Calculator

Announcements

Programming Languages

Programming Languages

A computer typically executes programs written in many different programming languages

Machine languages: statements are interpreted by the hardware itself

- A fixed set of instructions invoke operations implemented by the circuitry of the central processing unit (CPU)
- Operations refer to specific hardware memory addresses; no abstraction mechanisms

High-level languages: statements & expressions are interpreted by another program or compiled (translated) into another language

- Provide means of abstraction such as naming, function definition, and objects
- Abstract away system details to be independent of hardware and operating system

Python 3

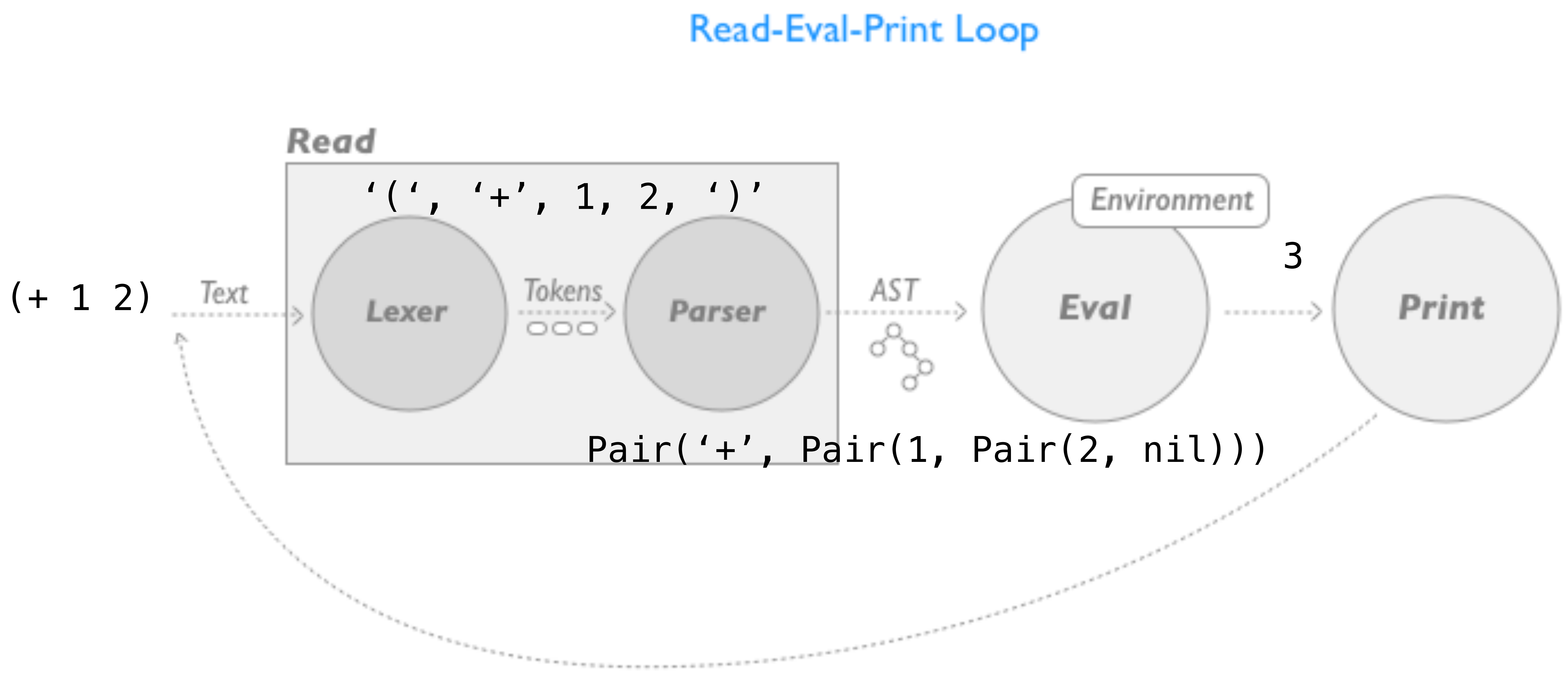
```
def square(x):
    return x * x
```

```
from dis import dis
dis(square)
```

Python 3 Byte Code

```
LOAD_FAST                0 (x)
LOAD_FAST                0 (x)
BINARY_MULTIPLY
RETURN_VALUE
```

Interpreters



Reading

Reading Scheme Lists

A Scheme list is written as elements in parentheses:

```
(<element_0> <element_1> ... <element_n>)
```

A Scheme list

Each <element> can be a combination or primitive

```
(+ (* 3 (+ (* 2 4) (+ 3 5))) (+ (- 10 7) 6))
```

The task of parsing a language involves coercing a string representation of an expression to the expression itself

(Demo)

http://composingprograms.com/examples/scalc/scheme_reader.py.html

Parsing

A Parser takes text and returns an expression


```
'(+ 1'                '(', '+', 1                         Pair('+', Pair(1, ...))
'   (- 23)'           '(', '-', 23, ')'                          printed as
'   (* 4 5.6))'       '(', '*', 4, 5.6, ')', ')'            (+ 1 (- 23) (* 4 5.6))
```

- Iterative process
- Checks for malformed tokens
- Determines types of tokens
- Processes one line at a time

- Tree-recursive process
- Balances parentheses
- Returns tree structure
- Processes multiple lines

Syntactic Analysis

Syntactic analysis identifies the hierarchical structure of an expression, which may be nested

Each call to scheme_read consumes the input tokens for exactly one expression

```
'(', '+', 1, '(', '-', 23, ')', '(', '*', 4, 5.6, ')', ')'
```

Base case: symbols and numbers

Recursive call: scheme_read sub-expressions and combine them

(Demo)

Scheme-Syntax Calculator

(Demo)

Calculator Syntax

The Calculator language has primitive expressions and call expressions. (That's it!)

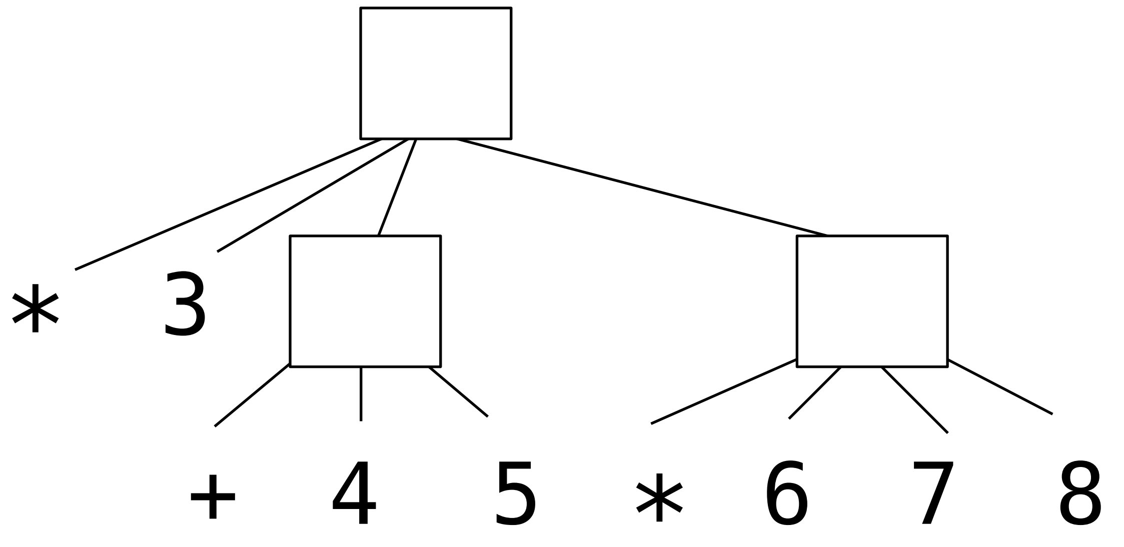
A primitive expression is a number: 2 -4 5.6

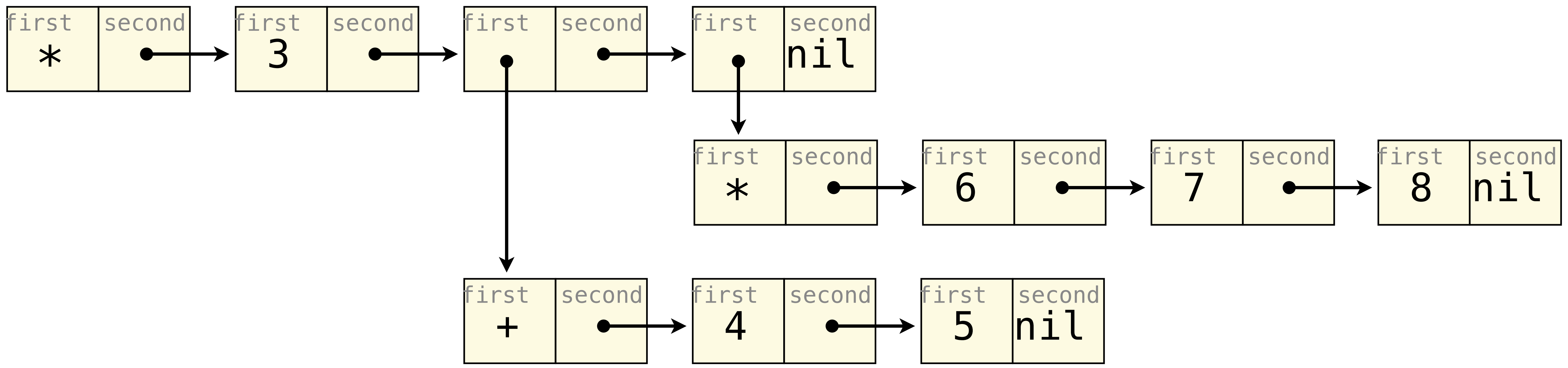
A call expression is a combination that begins with an operator (+, -, *, /) followed by 0 or more expressions: (+ 1 2 3) (/ 3 (+ 4 5))

Expressions are represented as Scheme lists (Pair instances) that encode tree structures.

Expression	**Expression Tree**	**Representation as Pairs**

```
(* 3
   (+ 4 5)
   (* 6 7 8))
```





Calculator Semantics

The value of a calculator expression is defined recursively.

Primitive: A number evaluates to itself.

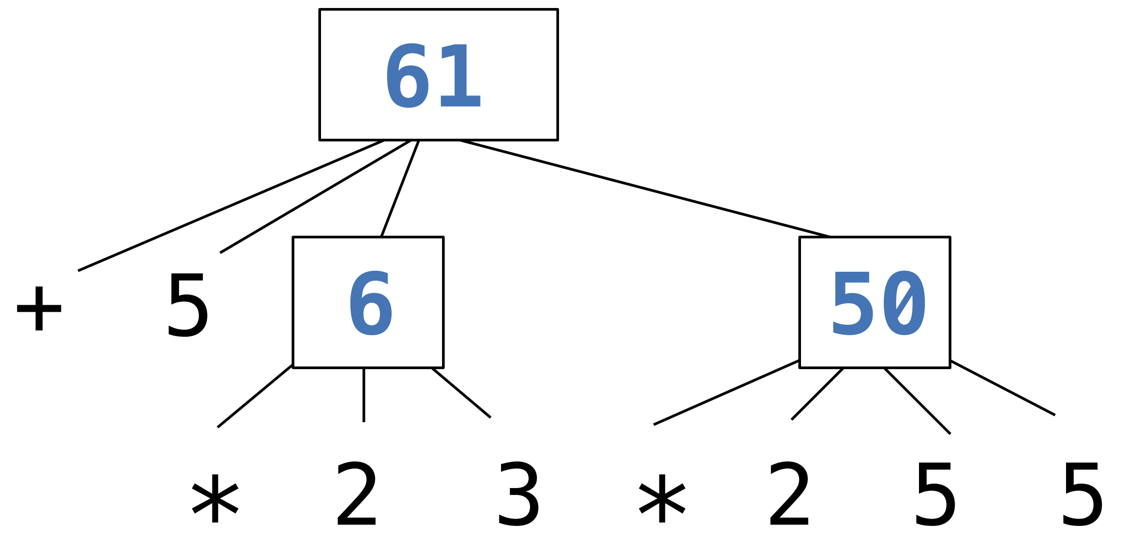
Call: A call expression evaluates to its argument values combined by an operator.

- **+**: Sum of the arguments
- *****: Product of the arguments
- **-**: If one argument, negate it. If more than one, subtract the rest from the first.
- **/**: If one argument, invert it. If more than one, divide the rest from the first.

Expression

```
(+ 5
   (* 2 3)
   (* 2 5 5))
```

Expression Tree



Evaluation

The Eval Function

The eval function computes the value of an expression, which is always a number

It is a generic function that dispatches on the type of the expression (primitive or call)

Implementation

```
def calc_eval(exp):
    if isinstance(exp, (int, float)):
        return exp
    elif isinstance(exp, Pair):
        arguments = exp.rest.map(calc_eval)
        return calc_apply(exp.first, arguments)
    else:
        raise TypeError
```

Recursive call returns a number for each operand

'+', '-', '*', '/'

A Scheme list of numbers

Language Semantics

A number evaluates...

to itself

A call expression evaluates...

to its argument values

combined by an operator

Applying Built-in Operators

The apply function applies some operation to a (Scheme) list of argument values

In calculator, all operations are named by built-in operators: +, -, *, /

Implementation

```
def calc_apply(operator, args):
    if operator == '+':
        return reduce(add, args, 0)
    elif operator == '-':
        ...
    elif operator == '*':
        ...
    elif operator == '/':
        ...
    else:
        raise TypeError
```

Language Semantics

+:

Sum of the arguments

-:

...

...

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Interactive Interpreters

Read-Eval-Print Loop

The user interface for many programming languages is an interactive interpreter

1. Print a prompt
2. **Read** text input from the user
3. Parse the text input into an expression
4. **Evaluate** the expression
5. If any errors occur, report those errors, otherwise
6. **Print** the value of the expression and repeat

(Demo)

Raising Exceptions

Exceptions are raised within lexical analysis, syntactic analysis, eval, and apply

Example exceptions

- **Lexical analysis:** The token 2.3.4 raises ValueError("invalid numeral")
- **Syntactic analysis:** An extra) raises SyntaxError("unexpected token")
- **Eval:** An empty combination raises TypeError("() is not a number or call expression")
- **Apply:** No arguments to - raises TypeError("- requires at least 1 argument")

(Demo)

Handling Exceptions

An interactive interpreter prints information about each error

A well-designed interactive interpreter should not halt completely on an error, so that the user has an opportunity to try again in the current environment

(Demo)

Interpreting Scheme

The Structure of an Interpreter

Eval

Base cases:
- Primitive values (numbers)
- Look up values bound to symbols

Recursive calls:
- Eval(operator, operands) of call expressions
- Apply(procedure, arguments)
- Eval(sub-expressions) of special forms

Requires an environment for symbol lookup

Apply

Base cases:
- Built-in primitive procedures

Recursive calls:
- Eval(body) of user-defined procedures

Creates a new environment each time a user-defined procedure is applied

Special Forms

Scheme Evaluation

The scheme_eval function choose behavior based on expression form:

- Symbols are looked up in the current environment
- Self-evaluating expressions are returned as values
- All other legal expressions are represented as Scheme lists, called combinations

```
(if <predicate> <consequent> <alternative>)

(lambda (<formal-parameters>) <body>)

(define <name> <expression>)

(<operator> <operand 0> ... <operand k>)
```

Special forms are identified by the first list element

Any combination that is not a known special form is a call expression

```
(define (demo s) (if (null? s) '(3) (cons (car s) (demo (cdr s))) ))

(demo (list 1 2))
```

Logical Forms

Logical Special Forms

Logical forms may only evaluate some sub-expressions

- **If** expression: (**if** <predicate> <consequent> <alternative>)
- **And** and **or**: (**and** <e1> ... <en>), (**or** <e1> ... <en>)
- **Cond** expression: (**cond** (<p1> <e1>) ... (<pn> <en>) (else <e>))

The value of an if expression is the value of a sub-expression:

- Evaluate the predicate
- Choose a sub-expression: <consequent> or <alternative>
- Evaluate that sub-expression to get the value of the whole expression

do_if_form

(Demo)

Quotation

Quotation

The quote special form evaluates to the quoted expression, which is not evaluated

(quote <expression>)   (quote (+ 1 2))   evaluates to the three-element Scheme list   (+ 1 2)

The <expression> itself is the value of the whole quote expression

'<expression> is shorthand for (quote <expression>)

(quote (1 2))   is equivalent to   '(1 2)

The scheme_read parser converts shorthand ' to a combination that starts with quote

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Lambda Expressions

Lambda Expressions

Lambda expressions evaluate to user-defined procedures

```
(lambda (<formal-parameters>) <body>)

(lambda (x) (* x x))
```

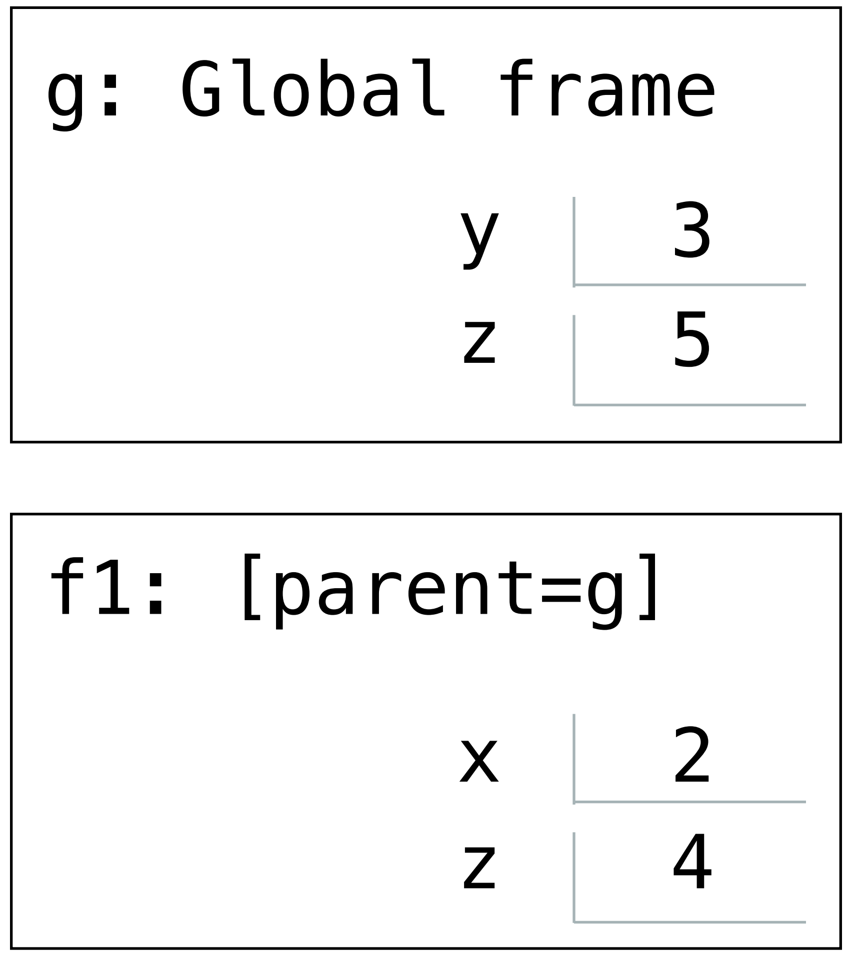
```
class LambdaProcedure:
    def __init__(self, formals, body, env):
        self.formals = formals ........ A scheme list of symbols
        self.body = body .............. A scheme list of expressions
        self.env = env ................ A Frame instance
```

Frames and Environments

A frame represents an environment by having a parent frame

Frames are Python instances with methods **lookup** and **define**

In Project 4, Frames do not hold return values



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Define Expressions

Define Expressions

Define binds a symbol to a value in the first frame of the current environment.

```
(define <name> <expression>)
```

1. Evaluate the <expression>
2. Bind <name> to its value in the current frame

```
(define x (+ 1 2))
```

Procedure definition is shorthand of define with a lambda expression

```
(define (<name> <formal parameters>) <body>)

(define <name> (lambda (<formal parameters>) <body>))
```

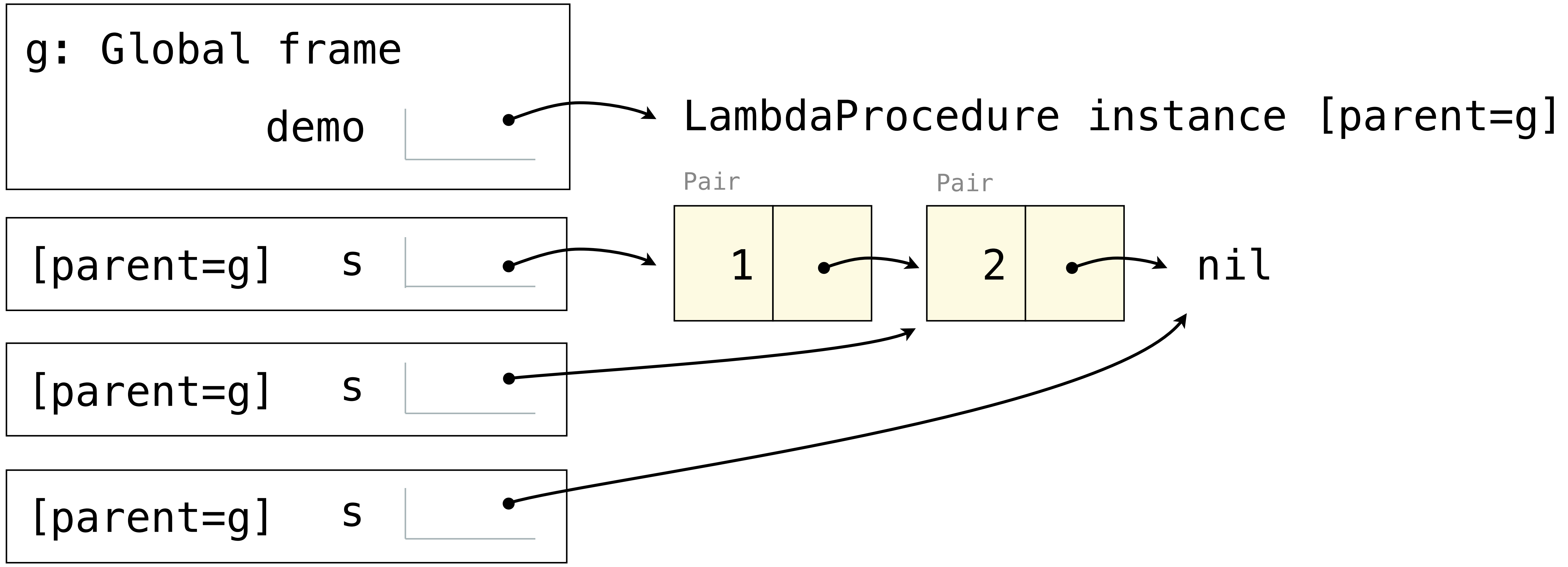
Applying User-Defined Procedures

To apply a user-defined procedure, create a new frame in which formal parameters are bound to argument values, whose parent is the **env** attribute of the procedure

Evaluate the body of the procedure in the environment that starts with this new frame

```
(define (demo s) (if (null? s) '(3) (cons (car s) (demo (cdr s)))))

(demo (list 1 2))
```



Eval/Apply in Lisp 1.5

```
apply[fn;x;a] =
     [atom[fn] → [eq[fn;CAR] → caar[x];
                  eq[fn;CDR] → cdar[x];
                  eq[fn;CONS] → cons[car[x];cadr[x]];
                  eq[fn;ATOM] → atom[car[x]];
                  eq[fn;EQ] → eq[car[x];cadr[x]];
                  T → apply[eval[fn;a];x;a]];
     eq[car[fn];LAMBDA] → eval[caddr[fn];pairlis[cadr[fn];x;a]];
     eq[car[fn];LABEL] → apply[caddr[fn];x;cons[cons[cadr[fn];
                                                caddr[fn]];a]]]
eval[e;a] = [atom[e] → cdr[assoc[e;a]];
     atom[car[e]]→
               [eq[car[e],QUOTE] → cadr[e];
               eq[car[e];COND] → evcon[cdr[e];a];
               T → apply[car[e];evlis[cdr[e];a];a]];
     T → apply[car[e];evlis[cdr[e];a];a]]
```