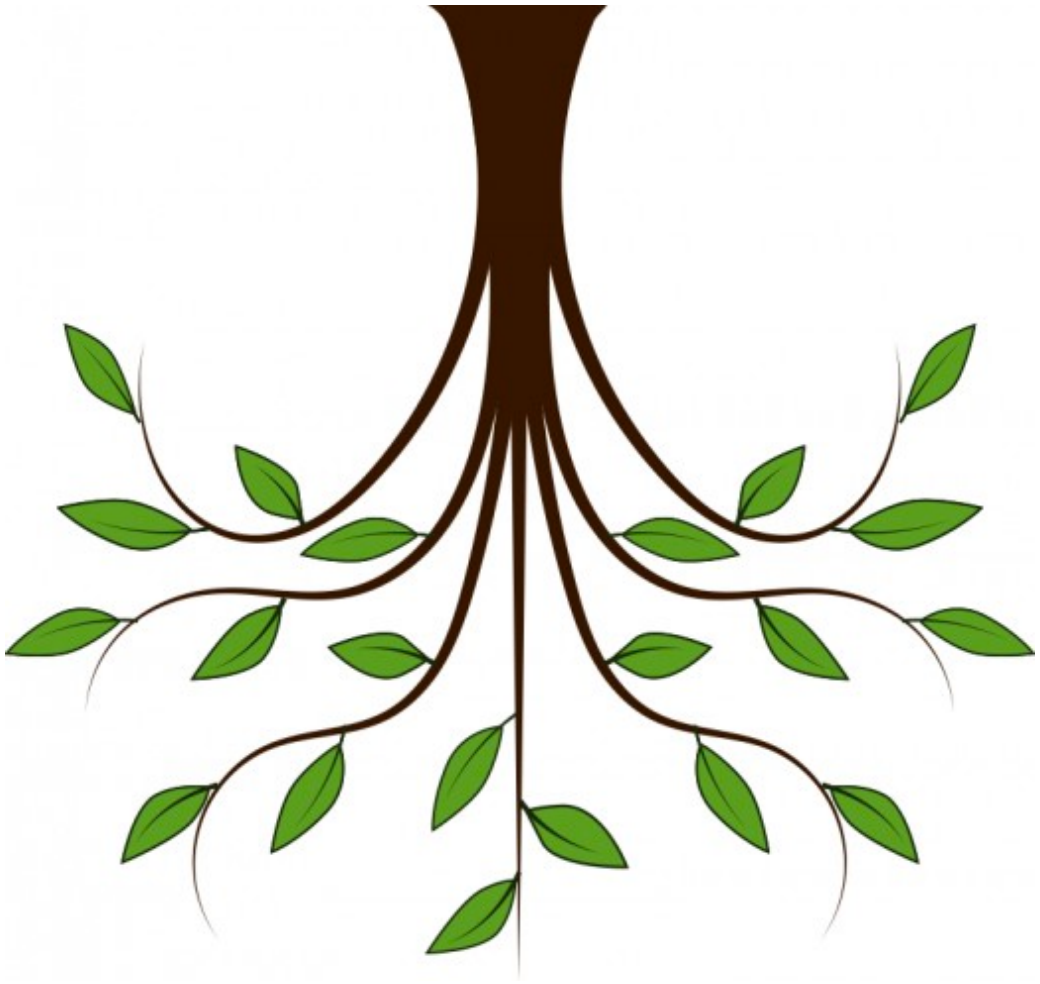
# Summary

- Trees created with a class are mutable!



# Summary

- Trees created with a class are mutable!
- BSTs allow us to organize our data in left child and right child based on value



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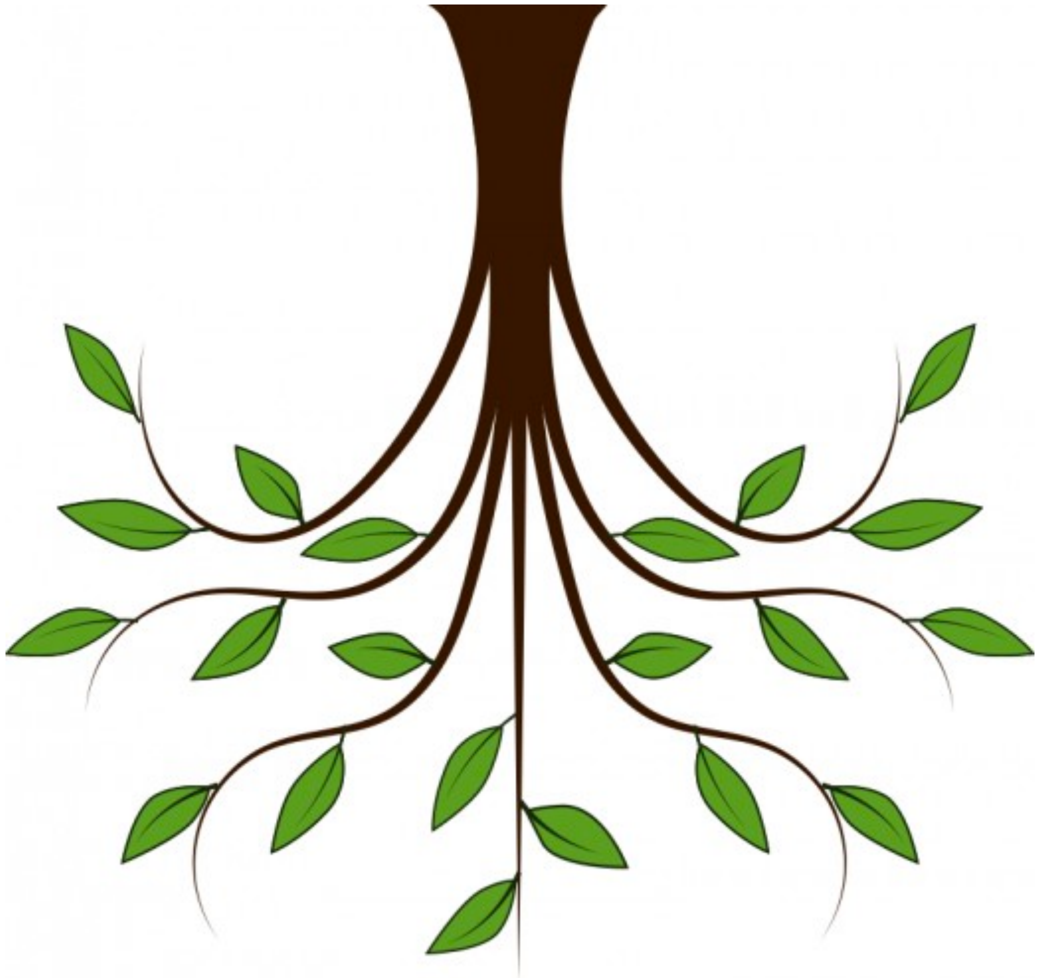
- BST allows for more efficient search



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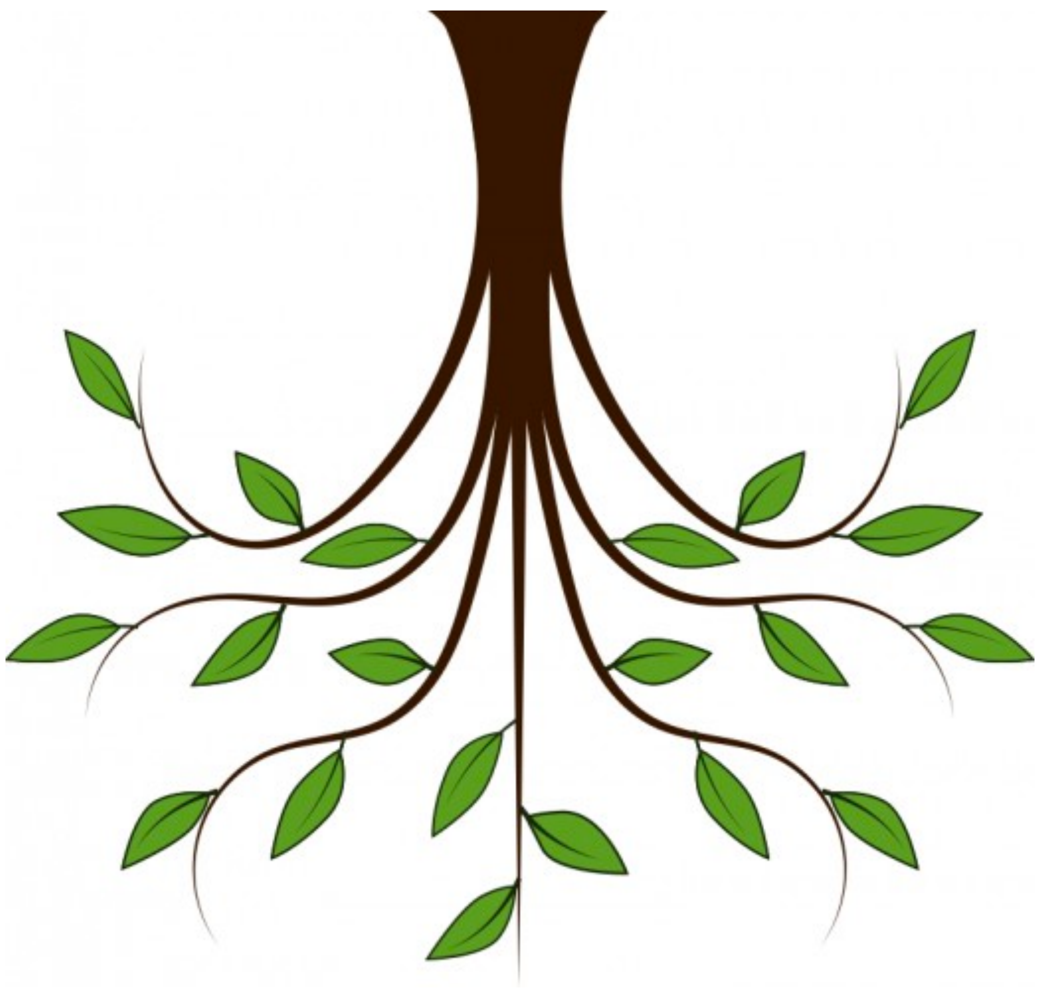
    - $\Theta(n)$ in regular tree



# Summary

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- BSTs allow us to organize our data in left child and right child based on value
- BST allows for more efficient search
    - **Θ(n)** in regular tree
    - **Θ(log n)** in BST