CS 61A
Summer 2017

Midterm Review

July 12, 2017

Instructions

Form a small group. Start on problem 1.1. Check off with a staff member or discuss your *solution process* with a nearby group when you think everyone in your group understands *how to solve* problem 1.1. Then repeat for 1.2, 1.3, . . .

You may not move to the next problem until you check off or discuss with another group and *everyone understands why the solution is what it is*. You may use any course resources at your disposal: the purpose of this review session is to have everyone learning together as a group.

1 Functions

1.1 Explain the difference between the following:

(a)
```
>>> def square(x):
...     return x * x
```

Defines a function called `square`.

(b)
```
>>> square(4)
```

Calls the function, `square`.

(c)
```
>>> square
```

Evaluates to the function that squares its input number.

1.2 What would Python display?

(a)
```
(lambda x: x(x))(lambda y: 4)
```

4

(b)
```
(lambda x, y: y(x))(mul, lambda a: a(3, 5))
```

```
15
```

1.3 Implement `make_alternator`.

```
def make_alternator(f, g):
    """
    >>> a = make_alternator(lambda x: x * x, lambda x: x + 4)
    >>> a(5)
    1
    6
    9
    8
    25
    """


    def alternator(n):
        i = 1
        while i <= n:
            if i % 2 == 1:
                print(f(i))
            else:
                print(g(i))
            i += 1
    return alternator
```

2 Lists & Tree Recursion

Mutative (*destructive*) operations change the state of a list by adding, removing, or otherwise modifying the list itself.

- `lst.append(element)`
- `lst.extend(lst)`
- `lst.pop(index)`
- `lst += lst` (**not** `lst = lst + lst`)
- `lst[i] = x`
- `lst[i:j] = lst`

Non-mutative (*non-destructive*) operations include the following.

- `lst + lst`
- `lst * n`
- `lst[i:j]`
- `list(lst)`

Recall: To execute assignment statements,

- Evaluate all expressions to the right of the = sign
- Bind all names to the left of the = to those resulting values

The **Golden Rule of Equals** describes how this rule behaves with composite values. *Composite values*, such as functions and lists, are connected by a pointer. When an expression evaluates to a composite value, we are returned the pointer to that value, rather than the value itself.

In an environment diagram, we can summarize this rule with,

Copy *exactly* what is in the box!

2.1 Write a list comprehension that accomplishes each of the following tasks.

(a) Square all the elements of a given list, `lst`.

```
[x ** 2 for x in lst]
```

(b) Compute the dot product of two lists `lst1` and `lst2`. *Hint*: The dot product is defined as $\text{lst1}[0] \cdot \text{lst2}[0] + \text{lst1}[1] \cdot \text{lst2}[1] + \ldots + \text{lst1}[n] \cdot \text{lst2}[n]$. The Python **`zip`** function may be useful here.

```
sum([x * y for x, y in zip(lst1, lst2)])
```

(c) Return a list of lists such that `lol = [[0], [0, 1], [0, 1, 2], [0, 1, 2, 3], [0, 1, 2, 3, 4]]`.

```
[[x for x in range(y)] for y in range(1, 6)]
```

(d) Return the same list as above, except now excluding every instance of the number 2: `lold = [[0], [0, 1], [0, 1], [0, 1, 3], [0, 1, 3, 4]])`.

```
[[x for x in range(y) if x != 2] for y in range(1, 6)]
```

2.2 Draw the environment diagram that results from running the following code.

```
pom = [16, 15, 13]
pompom = pom * 2
pompom.append(pom[:])
pom.extend(pompom)
```

https://goo.gl/ZU1V7h

2.3 Draw the environment diagram that results from running the following code.

```
bless, up = 3, 5
another = [1, 2, 3, 4]
one = another[1:]

another[bless] = up
another.append(one.remove(2))
another[another[0]] = one
one[another[0]] = another[1]
one = one + [another.pop(3)]
another[1] = one[1][1][0]
one.append([one.pop(1)])
```

https://goo.gl/FyMmbJ

2.4
```
def jerry(jerry):
    def jerome(alex):
        alex.append(jerry[1:])
        return alex
    return jerome

ben = ['nice', ['ice']]
jerome = jerry(ben)
alex = jerome(['cream'])
ben[1].append(alex)
ben[1][1][1] = ben
print(ben)
```

https://goo.gl/uhSClr

2.5 Implement `subset_sum`, which takes in a list of integers and a number k and returns whether there is a subset of the list that adds up to k? *Hint*: Use the **`in`** operator to determine if an element belongs to a list.

```
>>> 3 in [1, 2, 3]
True
>>> 4 in [1, 2, 3]
False

def subset_sum(seq, k):
    """
    >>> subset_sum([2, 4, 7, 3], 5)         # 2 + 3 = 5
    True
    >>> subset_sum([1, 9, 5, 7, 3], 2)
    False
    """


    if len(seq) == 0:
        return False
    elif k in seq:
        return True
    else:
        return subset_sum(seq[1:], k - seq[0]) or subset_sum(seq[1:], k)
```

2.6 Implement `subsets`, which takes in a list of values and an integer `n` and returns all subsets of the list of size exactly `n` in any order.

```
def subsets(lst, n):
    """
    >>> three_subsets = subsets(list(range(5)), 3)
    >>> for subset in sorted(three_subsets):
    ...     print(subset)
    [0, 1, 2]
    [0, 1, 3]
    [0, 1, 4]
    [0, 2, 3]
    [0, 2, 4]
    [0, 3, 4]
    [1, 2, 3]
    [1, 2, 4]
    [1, 3, 4]
    [2, 3, 4]
    """


    if n == 0:
        return [[]]
    if len(lst) == n:
        return [lst]
    with_first = [[lst[0]] + x for x in subsets(lst[1:], n - 1)]
    without_first = subsets(lst[1:], n)
    return with_first + without_first
```

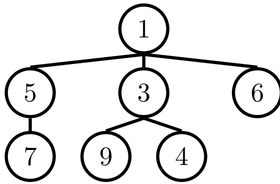
3 Recursion on Trees

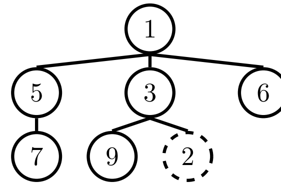
3.1 A **min-heap** is a tree with the special property that every node's value is less than or equal to the values of all of its branches.

```
def tree(root, branches=[]):
    return [root] + list(branches)

def root(tree):
    return tree[0]

def branches(tree):
    return tree[1:]
```





Implement `is_min_heap` which takes in a tree and returns whether the tree satisfies the min-heap property or not.

```
def is_min_heap(t):
    for b in branches(t):
        if root(t) > root(b) or not is_min_heap(b):
            return False
    return True
```

4 Growth

4.1 Give a tight asymptotic runtime bound for the following functions in $\Theta(\cdot)$ notation, or "Infinite" if the program does not terminate.

(a)
```
def one(n):
    while n > 0:
        n = n // 2
```

$\Theta(\log n)$

(b)
```
def two(n):
    for i in range(n):
        for j in range(i):
            print(str(i), str(j))
```

$\Theta(n^2)$

(c)
```
def three(n):
    i = 1
    while i <= n:
        for j in range(i):
            print(j)
        i *= 2
```

$\Theta(n)$

For each of the questions below, give a $\Theta(\cdot)$ bound on the asymptotic runtime.

4.2
```
def strange_add(n):
    if n == 0:
        return 1
    else:
        return strange_add(n - 1) + strange_add(n - 1)
```

$\Theta(2^n)$. To see this, try drawing out the call tree. Each level will create two new calls to `strange_add`, and there are n levels. Therefore, 2^n calls.

4.3
```
def belgian_waffle(n):
    i = 0
    total = 0
    while i < n:
        for j in range(n ** 2):
            total += 1
        i += 1
    return total
```

$\Theta(n^3)$. Inner loop runs n^2 times, and the outer loop runs n times. To get the total, multiply those together.

4.4
```
def flip(n):
    return -n

def pancake(n):
    if n < 1:
        return 1
    return flip(n) + pancake(n - 1) + pancake(n - 1)
```

$\Theta(2^n)$. `flip` contributes only a constant amount of time to each call so this is just like `strange_add`.

4.5
```
def toast(n):
    i = 0
    j = 0
    stack = 0
    while i < n:
        stack += pancake(n)
        i += 1
    while j < n:
        stack += 1
        j += 1
    return stack
```

$O(n \cdot 2^n)$. There are two loops: the first runs n times with each iteration taking $\Theta(2^n)$ time for a total of $\Theta(n \cdot 2^n)$. The second loop runs n times with each iteration taking constant time for a total of $\Theta(n)$. When computing the order of growth, however, we focus on the dominating term – in this case, $\Theta(n \cdot 2^n + n) \in \Theta(n \cdot 2^n)$.