CS61B Lecture #33

gs: Graph Structures: *DSIJ*, Chapter 12

Why Graphs?

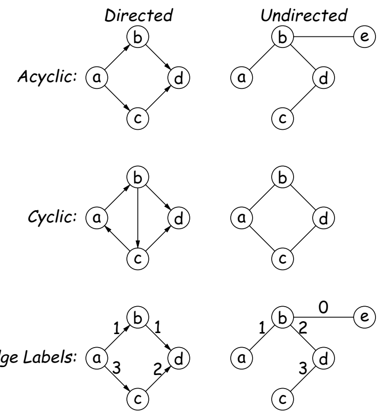
ng non-hierarchically related items

: pipelines, roads, assignment problems
ting processes: flow charts, Markov models
ting partial orderings: PERT charts, makefiles

Some Terminology

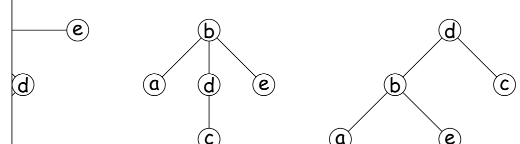
sists of
nodes (aka *vertices*)
dges: pairs of nodes.
h an edge between are *adjacent*.
g on problem, nodes or edges may have *labels* (or *weights*)
node set $V = \{v_0, \ldots\}$, and edge set E.
have an order (first, second), they are *directed edges,* a *directed graph (digraph),* otherwise an *undirected*
cident to their nodes.
ges *exit* one node and *enter* the next.
path without repeated edges leading from a node back lowing arrows if directed).
yclic if it has a cycle, else *acyclic*. Abbreviation: Di-lic Graph—*DAG*.

Some Pictures



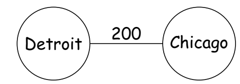
Trees are Graphs

onnected if there is a (possibly directed) path between f nodes.
e node of the pair is *reachable* from the other.
rooted) tree iff connected, and every node but the root one parent.
, acyclic, undirected graph is also called a *free tree.* free to pick the root; e.g.,

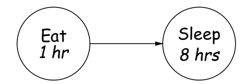


Examples of Use

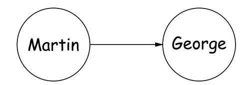
ecting road, with length.



be completed before; Node label = time to complete.

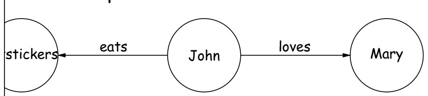


t

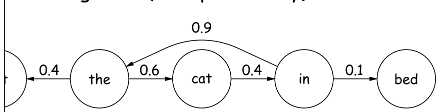


More Examples

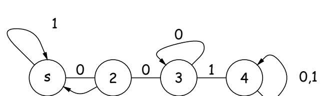
relationship



state might be (with probability)



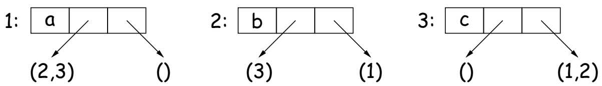
state in state machine, label is triggering input. (Start
state 4 means "there is a substring '001' somewhere in



Representation

to number the nodes, and use the numbers in edges.

resentation: each node contains some kind of list (e.g., array) of its successors (and possibly predecessors).



ollection of all edges. For graph above:

$$\{(1,2),(1,3),(2,3)\}$$

atrix: Represent connection with matrix entry:

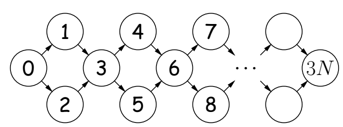
$$\begin{array}{c c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Traversing a Graph

hms on graphs depend on traversing all or some nodes.

se recursion because of cycles.

ic graphs, can get combinatorial explosions:



he root and do recursive traversal down the two edges
node: $\Theta(2^N)$ operations!

try to visit each node constant # of times (e.g., once).

ive Depth-First Traversal of a Graph

ng and combinatorial problems using the "bread-crumb" in earlier lectures for a maze.

k nodes as we traverse them and don't traverse previ- nodes.

to talk about *preorder* and *postorder*, as for trees.

```
Traverse(Graph G, Node v)
nmarked) {


e(v, w) ∈ G)
se(G, w);
```

```
void postorderTraverse(Graph G, Node v)
{
   if (v is unmarked) {
     mark(v);
     for (Edge(v, w) ∈ G)
       traverse(G, w);
     visit v;
   }
}
```


e Depth-First Traversal of a Graph (II)

n interested in traversing *all* nodes of a graph, not just able from one node.

epeat the procedure as long as there are unmarked

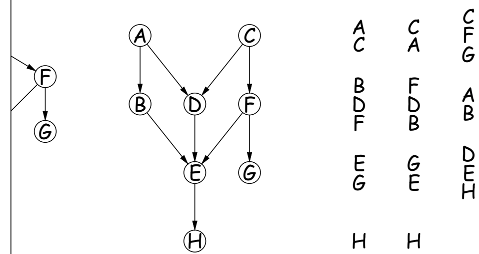
```
orderTraverse(Graph G) {
 ∈ nodes of G) {
eorderTraverse(G, v);


torderTraverse(Graph G) {
 ∈ nodes of G) {
storderTraverse(G, v);
```


Topological Sorting

n a DAG, find a linear order of nodes consistent with

r the nodes $v_0,\ v_1,\ \ldots$ such that v_k is never reachable
$> k$.

this. Also PERT charts.



eneral Graph Traversal Algorithm

```
OF_VERTICES fringe;

IAL_COLLECTION;
.isEmpty()) {
ringe.REMOVE_HIGHEST_PRIORITY_ITEM();

D(v)) {


dge(v,w) {
DS_PROCESSING(w))
 to fringe;
```

ECTION_OF_VERTICES, INITIAL_COLLECTION, etc.
es, expressions, or methods to different graph algo-

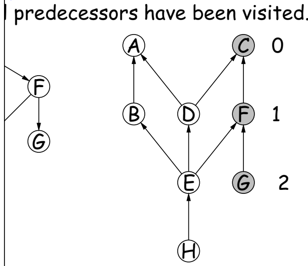
Sorting and Depth First Search

: Suppose we *reverse the links* on our graph.

ecursive DFS on the reverse graph, starting from node
le, we will find all nodes that must come *before* H.

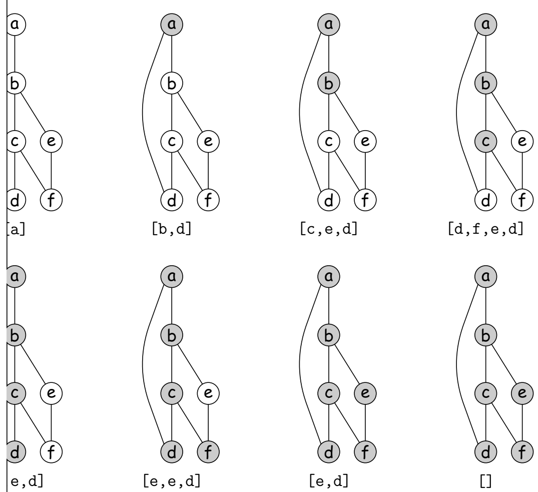
earch reaches a node in the reversed graph and there
ssors, we know that it is safe to put that node first.

postorder traversal of the reversed graph visits nodes
predecessors have been visited.



Numbers show post-order traversal order starting from G: everything that must come before G.

epth-First Traversal Illustrated



Example: Depth-First Traversal

every node reachable from v once, visiting nodes fur-
first.

```
x> fringe;

ck containing {v};
ge.isEmpty()) {
= fringe.pop();

ed(v)) {
;
v);
n edge(v,w) {
arked(w))
ge.push(w);
```


ortest Paths: Dijkstra's Algorithm

n a graph (directed or undirected) with non-negative
ompute shortest paths from given source node, s, to

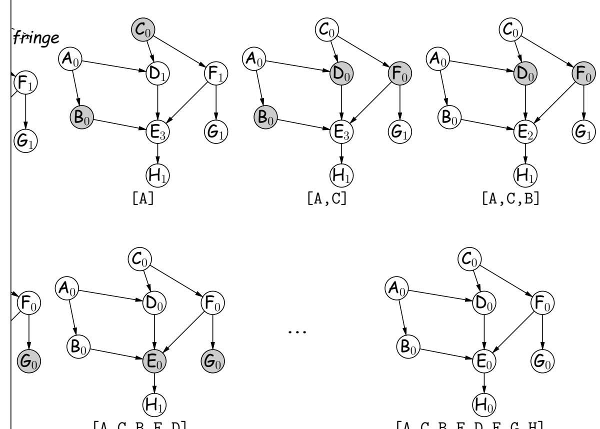
sum of weights along path is smallest.

le, keep estimated distance from s, ...

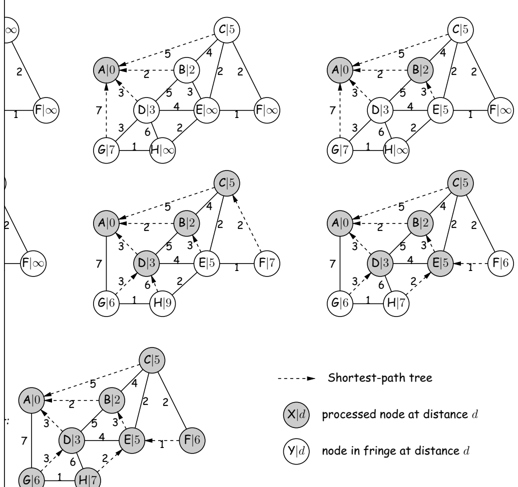
ceding node in shortest path from s.

```
Vertex> fringe;
v { v.dist() = ∞; v.back() = null; }

ty queue ordered by smallest .dist();
to fringe;
.isEmpty()) {
ringe.removeFirst();

e(v,w) {
() + weight(v,w) < w.dist())
() = v.dist() + weight(v,w); w.back() = v; }
```


Topological Sort in Action



Example


Shortest-path tree
processed node at distance d
node in fringe at distance d