CS61B Lectures #27

rts, heap sort

ay: *DS(IJ)*, Chapter 8; Next topic: Chapter 9.

Sorting by Selection: Heapsort

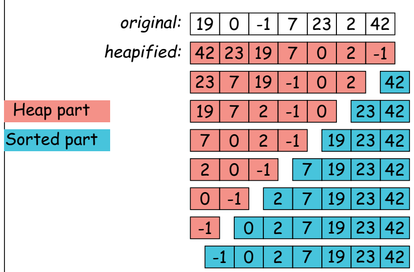
lecting smallest (or largest) element.

ea on a simple list or vector.

eady seen it in action: use heap.

$N)$ algorithm (N remove-first operations).

nove items from end of heap, we can use that area to esult:



ing By Selection: Initial Heapifying

ng heaps before, we created them by insertion in an y heap.

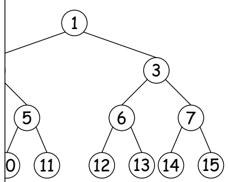
an array of unheaped data to start with, there is a dure:

```
ify(int[] arr) {
 = arr.length;
(int k = N / 2; k > 0; k -= 1) {
hile (2*k <= N) {
  int c = 2k or 2k+1, whichever is <= N
            and indexes larger value in arr;
  swap elements c-1 and k-1 of arr;
```

e procedure for re-inserting an element after the top he heap is removed, repeated $N/2$ times.

of being $\Theta(N \lg N)$, it's just $\Theta(N)$.

Cost of Creating Heap



1 node × 3 steps down

2 nodes × 2 steps down

4 nodes × 1 step down

orst-case cost for a heap with $h+1$ levels is

$h + 2^1 \cdot (h-1) + \ldots + 2^{h-1} \cdot 1$

$+ 2^1 + \ldots + 2^{h-1}) + (2^0 + 2^1 + \ldots + 2^{h-2}) + \ldots + (2^0)$

$-1) + (2^{h-1} - 1) + \ldots + (2^1 - 1)$

$-1-h$

$^h) = \Theta(N)$

he rest of heapsort still takes $\Theta(N \lg N)$, this does not symptotic cost.

Merge Sorting

ata in 2 equal parts; recursively sort halves; merge re-

analysis: $\Theta(N \lg N)$.

ernal sorting:

k data into small enough chunks to fit in memory and

atedly merge into bigger and bigger sequences.

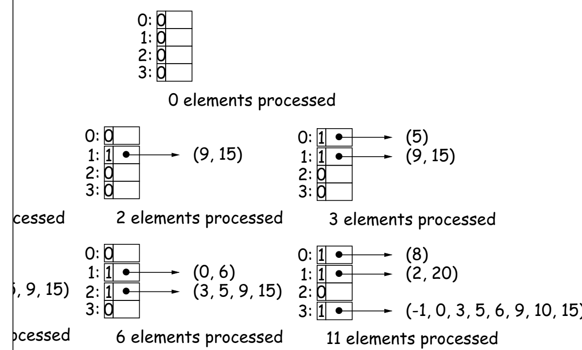
sequences of *arbitrary size* on secondary storage using e:

```
= new Data[K];
, set V[i] to the first data item of sequence i;
re is data left to sort:
k so that V[k] is smallest;
t V[k], and read new value into V[k] (if present).
```


ustration of Internal Merge Sort

ting, can use a binomial comb to orchestrate:

0, 6, 10, -1, 2, 20, 8)



icksort: Speed through Probability

ta into pieces: everything $>$ a *pivot* value at the high equence to be sorted, and everything $\leq$ on the low end.

rsively on the high and low pieces.

top when pieces are "small enough" and do insertion sort thing.

rtion sort has low constant factors. By design, no item of its will move out of its piece [why?], so when pieces nversions is, too.

ose pivot well. E.g.: *median* of first, last and middle uence.

Example of Quicksort

ple, we continue until pieces are size ≤ 4.

xt step are starred. Arrange to move pivot to dividing e.

insertion sort.

18 | -4 | -7 | 12 | -5 | 19 | 15 | 0 | 22 | 29 | 34 | -1*

-1 || 18 | 13 | 12 | 10 | 19 | 15 | 0 | 22 | 29 | 34 | 16*

-1 || 15 | 13 | 12* | 10 | 0 || 16 || 19* | 22 | 29 | 34 | 18

-1 || 10 | 0 || 12 || 15 | 13 || 16 || 18 || 19 || 29 | 34 | 22

ing is "close to" right, so just do insertion sort:

-1 | 0 | 10 | 12 | 13 | 15 | 16 | 18 | 19 | 22 | 29 | 34

Performance of Quicksort

time:

of pivots good, divide data in two each time: $\Theta(N \lg N)$ d constant factor relative to merge or heap sort.

of pivots bad, most items on one side each time: $\Theta(N^2)$.

in best case, so insertion sort better for nearly or- ut sets.

point: randomly shuffling the data before sorting makes *ery* unlikely!

Quick Selection

roblem: for given k, find k^{th} smallest element in data.

hod: sort, select element #k, time $\Theta(N \lg N)$.

constant, can easily do in $\Theta(N)$ time:

h array, keep smallest k items.

$\Theta(N)$ *time* for all k by adapting quicksort:

around some pivot, p, as in quicksort, arrange that pivot dividing line.

hat in the result, pivot is at index m, all elements $\leq$ indicies $\leq m$.

you're done: p is answer.

recursively select k^{th} from left half of sequence.

, recursively select $(k - m - 1)^{\text{th}}$ from right half of

Selection Example

just item #10 in the sorted version of array:

s:

4 | 37 | 4 | 49 | 10 | 40* | 59 | 0 | 13 | 2 | 39 | 11 | 46 | 31

0 to left of pivot 40:

4 | 37 | 4* | 11 | 10 | 39 | 2 | 0 || 40 || 59 | 51 | 49 | 46 | 60

to right of pivot 4:

4 || 37 | 13 | 11 | 10 | 39 | 21 | 31* || 40 || 59 | 51 | 49 | 46 | 60

4

to right of pivot 31:

4 || 21 | 13 | 11 | 10 || 31 || 39 | 37 || 40 || 59 | 51 | 49 | 46 | 60

9

ents; just sort and return #1:

4 || 21 | 13 | 11 | 10 || 31 || 37 | 39 || 40 || 59 | 51 | 49 | 46 | 60

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Selection Performance

rithm, if m roughly in middle each time, cost is

$$
\begin{aligned}
C(N) &= \begin{cases} 1, & \text{if } N = 1, \\ N + C(N/2), & \text{otherwise.} \end{cases} \\
&= N + N/2 + \ldots + 1 \\
&= 2N - 1 \in \Theta(N)
\end{aligned}
$$

case, get $\Theta(N^2)$, as for quicksort.

non-obvious algorithm, can get $\Theta(N)$ worst-case time e CS170).