Lecture 27: Theory of Computation

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Announcements

Roadmap

Introduction

Functions

Data

Mutability

Objects

Interpretation

Paradigms

Applications

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- This week (Applications), the goals are:

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  - To go beyond CS 61A and see examples of what comes next
  - To wrap up CS 61A!

Theoretical Computer Science

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- If today is interesting, consider CS 170 and CS 172

Computability Theory

What can computers do?

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  - It *must* either halt or not halt, there exists no third option

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3. *Conclude* that our assumption must be false
  - `very_bad` is valid Python, there is nothing wrong there
  - So it *must* be the case that our assumption is wrong
  - Therefore, there is no way to write `halts`, and the halting problem must be undecidable

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- In a reduction, we find a way to solve the halting problem using the solution to another problem
  - "If I can solve this problem, then I can also solve the halting problem" implies:
    - "I can't solve this problem, because I can't solve the halting problem."

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- So this successfully solves the halting problem!
    - “I can’t solve `computes_same`, because I can’t solve the halting problem.”

Complexity Theory

What can computers do efficiently?

Complexity

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linear runtime
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Orders of Growth

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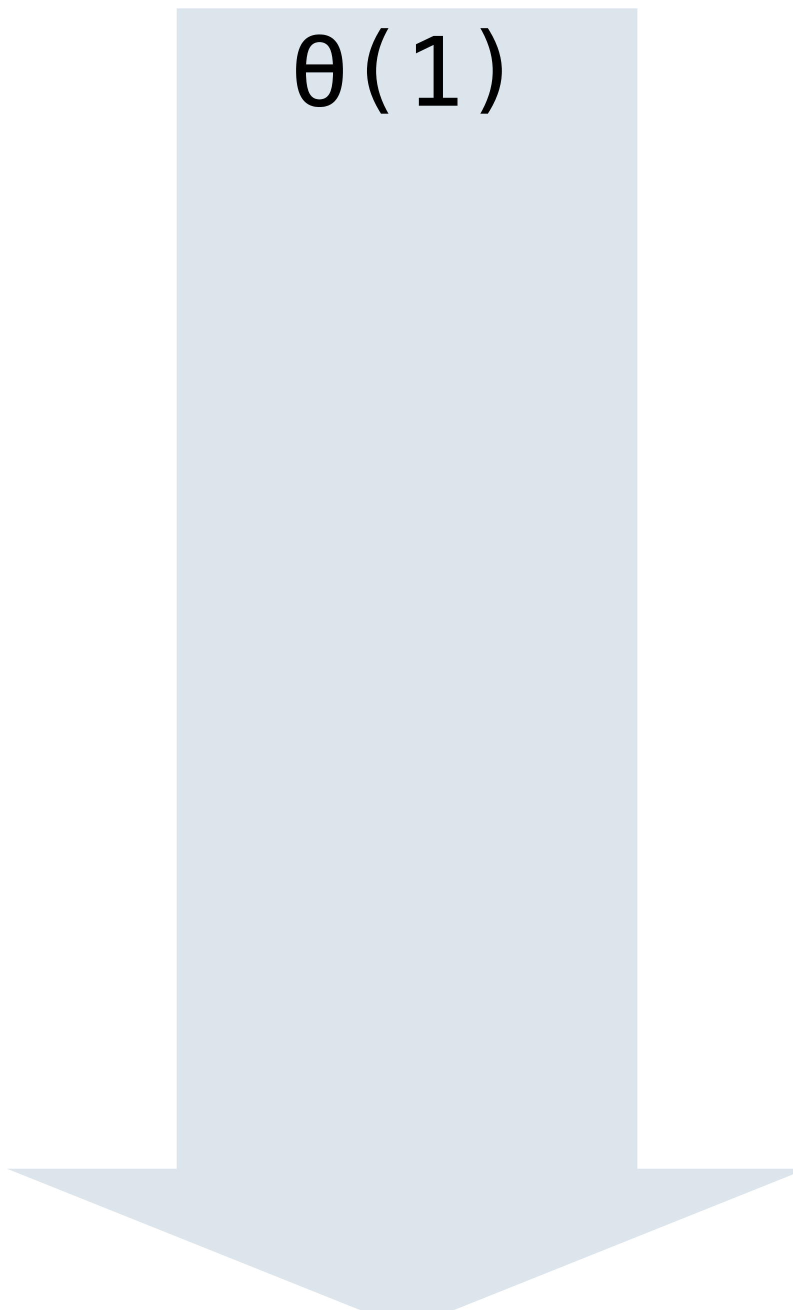

Orders of Growth

$\theta(1)$

$\theta(\log n)$

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Orders of Growth

iterative
Fibonacci

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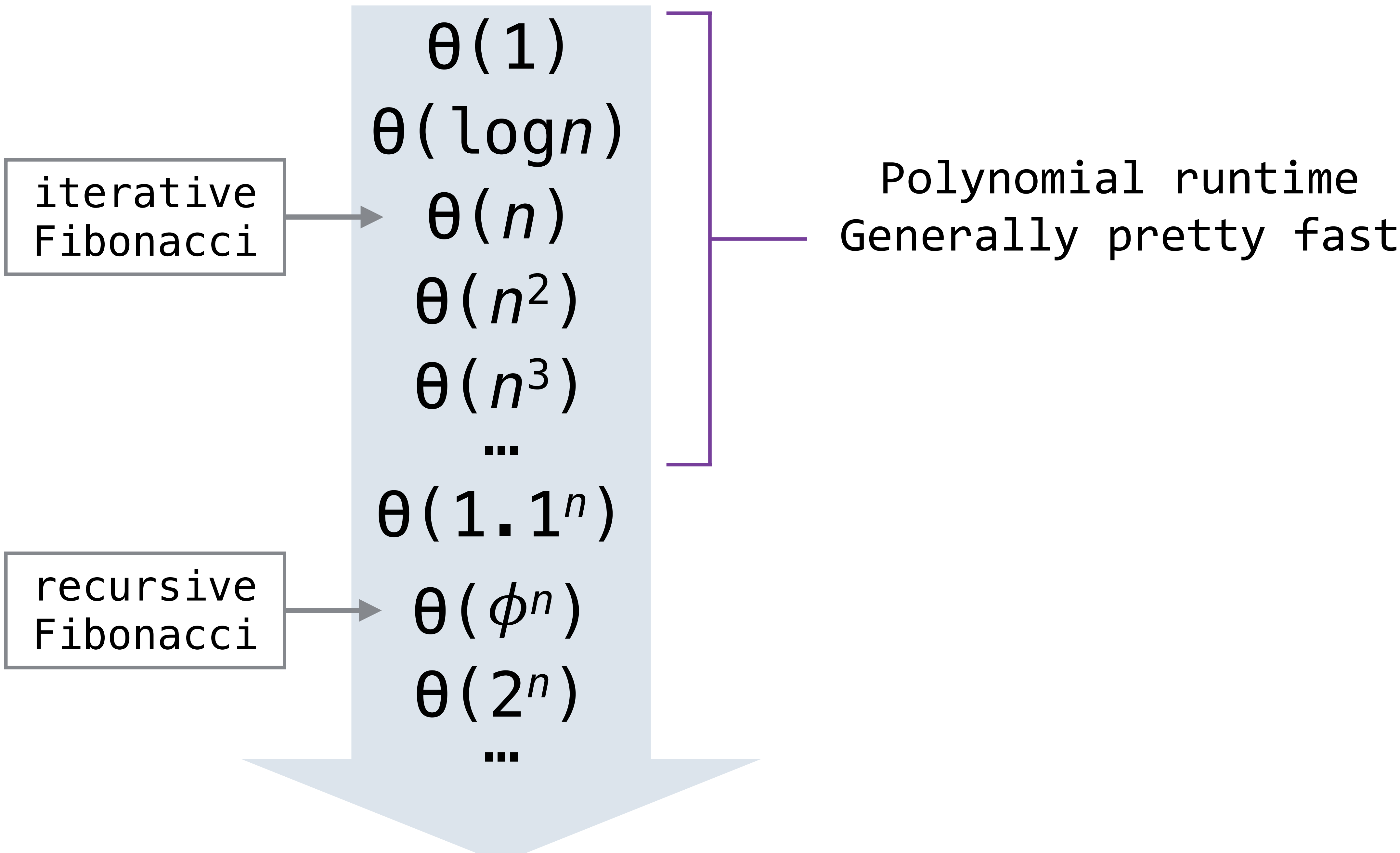
Polynomial runtime

Orders of Growth



Orders of Growth

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Polynomial runtime
Generally pretty fast
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Intractable

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- Ignoring the smaller differences allows us to develop more rigorous theory involving *complexity classes*

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  - If I tell you: “The n^{th} Fibonacci number is k”
  - Can you verify that this is correct in polynomial time?
- In this example, the answer is yes, because you can just run the iterative solution to check, so Fibonacci is also in NP

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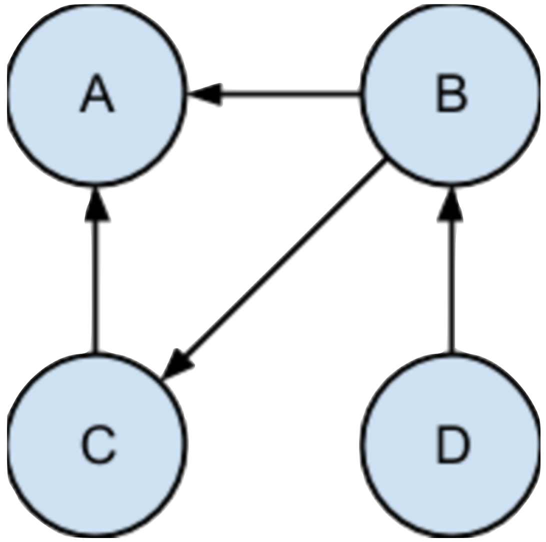

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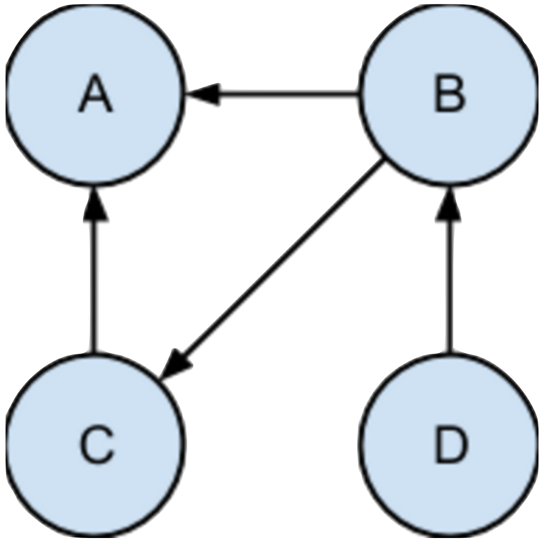

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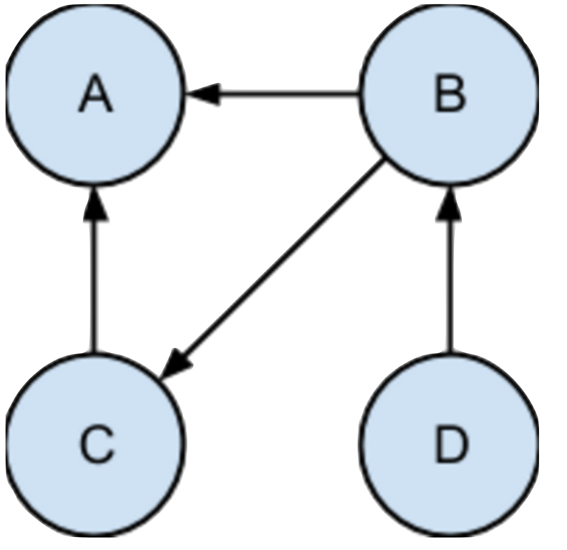

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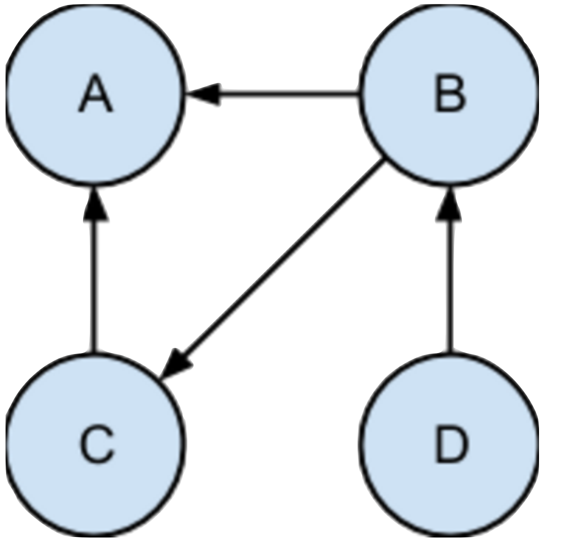

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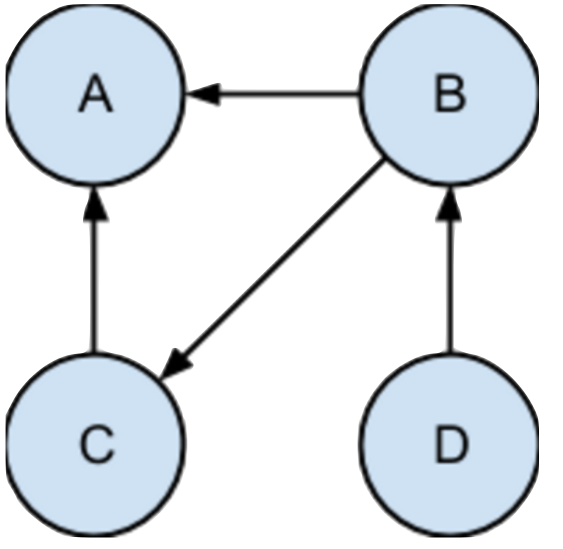

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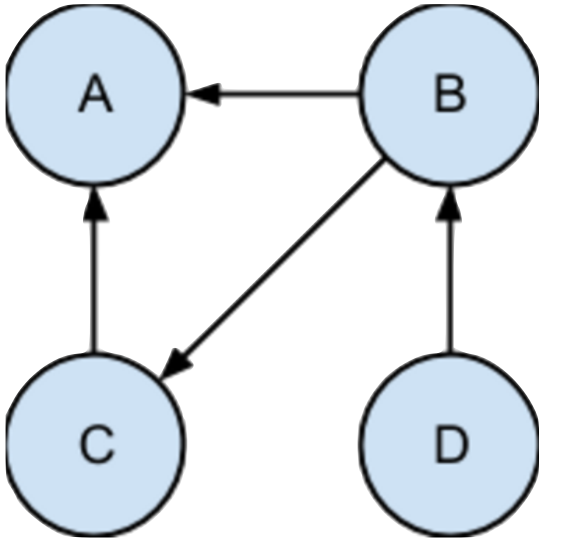

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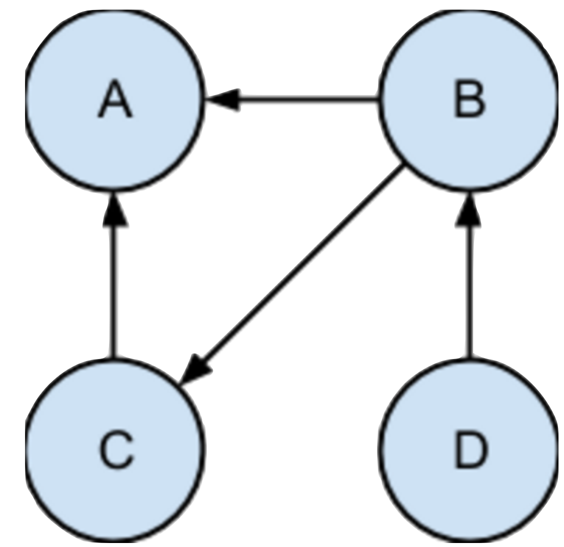

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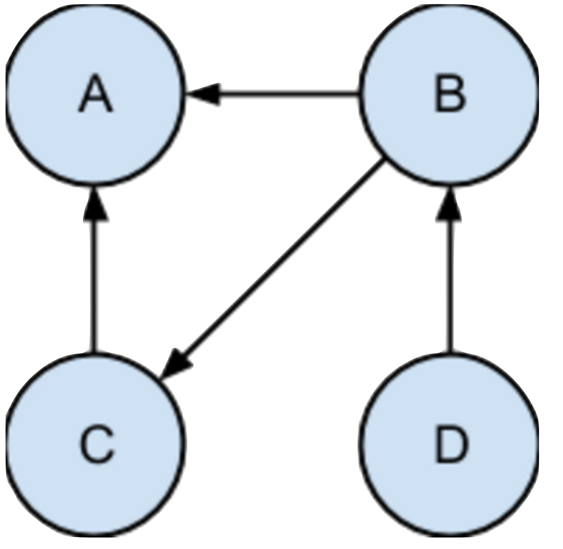

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- If I just proved that P = NP, how do I take over the world?

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- CS 170 and CS 172 go into more detail on this material