

Searching by "Generate and Test"

Considering the problem of searching a set of data stored in some data structure: "Is $x \in S$?"

We don't have a set S , but know how to recognize what we find it: "Is there an x such that $P(x)$?"

How to enumerate all possible candidates, can use *generate and test*: test all possibilities in turn.

Can be more clever: avoid trying things that won't work.

What if the set of possible candidates is infinite?

General Recursive Algorithm

PATH a sequence of knight moves starting at ROW, COL is all squares that have been hit already and up one square away from ENDROW, ENDCOL. B[i][j] is true if row i and column j have been hit on PATH so far. Return true if it succeeds, else false (with no change to PATH). Return false initially with PATH containing the starting square, and ending square (only) marked in B. */

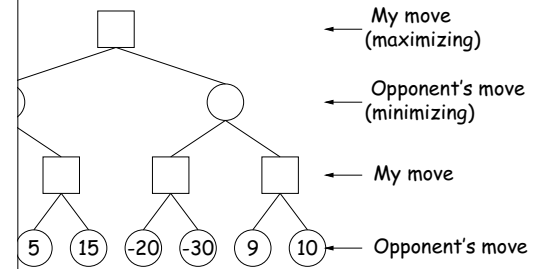
```

bool Path(boolean[][] b, int row, int col,
           int endRow, int endCol, List path) {
    if (row == 64) return isKnightMove(row, col, endRow, endCol);
    for (all possible moves from (row, col)) {
        boolean[] c = {row, col};
        b[c[0]][c[1]] = true; // Mark the square
        if (Path(b, r, c, endRow, endCol, path)) return true;
        b[c[0]][c[1]] = false; // Backtrack out of the move.
    }
    return false;
}

```

Game Trees

Represent the space of possible continuations of the game as a tree. At each position, each edge is a move.



The numbers at the bottom are the values of those final positions. Smaller numbers are of more value to *my opponent*.

I move? What value can I get if my opponent plays as optimally?

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Backtracking searches, game trees (DSIJ, Section 6.5)

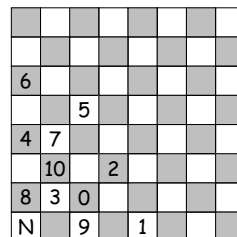
Backtracking Search

Backtracking search is one way to enumerate all possibilities.

Knight's Tour. Find all paths a knight can travel on a chessboard that touches every square exactly once and ends up where it started.

Below, the numbers indicate position numbers (knight starts at N)

(N) is stuck; how to handle this?



Another Kind of Search: Best Move

The problem of finding the *best* move in a two-person game.

Assign a *heuristic value* to each possible move and pick the best (using *static evaluation*). Examples:

- Number of black pieces – number of white pieces in checkers.
- Sum of white piece values – weighted sum of black piece values in chess (Queen=9, Rook=5, etc.)

Number of pieces to strategic areas (center of board).

Be careful of misleading. A move might give us more pieces, but set up a trap for a response from the opponent.

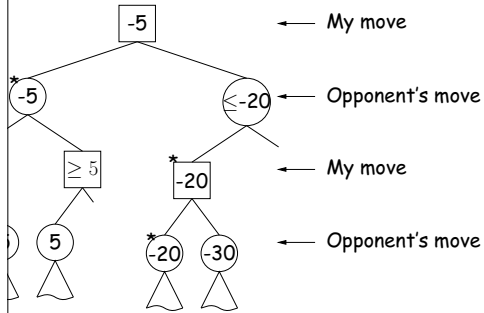
When you make a move, look at *opponent's* possible moves, assume he will pick the best one for him, and use that as the value.

How do you respond to his response?

How do you organize this sensibly?

Alpha-Beta Pruning

See this tree as we search it.



In this position, I know that the opponent will not choose to move to the right (since he already has a -5 move).

In this position, my opponent knows that I will never choose to move to the right (since I already have a -5 move).

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Overall Search Algorithm

For a move whose move it is (maximizing player or minimizing player), we search for a move estimated to be optimal in one direction or the other.

We do not search exhaustively down to a particular depth in the game tree. Instead, we guess values.

and β limits:

The maximizing player does not care about exploring a position further once its value is larger than what the minimizing player knows he can get (β), because the minimizing player will never allow that to come about.

The minimizing player won't explore a position whose value is less than what the maximizing player knows he can get (α).

The maximizing player will find a move with value α .

function `maxPlayerWon(current position, search depth $-\infty$, $+\infty$)`

player:

function `minPlayerWon(current position, search depth $-\infty$, $+\infty$)`

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One-Level Search for Minimizing Player

```

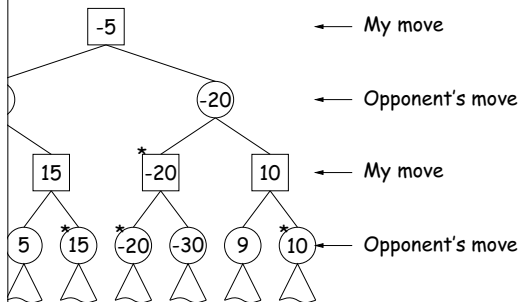
minPlayerWon(Position posn, double alpha, double beta) {
    MaxPlayerWon()
    artificial "Move" with value  $+\infty$ ;
    posn.minPlayerWon()
    artificial "Move" with value  $-\infty$ ;
    bestSoFar = artificial "Move" with value  $+\infty$ ;
    M = a legal move for minimizing player from posn) {
        MinPlayerWon next = posn.makeMove(M);
        setValue(heuristicEstimate(next));
        if (next.value() <= bestSoFar.value()) {
            bestSoFar = next;
            beta = min(beta, next.value());
            if (beta <= alpha) break;
        }
    }
    return bestSoFar;
}
    
```

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Game Trees, Minimax

The space of possible continuations of the game as a tree. At each position, each edge a move.



At each position, we guess the values we guess for the positions (larger means better). Starred nodes would be chosen.

The maximizing player chooses the child (next position) with maximum value; the minimizing player chooses the child with minimum value ("Minimax algorithm")

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Cutting off the Search

If you traverse the game tree to the bottom, you'd be able to find the best move (if it's possible).

But, in practice, it's often not possible near the end of a game.

Typically, game trees tend to be either infinite or impossibly large.

We can limit the search by using a maximum depth, and use a heuristic value computed on the fly (called a static valuation) as the value at that depth.

We can use iterative deepening (kind of breadth-first search), which searches at increasing depths until time is up.

More sophisticated searches are possible, however (take CS188).

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Pseudocode for Searching (One Level)

A naive kind of game-tree search is to assign some heuristic value to each position, looking at just the next possible move:

```

maxPlayerWon(Position posn, double alpha, double beta) {
    MaxPlayerWon()
    artificial "Move" with value  $+\infty$ ;
    posn.minPlayerWon()
    artificial "Move" with value  $-\infty$ ;
    bestSoFar = artificial "Move" with value  $-\infty$ ;
    M = a legal move for maximizing player from posn) {
        MinPlayerWon next = posn.makeMove(M);
        setValue(heuristicEstimate(next));
        if (next.value() >= bestSoFar.value()) {
            bestSoFar = next;
            alpha = max(alpha, next.value());
            if (beta <= alpha) break;
        }
    }
    return bestSoFar;
}
    
```

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Code for Searching (Minimizing Player)

```
Best move for minimizing player from POSN, searching
DEPTH. Any move with value <= ALPHA is also
legal. */
Position posn, int depth, double alpha, double beta) {
    if (depth == 0 || gameOver(posn))
        return simpleFindMin(posn, alpha, beta);
    double bestSoFar = artificial "Move" with value +∞;
    for (Move M = a legal move for minimizing player from posn) {
        Position next = posn.makeMove(M);
        Response response = findMax(next, depth-1, alpha, beta);
        if (response.value() <= bestSoFar.value()) {
            bestSoFar = response;
            next.setValue(response.value());
            beta = min(beta, response.value());
            if (beta <= alpha) break;
        }
    }
    return bestSoFar;
}
```

Code for Searching (Maximizing Player)

```
Best move for maximizing player from POSN, searching
DEPTH. Any move with value >= BETA is also
legal. */
Position posn, int depth, double alpha, double beta) {
    if (depth == 0 || gameOver(posn))
        return simpleFindMax(posn, alpha, beta);
    double bestSoFar = artificial "Move" with value -∞;
    for (Move M = a legal move for maximizing player from posn) {
        Position next = posn.makeMove(M);
        Response response = findMin(next, depth-1, alpha, beta);
        if (response.value() >= bestSoFar.value()) {
            bestSoFar = response;
            next.setValue(response.value());
            alpha = max(alpha, response.value());
            if (beta <= alpha) break;
        }
    }
    return bestSoFar;
}
```