

## 1 Higher-Order Functions

A **higher order function** (HOF) is a function that manipulates other functions by taking in functions as arguments, returning a function, or both.

### Functions as Arguments

One way a higher order function can manipulate other functions is by taking functions as input (an argument). Consider this higher order function called `negate`.

`negate` takes in a function `f` and a number `x`. It doesn't care what exactly `f` does, as long as `f` is a function, takes in a number and returns a number. Its job is simple: call `f` on `x` and return the negation of that value.

```
def negate(f, x):  
    return -f(x)
```

### Questions

- 1.1 Here are some possible functions that can be passed through as `f`.

```
def square(n):  
    return n * n
```

```
def double(n):  
    return 2 * n
```

What will the following Python statements display?

```
>>> negate(square, 5)
```

```
>>> negate(double, -19)
```

```
>>> negate(double, negate(square, -4))
```

### Functions as Return Values

Often, we will need to write a function that returns another function. One way to do this is to define a function inside of a function:

The return value of `outer` is the function `inner`. This is a case of a function returning a function. In this example, `inner` is defined inside of `outer`. Although this is a common pattern, we can also define `inner` outside of `outer` and still use the same return statement. However, note that in this second example (unlike the first example), `inner` doesn't have access to variables defined within the `outer` function, like `x`.

```
def outer(x):  
    def inner(y):  
        ...  
    return inner
```

```
def inner(y):  
    ...  
def outer(x):  
    return inner
```

## Questions

- 1.1 Use this definition of `outer` to fill in what Python would display when the following lines are evaluated.

```
def outer(n):
    def inner(m):
        return n - m
    return inner
```

```
>>> outer(61)
```

```
>>> f = outer(10)
>>> f(4)
```

```
>>> outer(5)(4)
```

- 1.2 Write a function `and_add` that takes a one-argument function `f` and a number `n` as arguments. It should return a function that takes one argument, and does the same thing as the function `f`, except also adds `n` to the result.

```
def and_add(f, n):
    """Return a new function. This new function takes an
    argument x and returns f(x) + n.
```

```
>>> def square(x):
...     return x * x
>>> new_square = and_add(square, 3)
>>> new_square(4)    # 4 * 4 + 3
19
"""
```

# Environment Diagrams

- 1.1 Draw the environment diagram for the following code:

```
def curry2(h):  
    def f(x):  
        def g(y):  
            return h(x, y)  
        return g  
    return f
```

```
make_adder = curry2(lambda x, y: x + y)  
add_three = make_adder(3)  
five = add_three(2)
```

## 4 Higher-Order Functions

1.2 Draw the environment diagram that results from running the following code:

```
n = 7
```

```
def f(x):  
    n = 8  
    return x + 1
```

```
def g(x):  
    n = 9  
    def h():  
        return x + 1  
    return h
```

```
def f(f, x):  
    return f(x + n)
```

```
f = f(g, n)  
g = (lambda y: y())(f)
```

- 1.3 *The following question is extremely difficult. Something like this would not appear on the exam. Nonetheless, it's a fun problem to try.*

Draw the environment diagram for the following code: (Note that using the + operator with two strings results in the second string being appended to the first. For example "C" + "S" concatenates the two strings into one string "CS")

```
y = "y"
h = y
def y(y):
    h = "h"
    if y == h:
        return y + "i"
    y = lambda y: y(h)
    return lambda h: y(h)
y = y(y)(y)
```