## 1 Balanced Search Trees

(a) Convert the red-black tree into a 2-4 tree. (Solid nodes are black.)

(b) Insert 13 into the resulting 2-4 tree. Assume that, if a node has 4 keys, we choose to push up the right of the 2 middle keys (so the $2^{\text {nd }}$ key from the right).
(c) Convert the resulting 2-4 tree into a valid left-leaning red-black tree.
(d) Given a $(2,4)$ tree containing N keys, how would you obtain the keys in sorted order in worst case $\mathrm{O}(\mathrm{N})$ time? We don't need actual code-pseudo code or an unambiguous description will do.
(e) If a $(2,4)$ tree has depth $h$ (that is, the leaves are at distance $h$ from the root), what is the maximum number of comparisons done in the corresponding red-black tree to find whether a certain key is present in the tree?

## 2 Tries

(a) List the words encoded by the following trie.

(b) Draw the trie after inserting the words indent, inches, and trie.

## 3 Skip Lists

Draw the resulting skip list after adding the following numbers at the specified random height. Then highlight the links used to find 148.

| Number | 41 | 48 | 59 | 77 | 40 | 131 | 148 | 54 | 139 | 179 | 43 | 128 | 161 | 189 | 170 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Height | 1 | 1 | 1 | 4 | 2 | 2 | 1 | 3 | 1 | 1 | 3 | 2 | 3 | 1 | 2 |

