CS 61A Fall 2016

Structure and Interpretation of Computer Programs

MIDTERM 2 SOLUTIONS

INSTRUCTIONS

- You have 2 hours to complete the exam.
- The exam is closed book, closed notes, closed computer, closed calculator, except one hand-written 8.5" × 11" crib sheet of your own creation and the official CS 61A study guides.
- Mark your answers **on the exam itself**. We will *not* grade answers written on scratch paper.

Last name	
First name	
Student ID number	
CalCentral email (`_@berkeley.edu`)	
TA	
Name of the person to your left	
Name of the person to your right	
All the work on this exam is my own. **(please sign)**	

1. (8 points) Halloween

For each of the expressions in the table below, write the output displayed by the interactive Python interpreter when the expression is evaluated. The output may have multiple lines. If an error occurs, write "Error", but include all output displayed before the error. The `Link` class appears on page 2 of the midterm 2 study guide.

The first two rows have been provided as examples.

Recall: The interactive interpreter displays the value of a successfully evaluated expression, unless it is `None`.

Assume that you have started `python3` and executed the following statements:

```
class Party:
    guests = Link.empty

    def __init__(self, time):
        Party.guests = Link(time+1, Party.guests)

    def attend(self):
        self.guests.rest = Link(self.guests.rest)
        return self.guests

class Costume(Party):
    def __init__(self, bow, tie):
        Party.guests.rest = Link(bow)
        self.ie = Link(self)

    def attend(self):
        print(repr(self.ie))
        Party.attend = lambda self: Party(9).guests

    def __repr__(self):
        print('Nice')
        return 'Costume'
```

Expression	**Interactive Output**
`Link(1, Link.empty)`	Link(1)
`Link(1, Link(2))`	Link(1, Link(2))
`Party(1).guests`	`Link(2)`
`Party(3).attend()`	`Link(4, Link(Link(2)))`
`Costume(5, 6).attend()`	`Nice` `Link(Costume)`
`Party(7).attend()`	`Link(10, Link(8, Link(4, Link(5))))`

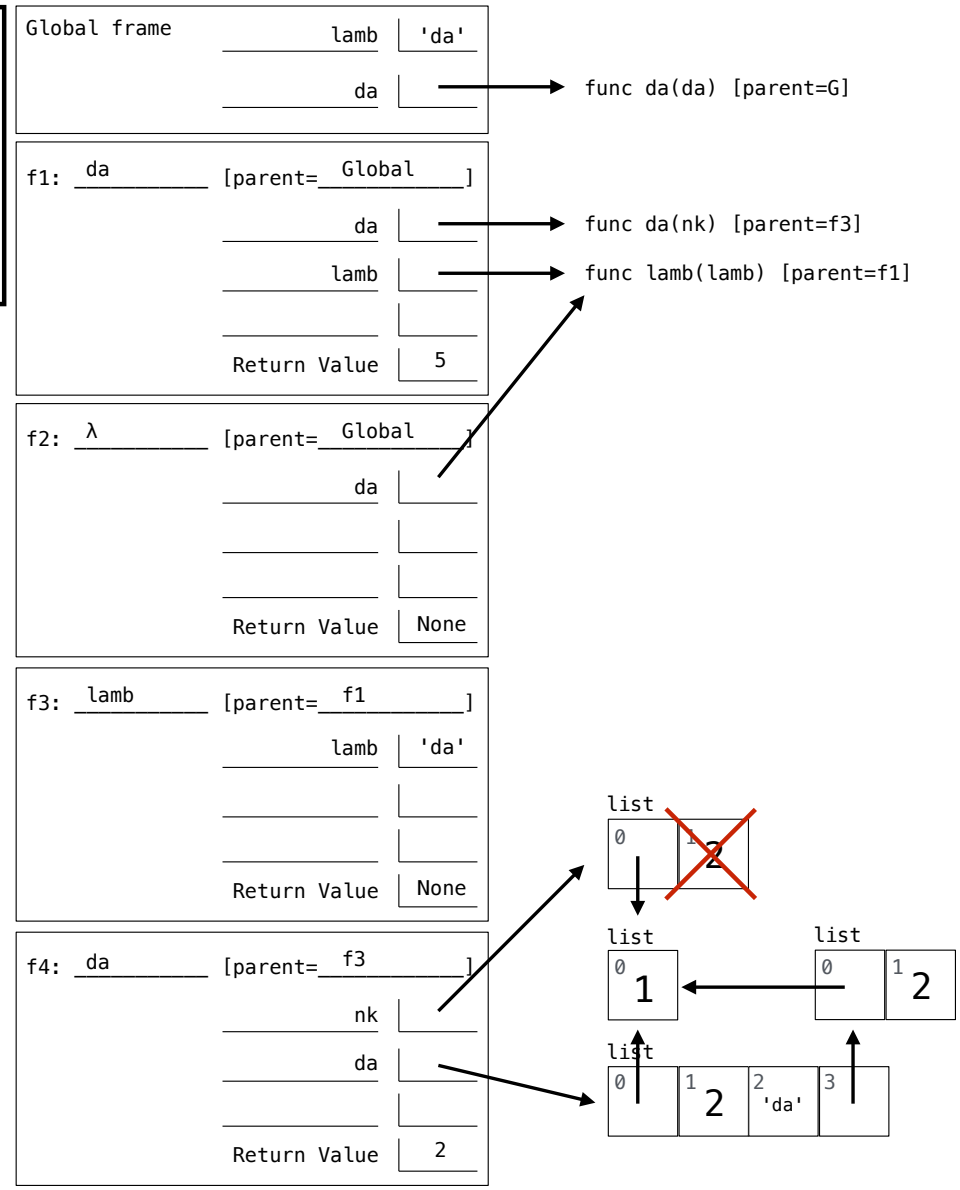
2. (8 points) A List with a Twist

Fill in the environment diagram that results from executing the code below until the entire program is finished, an error occurs, or all frames are filled. *You may not need to use all of the spaces or frames.*

A complete answer will:

- Add all missing names and parent annotations to frames.
- Add all missing values created or referenced during execution.
- Show the return value for each local frame.
- Use box-and-pointer notation for list values. You do not need to write index numbers or the word "list".

```
1  lamb = 'da'
2  def da(da):
3    def lamb(lamb):
4      nonlocal da
5      def da(nk):
6        da = nk + ['da']
7        da.append(nk[0:2])
8        return nk.pop()
9    da(lamb)
10   return da([[1], 2]) + 3
11
12 da(lambda da: da(lamb))
```



3. (6 points) Return Policy

Implement `quota`, which takes a one-argument function `f` and a non-negative integer `limit`. The function it returns has the same behavior as `f`, except that each value is only returned up to `limit` times. After that, the function returns an "Over quota" message instead, and the `limit` is decreased by 1 for future calls.

```
def quota(f, limit):
    """A decorator that limits the number of times a value can be returned.

    >>> square = lambda x: x * x
    >>> square = quota(square, 3)
    >>> square(6)                     # 1st call with return value 36
    36
    >>> [square(5) for x in range(3)] # 3 calls when the limit is 3
    [25, 25, 25]
    >>> square(5)                     # 4th call with return value 25
    'Over quota! Limit is now 2'
    >>> square(-6)                    # 2nd call with return value 36
    36
    >>> square(-6)                    # 3rd call when the limit is 2
    'Over quota! Limit is now 1'
    >>> square(7)                     # 1st call when the limit is 1
    49
    >>> square(5)                     # 5th call with return value 25
    'Over quota! Limit is now 0'
    """

    values = []

    def limited(n):

        nonlocal limit


        y = f(n)


        count = len([x for x in values if x == y])

        values.append(y)


        if count < limit:


            return y


        limit = limit - 1


        return 'Over quota! Limit is now ' + str(limit)

    return limited
```

4. (6 points) A Classy Election

Implement the `VotingMachine` and `Ballot` classes based on the doctest below. The voting machine must determine which choice has the most votes (the `winner`) and detect if a ballot is used more than once. In case of a tie, the `winner` among choices with maximal votes is the one that most recently received a vote. `Ballot.vote` takes a string, and a `VotingMachine` must handle an arbitrary number of choices.

```
class VotingMachine:
    """A machine that creates and records ballots.

    >>> machine = VotingMachine(4)
    >>> a, b, c, d = machine.ballots
    >>> d.vote('Bruin')
    'Bruin is winning'
    >>> b.vote('Bruin')
    'Bruin is winning'
    >>> c.vote('Bear')
    'Bear is losing'
    >>> a.vote('Bear')
    'Bear is winning'
    >>> c.vote('Tree')
    'Fraud: multiple votes from the same ballot!'
    >>> machine.winner
    'Bear'
    """
    def __init__(self, k):
        self.ballots =   [Ballot(self) for i in range(k)]
        self.votes = {}

    def record(self, ballot, choice):
        if ballot.used:
            return 'Fraud: multiple votes from the same ballot!'


        ballot.used = True


        self.votes[choice] = self.votes.get(choice, 0) + 1


        if max(self.votes.values()) > self.votes[choice]:
            return choice + ' is losing'
        else:

            self.winner = choice
            return choice + ' is winning'
class Ballot:

    used = False

    def __init__(self, machine):
        self.machine = machine
    def vote(self, x):

        return self.machine.record(self, x)
```

5. (6 points) Trick or Tree

Implement `path`, which takes a linked list `s` and a `Tree` instance `t`. It returns whether `s` is a path from the root of `t` to some leaf. The `Tree` and `Link` classes are on page 2 of the midterm 2 study guide.

Restrictions:

- You may not call the built-in `len` function on a linked list or invoke its `__len__` method.
- You may not apply element selection (e.g., `s[2]`) on a linked list or invoke its `__getitem__` method.

```
from tree import *
from link import *

def path(s, t):
    """Return whether Link S is a path from the root to a leaf in Tree T.

    >>> t = Tree(1, [Tree(2), Tree(3, [Tree(4), Tree(5)]), Tree(6)])
    >>> a = Link(1, Link(3, Link(4)))  # A full path
    >>> path(a, t)
    True
    >>> b = Link(1, Link(3))           # A partial path
    >>> path(b, t)
    False
    >>> c = Link(1, Link(2, Link(7)))  # A path and an extra value
    >>> path(c, t)
    False
    >>> d = Link(3, Link(4))           # A path of a branch
    >>> path(d, t)
    False
    """

    if s is Link.empty or t.root != s.first:


        return False


    if t.is_leaf() and s.rest is Link.empty:


        return True


    return any([path(s.rest, b) for b in t.branches])
```

6. (6 points) Left it Right There

(a) (4 pt) Implement `binary`, which takes a list `s` of unique integers. It returns a *binary search tree* containing all of those integers, represented as a `BTree` instance or `BTree.empty`. The values in any path of this tree must appear in the same order as they did in `s`. The `BTree` class is on page 2 of the midterm 2 study guide.

```
from tree import *

def binary(s):
    """Construct a binary search tree from S for which all paths are in order.

    >>> binary([3, 5, 1])
    BTree(3, BTree(1), BTree(5))
    >>> binary([4, 3, 7, 6, 2, 9, 8])
    BTree(4, BTree(3, BTree(2)), BTree(7, BTree(6), BTree(9, BTree(8))))
    """

    assert len(s) == len(set(s)), 'All elements of s should be unique'


    if not s:


        return BTree.empty


    root = s[0]


    left = [x for x in s if x < root]


    right = [x for x in s if x > root]


    return BTree(root, binary(left), binary(right))
```

(b) (1 pt) Circle the Θ expression that describes the **smallest** possible height of the tree returned by `binary(s)` for a list `s` of length n. The height of a tree is the length of the longest path from its root to a leaf.

$\Theta(1)$ $\Theta(\log n)$ $\Theta(n)$ $\Theta(n^2)$ $\Theta(2^n)$ None of these

(c) (1 pt) Circle the Θ expression that describes the **largest** possible height of the tree returned by `binary(s)` for a list `s` of length n. The height of a tree is the length of the longest path from its root to a leaf.

$\Theta(1)$ $\Theta(\log n)$ $\Theta(n)$ $\Theta(n^2)$ $\Theta(2^n)$ None of these

7. (10 points) Summer Camp

(a) (6 pt) Implement `sums`, which takes two positive integers `n` and `k`. It returns a list of lists containing all the ways that a list of `k` positive integers can sum to `n`. Results can appear in any order.

```
def sums(n, k):
    """Return the ways in which K positive integers can sum to N.

    >>> sums(2, 2)
    [[1, 1]]
    >>> sums(2, 3)
    []
    >>> sums(4, 2)
    [[3, 1], [2, 2], [1, 3]]
    >>> sums(5, 3)
    [[3, 1, 1], [2, 2, 1], [1, 3, 1], [2, 1, 2], [1, 2, 2], [1, 1, 3]]
    """
    if k == 1:
        return [[n]]
    y = []
    for x in range(1, n): # range(1, n-k+1) is OK
        y.extend([s + [x] for s in sums(n-x, k-1)])
    return y

    OR

    if n == 0 and k == 0:
        return [[]]
    y = []
    for x in range(1, n+1):
        y.extend([s + [x] for s in sums(n-x, k-1)])
    return y
```

(b) (4 pt) *Why so many lines?* Implement `f` and `g` for this alternative version of the `sums` function.

```
f = lambda x, y: (x and [[x] + z for z in y] + f(x-1, y)) or []

def sums(n, k):
    """Return the ways in which K positive integers can sum to N.

    >>> sums(2, 2)
    [[1, 1]]
    >>> sums(4, 2)
    [[3, 1], [2, 2], [1, 3]]
    >>> sums(5, 3)
    [[3, 1, 1], [2, 2, 1], [2, 1, 2], [1, 3, 1], [1, 2, 2], [1, 1, 3]]
    """
    g = lambda w: (w and f(n, g(w-1))) or [[]]
    return [v for v in g(k) if sum(v) == n]
```