

Lecture 24: Logic II

Brian Hou
August 2, 2016

Announcements

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- **Potluck II** on 8/10! 5–8pm (or later) in Wozniak Lounge

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- **Potluck II** on 8/10! 5–8pm (or later) in Wozniak Lounge
 - Bring food and board games!

Roadmap

Introduction

Functions

Data

Mutability

Objects

Interpretation

Paradigms

Applications

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- This week (Paradigms), the goals are:

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- This week (Paradigms), the goals are:
 - To study examples of paradigms that are very different from what we have seen so far

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Paradigms

Applications

- This week (Paradigms), the goals are:
 - To study examples of paradigms that are very different from what we have seen so far
 - To expand our definition of what counts as programming

Anagram

Did you mean: *nag a ram?*

Anagrams

Anagrams

cat

Anagrams

cat

at

Anagrams

at

cat

at

ta

Anagrams

cat

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cat

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ta

Anagrams

cat

at

act

cat

at

ta

Anagrams

cat

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ta

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cat

act

atc

Anagrams

cat

at

at

ta

cat

act

atc

cta

Anagrams

cat

at

at

ta

cat

act

atc

cta

tca

Anagrams

cat

at

at

ta

cat

act

atc

cta

tca

tac

Imperative Anagrams

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```
def anagram(s):
```

Imperative Anagrams

```
def anagram(s):  
    if len(s) == 0:
```

Imperative Anagrams

```
def anagram(s):  
    if len(s) == 0:  
        return [[]]
```

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```
def anagram(s):  
    if len(s) == 0:  
        return [[]]  
    result = []
```

Imperative Anagrams

```
def anagram(s):  
    if len(s) == 0:  
        return []  
    result = []  
    anagrams = anagram(s[1:])
```

Imperative Anagrams

```
def anagram(s):  
    if len(s) == 0:  
        return []  
    result = []  
    anagrams = anagram(s[1:])  
    for x in anagrams:
```

Imperative Anagrams

```
def anagram(s):  
    if len(s) == 0:  
        return []  
    result = []  
    anagrams = anagram(s[1:])  
    for x in anagrams:  
        for i in range(0, len(x) + 1):
```

Imperative Anagrams

```
def anagram(s):  
    if len(s) == 0:  
        return []  
    result = []  
    anagrams = anagram(s[1:])  
    for x in anagrams:  
        for i in range(0, len(x) + 1):  
            new_anagram = x[:i] + [s[0]] + x[i:]
```

Imperative Anagrams

```
def anagram(s):  
    if len(s) == 0:  
        return []  
    result = []  
    anagrams = anagram(s[1:])  
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        for i in range(0, len(x) + 1):  
            new_anagram = x[:i] + [s[0]] + x[i:]  
            result.append(new_anagram)
```

Imperative Anagrams

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def anagram(s):  
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            new_anagram = x[:i] + [s[0]] + x[i:]  
            result.append(new_anagram)  
    return result
```

Imperative Anagrams

(demo)

```
def anagram(s):
    if len(s) == 0:
        return [[]]
    result = []
    anagrams = anagram(s[1:])
    for x in anagrams:
        for i in range(0, len(x) + 1):
            new_anagram = x[:i] + [s[0]] + x[i:]
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    return result
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Declarative Anagrams

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```
logic> (fact (insert ?a ?r (?a . ?r)))
```

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logic> (fact (insert ?a (?b . ?r) (?b . ?s)))
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logic> (fact (insert ?a (?b . ?r) (?b . ?s))  
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logic> (fact (insert ?a ?r (?a . ?r)))
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logic> (fact (insert ?a (?b . ?r) (?b . ?s))  
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logic> (fact (insert ?a ?r (?a . ?r)))
```

```
logic> (fact (insert ?a (?b . ?r) (?b . ?s))  
          (insert ?a ?r ?s))
```

```
logic> (fact (anagram () ()))
```

Declarative Anagrams

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logic> (fact (insert ?a ?r (?a . ?r)))
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logic> (fact (insert ?a (?b . ?r) (?b . ?s))  
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logic> (fact (anagram () ()))
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```
logic> (fact (anagram (?a . ?r) ?b))
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logic> (fact (insert ?a (?b . ?r) (?b . ?s))  
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logic> (fact (insert ?a ?r (?a . ?r)))
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```
logic> (fact (anagram () ()))
```

```
logic> (fact (anagram (?a . ?r) ?b)  
         (anagram ?r ?s)  
         (insert ?a ?s ?b))
```

```
logic> (query (anagram ?s (s t a r)))
```

Palindromes

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- A palindrome is a sequence that is the same when read backward and forward

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logic> (fact (palindrome ?s))
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```
logic> (fact (palindrome ?s)  
          (reverse ?s ?s))
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```
logic> (fact (palindrome ?s)
          (reverse ?s ?s))
logic> (fact (reverse () ()))
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 - Examples: "racecar", "senile felines", "too hot to hoot"

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logic> (fact (palindrome ?s)
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logic> (fact (reverse () ()))
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```
logic> (fact (reverse (?first . ?rest) ?rev))
```

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logic> (fact (reverse (?first . ?rest) ?rev)
          (reverse ?rest ?rest-rev))
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 - Examples: "racecar", "senile felines", "too hot to hoot"

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logic> (fact (palindrome ?s)
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logic> (fact (reverse (?first . ?rest) ?rev)
          (reverse ?rest ?rest-rev)
          (append ?rest-rev (?first) ?rev))
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- In declarative programming, we tell the computer what a solution looks like, rather than how to get the solution

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- In declarative programming, we tell the computer what a solution looks like, rather than how to get the solution
- If we describe a solution in two different ways, will the computer take the same amount of time to compute a solution?
 - Probably not...

Reverse

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```
logic> (fact (reverse () ()))
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logic> (fact (reverse (?first . ?rest) ?rev)  
          (reverse ?rest ?rest-rev)  
          (append ?rest-rev (?first) ?rev)))
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```
logic> (fact (reverse (?first . ?rest) ?rev)
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```

```
logic> (fact (accrev (?first . ?rest) ?acc ?rev))
```

Reverse

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logic> (fact (reverse () ()))
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logic> (fact (reverse (?first . ?rest) ?rev)
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logic> (fact (accrev (?first . ?rest) ?acc ?rev)
          (accrev ?rest (?first . ?acc) ?rev)))
```

```
logic> (fact (accrev () ?acc ?acc))
```

```
logic> (fact (accrev ?s ?rev))
```

Reverse

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logic> (fact (reverse (?first . ?rest) ?rev)
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          (accrev ?rest (?first . ?acc) ?rev))
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logic> (fact (accrev () ?acc ?acc))
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```

Break!

Arithmetic

Number Representation

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- Logic does not have numbers, but does have Scheme lists

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- Let's create our own number representation!

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- Let's create our own number representation!
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- We can represent the numbers
 - 0, 1, 2, 3, ... as
 - 0, (+ 1 0), (+ 1 (+ 1 0)), (+ 1 (+ 1 (+ 1 0))), ...

Number Representation

- Logic does not have numbers, but does have Scheme lists
- Let's create our own number representation!
 - We'll limit ourselves to non-negative integers
- We can represent the numbers
 - 0, 1, 2, 3, ... as
 - 0, (+ 1 0), (+ 1 (+ 1 0)), (+ 1 (+ 1 (+ 1 0))), ...
- This is still a **symbolic** representation! Logic doesn't know that these are Scheme expressions that would evaluate to that number

Addition

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- Mathematical facts:

Addition

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- $0 + n = n$

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```
logic> (fact (+ 0 ?n ?n))
```

Addition

- Mathematical facts:

- $0 + n = n$

- In order for $(x + 1) + y = (z + 1)$ to be true, $x + y = z$

```
logic> (fact (+ 0 ?n ?n))
```

```
logic> (fact (+ (+ 1 ?x) ?y (+ 1 ?z)))
```

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```
logic> (fact (+ 0 ?n ?n))
```

```
logic> (fact (+ (+ 1 ?x) ?y (+ 1 ?z))  
           (+ ?x ?y ?z))
```

```
logic> (query (+
```

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logic> (fact (+ (+ 1 ?x) ?y (+ 1 ?z))  
           (+ ?x ?y ?z))
```

```
logic> (query (+  
              (+ 1 (+ 1 (+ 1 0))))
```

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logic> (query (+  
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        (+ 1 (+ 1 0)))
```

Addition

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```
logic> (query (+  
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              (+ 1 (+ 1 0))  
              ?z))
```

Multiplication

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```
logic> (fact (* 0 ?n 0))
```

Multiplication

- Mathematical facts:

- $0 * n = 0$

- In order for $(x + 1) * y = z$ to be true, $x * y + y = z$

```
logic> (fact (* 0 ?n 0))
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Multiplication

- Mathematical facts:

- $0 * n = 0$

- In order for $(x + 1) * y = z$ to be true, $x * y + y = z$

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logic> (fact (* 0 ?n 0))
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```
logic> (fact (* (+ 1 ?x) ?y ?z))
```

Multiplication

- Mathematical facts:

- $0 * n = 0$

- In order for $(x + 1) * y = z$ to be true, $x * y + y = z$

```
logic> (fact (* 0 ?n 0))
```

```
logic> (fact (* (+ 1 ?x) ?y ?z)  
          (+ ?xy ?y ?z))
```

Multiplication

- Mathematical facts:

- $0 * n = 0$

- In order for $(x + 1) * y = z$ to be true, $x * y + y = z$

```
logic> (fact (* 0 ?n 0))
```

```
logic> (fact (* (+ 1 ?x) ?y ?z)
          (+ ?xy ?y ?z)
          (* ?x ?y ?xy))
```

Multiplication

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- Mathematical facts:

- $0 * n = 0$

- In order for $(x + 1) * y = z$ to be true, $x * y + y = z$

```
logic> (fact (* 0 ?n 0))
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          (+ ?xy ?y ?z)
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```

Multiplication

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- Mathematical facts:

- $0 * n = 0$

- In order for $(x + 1) * y = z$ to be true, $x * y + y = z$

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logic> (fact (* (+ 1 ?x) ?y ?z)
```

```
          (+ ?xy ?y ?z)
```

```
          (* ?x ?y ?xy))
```

```
logic> (query (* (+ 1 (+ 1 (+ 1 0)))) ?y
```

Multiplication

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- Mathematical facts:

- $0 * n = 0$

- In order for $(x + 1) * y = z$ to be true, $x * y + y = z$

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logic> (fact (* 0 ?n 0))
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```

Subtraction and Division

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 - Subtraction is the inverse of addition

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```
logic> (fact (- ?x ?y ?z))
```

Subtraction and Division

- Mathematical facts:
 - Subtraction is the inverse of addition
 - In order for $x - y = z$, $y + z = x$

```
logic> (fact (- ?x ?y ?z)
          (+ ?y ?z ?x))
```

Subtraction and Division

- Mathematical facts:
 - Subtraction is the inverse of addition
 - In order for $x - y = z$, $y + z = x$
 - Division is the inverse of multiplication

```
logic> (fact (- ?x ?y ?z)
          (+ ?y ?z ?x))
```

Subtraction and Division

- Mathematical facts:
 - Subtraction is the inverse of addition
 - In order for $x - y = z$, $y + z = x$
 - Division is the inverse of multiplication
 - In order for $x / y = z$, $y * z = x$ (assuming x is divisible by y)

```
logic> (fact (- ?x ?y ?z)
          (+ ?y ?z ?x))
```

Subtraction and Division

- Mathematical facts:
 - Subtraction is the inverse of addition
 - In order for $x - y = z$, $y + z = x$
 - Division is the inverse of multiplication
 - In order for $x / y = z$, $y * z = x$ (assuming x is divisible by y)

```
logic> (fact (- ?x ?y ?z)
          (+ ?y ?z ?x))
```

```
logic> (fact (/ ?x ?y ?z))
```

Subtraction and Division

- Mathematical facts:
 - Subtraction is the inverse of addition
 - In order for $x - y = z$, $y + z = x$
 - Division is the inverse of multiplication
 - In order for $x / y = z$, $y * z = x$ (assuming x is divisible by y)

```
logic> (fact (- ?x ?y ?z)
          (+ ?y ?z ?x))
```

```
logic> (fact (/ ?x ?y ?z)
          (* ?y ?z ?x))
```

Subtraction and Division

(demo)

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```
logic> (query (?op ?arg1 ?arg2  
              (+ 1 (+ 1 (+ 1 (+ 1 (+ 1 (+ 1 0)))))))
```

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- However, just because the computer is the one solving the problem doesn't mean that we can write any declarative program and it will "just work"
- As declarative programmers, we (eventually) should understand how the underlying problem solver works
- This semester, just focus on writing declarative programs; no need to worry about the underlying solver yet!