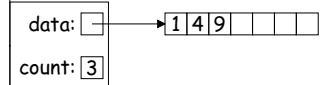


Implementing Lists (I): ArrayLists

Concrete types in Java library for interface List are ArrayList and LinkedList:

As you expect, an ArrayList, A, uses an array to hold data. For example, a list containing the three items 1, 4, and 9 might be implemented like this:



When you add four more items to A, its data array will be full, and the data will have to be replaced with a pointer to a new array that starts with a copy of its previous values.

For best performance, how big should this new array be?

Increasing the size by 1 each time it gets full (or by any constant factor) means the cost of N additions will scale as $\Theta(N^2)$, which makes ArrayList look much worse than LinkedList (which uses an amortized implementation).

The List Interface

Section

represent *indexed sequences* (generalized arrays)

Methods to those of Collection:

Helper tests: indexOf, lastIndexOf.

get(i), listIterator(), sublist(B, E).

add and addAll with additional index to say *where* to add. remove for removal operations. set operation to go with

Iterator<Item> extends Iterator<Item>:

hasNext and hasNextPrevious.

add, remove, and set allow one to iterate through a list, inserting, removing, or changing as you go.

Question: What advantage is there to saying List L is an ArrayList L or LinkedList L?

Amortization: Expanding Vectors (II)

	Resizing Cost	Cumulative Cost	Resizing Cost per Item	Array Size After Insertions
	0	0	0	1
	2	2	1	2
	4	6	2	4
	0	6	1.5	4
	8	14	2.8	8
	0	14	2.33	8
	⋮	⋮	⋮	⋮
	0	14	1.75	8
	16	30	3.33	16
	⋮	⋮	⋮	⋮
	0	30	1.88	16
	⋮	⋮	⋮	⋮
-1	0	$2^{m+2} - 2$	≈ 2	2^{m+1}
	2^{m+2}	$2^{m+3} - 2$	≈ 4	2^{m+2}

Work out (*amortize*) the cost of resizing, we average at 2 time units on each item: "amortized insertion time is 4 to add N elements is $\Theta(N)$, *not* $\Theta(N^2)$."

Amortization: Expanding Vectors

array for expanding sequence, best to *double* the size of the array. Here's why.

When you double its size and moving s elements to the new array, the time is proportional to $2s$.

There is an additional $\Theta(1)$ cost for each addition to the array, so the total cost is actually assigning the new value into the array.

Work out these costs for inserting a sequence of N items, it turns out to be proportional to N , as if each addition took constant time, even though some of the additions actually take time proportional to N all by themselves!

Application to Expanding Arrays

When you add to our array, the cost, c_i , of adding element $\#i$ when the array already has space for it is 1 unit.

When the array does not initially have space when adding items 1, 2, 4, 8, ... then the cost is 2^n for all $n \geq 0$. So,

$c_i \geq 0$ and is not a power of 2; and

$c_i = 2^n$ when i is a power of 2 (copy i items, clear another i items, then add item $\#i$).

For operation $\#2^n$ we're going to need to have saved up at least 2^{n+1} units of potential to cover the expense of expanding the array. And we have this operation and the preceding $2^{n-1} - 1$ operations which to save up this much potential (everything since the last doubling operation).

Let $a_i = 5$ for $i > 0$. Apply $\Phi_{i+1} = \Phi_i + (a_i - c_i)$, and it happens:

4	5	6	7	8	9	10	11	12	13	14	15	16	17	
9	1	1	1	17	1	1	1	1	1	1	1	33	1	<i>Pretending each cost is 5 never underestimates true cumulative time.</i>
5	5	5	5	5	5	5	5	5	5	5	5	5	5	
6	2	6	10	14	2	6	10	14	18	22	26	30	2	

Accounting Amortized Time: Potential Method

For each operation, associate a *potential*, $\Phi_i \geq 0$, to the i^{th} operation. Φ_i keeps track of "saved up" time from cheap operations that can be "spent" on later expensive ones. Start with $\Phi_0 = 0$.

Work out that the cost of the i^{th} operation is actually a_i , the *amortized cost*, defined

$$a_i = c_i + \Phi_{i+1} - \Phi_i,$$

is the real cost of the operation. Or, looking at potential:

$$\Phi_{i+1} = \Phi_i + (a_i - c_i)$$

For operations, we artificially set $a_i > c_i$ so that we can increase the potential (i.e., $\Phi_{i+1} > \Phi_i$).

For expensive operations, we typically have $a_i \ll c_i$ and greatly decrease Φ (but it goes negative—may not be "overdrawn").

Work out all this so that a_i remains as we desired (e.g., $O(1)$ for array), without allowing $\Phi_i < 0$.

Work out that we choose a_i so that Φ_i always stays ahead of c_i .